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REINFORCED CONCRETE DESIGN

INTRODUCTION TO
REINFORCED CONCRETE DESIGN

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PREFACE

The purpose of the authors has been to present the fundamentals of reinforced concrete design as simply and completely as possible. The method of the transformed section, more familiar in European than in American texts, is used for the development of the theory as it is believed to be by far the clearest and most logical approach. It has the great advantage that instead of leaving the student with a mass of formulas which are often difficult to visualize, it impresses on his mind the basic concepts of the subject and frees him from dependence on texts and equations.

The usual formulas are presented as the basis of diagrams and tables, indispensable as time-savers in practice. The computations that illustrate the application of the theory are arranged systematically in the form usual in office work, with parallel comments in the text. This manner of presentation enables the reader to grasp the problem as a logical whole and gives the student a clear idea of the proper manner of presenting design calculations and results. It is hoped that this arrangement will free the instructor from the drudgery of detailed presentation of designs and enable him to devote the class hour to general discussion of the important features.

The computations cover a wide range of construction: retaining walls, slab and beam bridges, floors, columns and footings for buildings, and the hingeless arch. It is hoped that the discussion paralleling these examples will serve to make plain many matters not usually explained in text books.

Enough is included about the modern theories of concrete, formwork, drawing and detailing to give a good background of knowledge in matters where real proficiency can come only with experience.

By combining the viewpoint of the teacher and the practicing engineer the authors have endeavored to direct the work of the student to practical ends with no sacrifice of theory. While the book is primarily for the student of engineering, it is believed that it will prove useful to the practitioner by reason of its compact and complete presentation of specific problems with discussion

of the reasons for the various operations. Unless he is a specialist in this field he will find particularly useful the articles dealing with the analysis of rigid frames by the slope deflection method and those treating of arch design.

It is assumed that the reader is conversant with the principles of applied mechanics and understands the elements of design in steel and wood, such knowledge being almost a necessity as a preliminary to the study of reinforced concrete. However, for the sake of completeness and for an aid to rapid review, these fundamental principles are outlined briefly and simply in the text.

The authors wish to express their appreciation to the many friends who have aided their work. It has been their intention to give full credit in the text to all to whom they stand indebted for material and for ideas. With the passage of time a great deal of fundamental information has become common property and the sources are too often not recorded. It is hoped that no borrowings have been inadvertently and wrongfully assumed to belong in that class.

H. S.
W. W. C.

BOSTON, MASS., *August*, 1926.

A HINT TO THE STUDENT

The engineer thinks in pictures at all stages of the analysis and design of structures. It is a practice the student should carefully cultivate.

As a basis for work in reinforced concrete design there are two fundamental pictures to be fixed in mind: that of a free rigid body at rest acted on by a system of coplanar forces, conforming to the conditions $\Sigma X = 0$, $\Sigma Y = 0$ and $\Sigma M = 0$; that of the free body at rest formed by isolating a portion of a reinforced concrete member for purpose of analysis. Form the habit of expressing the problems of design simply and clearly in terms of these basic diagrams.

The process of studying this or any other technical book consists in committing to memory with great exactness a series of abstract laws and building up in mind a series of definite pictures of the force systems, the structures and so on, which furnish concrete expressions of these laws.

Read through any article of this text for the first time with the purpose of seeing the general outlines of the picture there presented. Do not try to fill in the details word by word at first. If the meaning of a sentence is not clear, pass on to the next. The explanation of the difficulty may be there. After the outline is seen, perhaps dimly, read through the article again with more attention to detail. Use scratch paper and pencil liberally. Make many sketches. No problem can be understood until all the elements are clearly placed. A book with empty margins has never been properly studied. Here should show neat notes and sketches in amplification and explanation. When a statement or equation is obscure determine what would be a correct statement and compare with that given.

This book should be studied in connection with the larger reference works in this field, such as

"Concrete Plain and Reinforced," Taylor, Thompson and Smulski.

"Principles of Reinforced Concrete Construction," Turneaure and Maurer.

"Reinforced Concrete and Masonry Structures," Hool and Kinne.[1]

The first two books contain much valuable information on the many tests that have been made to verify theoretical reasoning. These test data should be carefully studied.

Lastly — do not fail to read carefully Professor Geo. F. Swain's little book on "How to Study."[1]

H. S.

[1] McGraw-Hill Book Co.

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INTRODUCTION TO REINFORCED CONCRETE DESIGN

CHAPTER I

INTRODUCTION

1. The design of reinforced concrete structures involves two major problems: first, the determination of the type and general features of the structure required for the purpose in hand; second, the detailed proportioning of the various members, such as slabs, beams, columns and footings, which make up the whole. For example, the engineer who is planning a reinforced concrete factory must study the requirements of the manufacturing process to be housed therein and lay out a building whose arrangement as regards floor plan, column spacing, story height, lighting, elevator service and so on, makes possible the utmost efficiency of production. The factory must be fitted to the manufacturing process. The general layout being settled, the engineer next proportions the reinforced concrete skeleton and records this design in the structural drawings. It is evident that these two major problems are closely interrelated; that decisions as to details of arrangement must constantly be based upon knowledge of the possibilities, limitations and economical use of the structural materials. Furthermore, the designer is responsible not only for the adequacy and strength of the structure but also for its durability, economy and good appearance.

It is not within the scope of this book to consider the first of these major problems of design nor to do more than introduce the reader to the elements of the second. Experience in active practice is necessary to give the knowledge and judgment necessary for the successful planning of structures since that requires familiarity with construction methods and with the costs of labor, of material, and of finished structures in whole and in detail. This elementary text is limited to a brief outline of the methods

of making strong and durable concrete, and to a somewhat more thorough study of the application of the principles of theoretical mechanics to the proportioning of structural members, in conformity to the general usage of modern practice.

2. Joint Committee. In the United States modern practice has been standardized to conform fairly closely with the recommendations made by a "Joint Committee on Standard Specifications for Concrete and Reinforced Concrete," composed of representatives from five national engineering groups.[1] The first Joint Committee was organized in 1904 and ended its existence on the presentation of its third and final report in 1916. In 1919 the present Joint Committee was organized and has presented two reports, a "Tentative Specification" in 1921 which was in the nature of a progress report, and in 1924, one entitled "Standard Specifications for Concrete and Reinforced Concrete." It is understood that a revision of this last in the light of the current discussion may be expected in a year or two. None of these reports have official authority but stand simply as the recommendations of the individuals making up the committee. This latest document differs so much from the 1916 report and makes so many recommendations that are considered radical by conservative designers that its rules should not be followed as a whole, at present, without due study.

Portions of the 1916 and 1924 reports are reprinted in this text and in its appendix, and some consideration is given to certain of the questions raised by their varying requirements. Whenever the Joint Committee is mentioned in the pages that follow, reference is to the present committee and its current (1924) report, unless otherwise stated.

3. Concrete and Reinforced Concrete. Concrete is artificial stone made by cementing together into a solid mass a mixture of inert material such as sand and broken stone, gravel or other aggregate. The cementing material almost universally used for reinforced concrete work is Portland cement, the only exception of note being the alumina cements recently put on the market. Both these cements are extremely fine powders, made from definite but differing proportions of argillaceous and calcareous materials,

[1] The American Society of Civil Engineers, American Society for Testing Materials, American Railway Engineering Association, American Concrete Institute and the Portland Cement Association.

which, when wet with the proper amount of water, become chemically active and harden.

Concrete is easily given any desired shape by pouring the wet mixture of materials into suitable forms where the mass hardens. When the various ingredients are properly proportioned and mixed together the resulting product is hard, durable, strong in compression and shear, very weak in tension, brittle, and, when not reinforced, adapted for use only in relatively massive members subject to compression. In combination with steel rods properly placed to resist the tensile stresses, concrete may be used for all types of structural members. This reinforcement is made possible by the adhesion of the concrete to the steel which prevents slipping between the two materials, and forces the member to act as a unit as it deforms under load.

Experience has shown that, generally speaking, steel embedded sufficiently in concrete is fully protected against corrosion and against fire. The required depth of protective concrete covering varies with the shape of the piece, the aggregate and the intensity of the exposure.

4. Historical Note. There are in existence today examples of concrete construction dating back to Roman times and even earlier. The cement used by these early builders was not a true cement but a mixture of hydrated lime and volcanic ash, a product known today as slag or Puzzolan cement. The first true hydraulic cementing material, that is, one that hardens under water, was made about 1756 by the English engineer, John Smeaton, as a result of his searches for a proper binding material for building the third Eddystone Lighthouse. This product is known today as hydraulic lime. Another Englishman, James Parker, in 1796, made the first natural cement by calcining and grinding an argillaceous limestone. In 1824 Joseph Aspdin of Leeds patented Portland cement, a much superior product, though crude judged by the more refined products of today. The name Portland was chosen on account of the resemblance of the hardened cement to the building stone quarried on the Isle of Portland. The industry did not begin to develop actively either in England or on the continent until about the middle of the last century.

In the United States natural cement was first made in 1818 by Canvas White and Portland cement in 1872 by David O. Saylor. The manufacture of Portland cement lagged behind

that of its lower priced rival until the modern method of manufacture (burning the cement clinker in rotary kilns) was introduced in 1892. Quickly the production of Portland cement mounted until now it ranks as one of the ten leading industries, an increase that tells eloquently of the increase in reinforced concrete construction.

In 1908, Bied in France and Spackman in the United States took out patents covering a high-alumina cement that so far surpasses Portland cement in several important respects that its advent may mark an advance comparable to that made by the introduction of Portland cement. The development of the new product took place however in France where it has been manufactured in increasingly large quantities since the war. Since 1924 it has been made in this country by a single company.[1]

The beginnings of reinforced concrete go back to 1850 when the Frenchman, Lambot, constructed a small boat of that material. In England, W. B. Wilkinson patented a true reinforced concrete floor slab in 1854. Seven years later François Coignet published his statement of the principles of the new construction. In the same year, 1861, Joseph Monier, a Parisian gardener, used metal frames as reinforcement for garden tubs and pots, and before 1870 had taken out a series of patents. There was comparatively little construction however until the German engineers, Wayss and Bauschinger, investigated and reported on the Monier system in 1887. From that time the use of reinforced concrete spread rapidly, the greatest developments in theory and practice being made by Austrian engineers. Melan's system, employing structural steel shapes as reinforcement, was developed in the early 90's, at the same time as that of Hennebique, whose methods, of all the pioneers, probably most nearly resemble those of today.

In the United States the pioneer was W. E. Ward, who built a reinforced concrete house in Port Chester, New York, in 1872. Thaddeus Hyatt published the results of tests on various types of beams in 1877. About the same time E. L. Ransome and his co-workers were beginning their work on the Pacific coast, erecting several notable buildings in California in the two following decades. The Melan system was introduced into this country from Europe in 1894. Edwin Thacher began his distinguished career as a bridge builder with a Melan type arch in 1896.

[1] The Atlas Lumnite Cement Co., New York City.

During all this period structures of reinforced concrete had been modelled largely on those of the more familiar wood and steel. In 1906 Mr. C. A. P. Turner of Minneapolis devised the girderless or flat slab type of floor, the Mushroom floor, as he termed it. This innovation marked a great step forward in utilizing the materials in the most advantageous and economical manner, recognizing to the full the monolithic character of the structure. At this date the extensive use of reinforced concrete was in full swing, a use that has increased tremendously and still increases from year to year.

In so new and rapidly developing a field as that of reinforced concrete it was inevitable that construction should often be in advance of theory. This was notably the case with the flat slab floor which is still designed by methods largely "rule of thumb." For the most part however the fundamental principles may be considered as definitely known and agreed upon, having proved themselves by a long series of satisfactory structures which in many cases have endured extremely large overloading with few signs of distress. However, there are still many details to be determined and the status of the theory is far less clearly settled than is that of steel design.

CHAPTER II

CONCRETE MATERIALS

5. Concrete is "a compound of gravel, broken rock or other aggregate, bound together by means of hydraulic cement, coal tar, asphaltum, or other cementing materials. Generally when a qualifying term is not used Portland cement concrete is understood."[1] In order to secure satisfactory concrete it is usually necessary to separate the aggregates into two portions by size; hence the Joint Committee definition: "a mixture of Portland cement, fine aggregate, coarse aggregate and water." The 1924 report deals only with Portland cement concrete.

6. Reinforced Concrete. Plain concrete, being brittle and weak in tension, is suitable only for relatively massive members subject to compression. Combination structural members made of concrete reinforced by steel bars, placed so as to carry the tensile stresses, are sturdy and reliable. The name reinforced concrete cannot be applied to a combination piece of steel and concrete unless both materials assist in carrying the load and the whole acts as a unit. Of the three fundamental types of structural members, beams, columns, and ties, only beams and columns can ever be said to be of reinforced concrete.

The advantages of reinforced concrete as a structural material are evident, each element making up for the deficiencies of the other, the steel supplying the tensile strength and toughness and the concrete supplying the compressive strength besides protecting the steel from corrosion and from fire.

7. Portland Cement. The usual description of Portland cement is "the product obtained by finely pulverizing clinker produced by calcining to incipient fusion an intimate and properly proportioned mixture of argillaceous and calcareous materials with no additions subsequent to calcination except water and calcined or uncalcined gypsum." It differs from natural cement ("the finely pulverized product resulting from the calcination of an argillaceous limestone at a temperature sufficient only to drive

[1] Definition adopted in 1923 by the American Concrete Institute.

off the carbonic acid gas ") in being slower setting, much stronger, more uniform and reliable. Portland cement is to a very considerable degree a standardized article of commerce and practically all brands can be depended upon to satisfy the standard tests.[1] It is customary, however, on all work of importance to submit the cement to test.

8. Alumina Cements. The new high-alumina cements are made by reducing to a powder a fused mixture of bauxite (aluminum ore) and limestone. Concrete made with these cements sets, that is, changes from a plastic to a stiff state, in about the same time as Portland cement concrete and then proceeds to harden and gain in strength very rapidly, attaining in 24 hours a compressive strength equal to or greater than that gained by Portland cement concrete in 28 days. This 24-hour strength is, approximately, 75 per cent of that reached at 28 days. This rapid gain in strength is accompanied by a considerable development of heat, sufficient to protect the mass from freezing until high strength is attained under weather conditions which would entirely prevent Portland cement from setting or hardening. Another important advantage, that which led to the development of this cement in France, is that concrete made with alumina cement apparently resists the action of sea water and alkalis which often disintegrate Portland cement concrete.

It is probable that the limited supplies of raw material suitable for making high alumina cements will always keep their cost in North America far above that of Portland cement. Consequently, they will be used only for a limited class of work where their high strength and quick hardening justify the increased expenditure. At present there is no reason to believe they will ever replace Portland cement for ordinary construction.

9. Fine Aggregate. "Fine aggregate shall consist of sand, or other approved inert materials with similar characteristics, or a combination thereof, having clean, hard, strong, durable uncoated grains and free from injurious amounts of dust, lumps, soft or flaky particles, shale, alkali, organic matter, loam or other

[1] "Standard Specifications and Tests for Portland Cement," (Serial Designation C9–21) issued by the American Society for Testing Materials and adopted as standard by the United States Government, the American Engineering Standards Committee, etc. Reprinted in Appendix II of the 1924 Joint Committee Report.

deleterious substances." (Joint Committee.) Generally fine aggregate is considered to consist of particles smaller than one-quarter of an inch in diameter.

The size and grading of an aggregate are studied by means of standard sieves,[1] made of wire cloth, the smaller sizes of which (No. 4 and finer) are designated by the number of openings per linear inch and the larger sizes by dimension of openings. The Joint Committee recommends that not less than 85 per cent of the fine aggregate shall pass the No. 4 sieve (size of opening, 0.187 in.), and not more than 30 per cent nor less than 10 per cent the No. 50 sieve. From this specification it is plain that sand made up of grains all of one size is not satisfactory. This is because a graded sand will compact more than a uniform one, the smaller grains fitting in between the larger, thereby giving a denser and stronger mortar.

The fine aggregate may be tested for the presence of fine silt, loam, clay and other water-soluble material by the decantation test[2] and for organic impurities by the colorimetric test.[3] In the decantation test the fine aggregate is placed in a pan and sufficient water is added to cover the sample. The pan and its contents are agitated vigorously for 15 seconds, and then after waiting 15 seconds to allow the heavier suspended particles to settle, the water is poured off. This operation is repeated until the wash water is clear. The Joint Committee limits the loss in weight by this test to 3 per cent in general. The colorimetric test consists in placing a sample of the material in a bottle partly filled with a sodium hydroxide solution which turns brown if organic matter is present, the depth of shade measuring the amount of the impurity. The limit set as a "standard color" is that produced by tannic acid when present in the proportion of one part in 4000.

The most useful tests are those of the strength of mortar (de-

[1] See "Standard Method of Test for Sieve Analysis of Aggregates for Concrete" (Serial Designation C41–24) of the American Society for Testing Materials, reprinted in Appendix VIII of the 1924 Joint Committee Report.

[2] "Tentative Method of Decantation Test for Sand and other Fine Aggregates" (Serial Designation D136–22T) of the American Society for Testing Materials, reprinted in Appendix IX of the 1924 Joint Committee Report.

[3] "Standard Method of Test for Organic Impurities in Sand for Concrete" (Serial Designation C40–22) of the A.S.T.M., reprinted as Appendix X of the 1924 Joint Committee Report.

fined as a mixture of cement, fine aggregate and water) or of concrete made with the given fine aggregate. The Joint Committee specifies that "fine aggregate shall be of such quality that mortar briquettes, cylinders or prisms, consisting of one part by weight of Portland cement and three parts by weight of fine aggregate · · · will show a tensile or compressive strength at ages of 7 and 28 days" preferably "not less than 100 per cent" of that of 1 : 3 standard Ottawa sand mortar of the same plasticity made with the same cement.[1]

It is still common in some localities to specify that the sand grains shall be sharp and to test the cleanliness of the sand by rubbing a little of it in the palm. Sharpness of grain, however, is not a necessary characteristic at all, nor are the feeling and appearance of sand sufficient guides to its quality. Unless it is known that any given sand has been used successfully in concrete work, it should be carefully tested as here described.

10. Coarse Aggregate. "Coarse aggregates shall consist of crushed stone, gravel or other approved inert materials with similar characteristics, or combinations thereof, having clean, hard, strong, durable, uncoated particles, free from injurious amounts of soft, friable, thin elongated or laminated pieces, alkali, organic or other deleterious matter." (Joint Committee.) "Coarse aggregate shall range in size from fine to coarse" in general within the limits indicated by the table on the following page.

The maximum size of coarse aggregate is rarely over 3 inches, $1\frac{1}{2}$ in. or 1 in. being the usual limit set for reinforced concrete work. In massive construction larger stones are often placed in the mass by hand or derrick, care being taken that these larger pieces, or "plums," are not too close together nor too near the face of the concrete. The Joint Committee uses the term rubble concrete for that in which are embedded stones larger than three inches and less than 100 pounds in weight, and cyclopean concrete for that with stones weighing more than 100 pounds.

[1] For testing methods, see the American Society for Testing Materials Specification C9–21 referred to in footnote, page 7, and also "Tentative Methods of Making Compression Tests of Concrete" (Serial Designation C39–21T) of the A.S.T.M., reprinted as Appendix XII of the 1924 Joint Committee Report. The standard Ottawa sand is a natural sand from Ottawa, Illinois, screened to pass a No. 20 sieve and retained on a No. 30 sieve. It is used as a standard on account of its uniformity.

Nominal maximum size of aggregate in inches	Percentage by weight passing through standard sieves with square openings						Percentage passing not more than	
	3 in.	2 in.	1½ in.	1 in.	¾ in.	½ in.	No. 4 sieve	No. 8 sieve
3	95	....	40–75	......	....		10	5
2	..	95	...	40–75	....		10	5
1½	...	....	95	...	40–75	.	10	5
1	..	...	......	95	..		10	5
¾	..	...	......	..	95		10	5
½	....	....	......	....	...	95	10	5

11. Water. "Water for concrete shall be clean and free from injurious amounts of oil, acid, alkali, organic matter or other deleterious substance." (Joint Committee.)

12. Reinforcement. The reinforcement for concrete usually consists of steel rods, round and square, sometimes made up in the form of wire fabric for use in slabs. For columns and arches the reinforcement often consists of built-up members of structural steel shapes. The following standard sizes of bars are in use and none others should ever be called for:

Size of bar in inches		Area in square inches
Round	Square	
¼	....	0.049
⅜	....	0.110
½	....	0.196
....	½	0.250
⅝	....	0 306
¾	....	0.441
⅞	....	0.601
1	....	0.785
....	1	1 000
....	1⅛	1.265
....	1¼	1.562

In European practice plain bars are commonly used. In the United States preference is given to deformed bars that are rolled with small projections to engage the concrete and prevent slipping between the two materials. Many styles of such rods are made. Square twisted bars are also used.

The Joint Committee specifications provide for three grades of bars rolled from billet steel, structural, intermediate and hard, and also for bars rolled from steel rails, giving preference to intermediate grade billet steel.[1]

[1] The Joint Committee specifies that steel shall conform to the requirements of the American Society for Testing Materials as follows: "Standard Specifications for Billet-Steel Concrete Reinforcement Bars" (Serial Designation A15–14), "Standard Specifications for Rail-Steel Concrete Reinforcement Bars" (A16–14), "Standard Specifications for Structural Steel for Bridges" (A7–24), "Standard Specifications for Structural Steel for Buildings" (A9–24), "Tentative Specifications for Cold-drawn Steel Wire for Concrete Reinforcement" (A82–21T). Cast iron used in composite columns shall conform to "Standard Specifications for Cast Iron Pipe and Special Castings" (A44–04). These several specifications are reprinted as appendices to the Joint Committee report 1924.

CHAPTER III

PROPORTIONING CONCRETE

13. All reinforced concrete design proceeds on the assumption that the concrete is of definite strength and uniform quality. Until recently the realization of this assumption has been a difficult and costly matter of laboratory study and unremitting expert supervision, something warranted only on important projects. Most of the concrete made has been, and still is, very variable in quality, and this variability has made it necessary to assume low strength on which to base design stresses. Demonstration of the practicability of attaining uniformity caused the 1924 Joint Committee to specify modern methods of control of concrete making and also higher working stresses. This results in a lowering of the previously uneconomical high factor of safety which had been indicated as advisable because of more or less careless and ineffective construction methods. Obviously it is of the utmost importance that the concrete measure up to the standards set by the design specifications. A vast amount of research is being carried on and marked advance has been made towards mastering the art and science of making good concrete, progress so definite that it is now possible to study the available aggregates and proportion, or design, the mix with considerable accuracy to attain a certain specified strength.

A thorough study of concrete is beyond the scope of this text. A brief discussion is given to enable the student to understand the main principles of modern concrete making, with references to guide him to the sources of information should he wish to gain enough understanding of the subject to enable him to apply the principles successfully.

14. Theories of Proportioning. Until within a few years the only accepted principles governing the proportioning of concrete were three:

That for any given combination of aggregates, strength, impermeability and durability increase with increased proportions of cement, the consistency remaining the same;

That for any given aggregates, the proportion of cement being fixed and consistency being constant, maximum strength, impermeability and durability are obtained with that combination of ingredients giving the densest mixture;

That the quality of the concrete is best when mixed with enough water to give a plastic or mushy consistency, an excess of water resulting in a weak concrete.

Accordingly four general methods, with variations, were developed, each aiming to determine the proportions of the several ingredients that result in a mass containing the maximum amount of solid matter per unit volume:

Method of Void Determinations;
Method of Arbitrary Proportions;
Method of Mechanical Analysis;
Method of Trial Mixes.

Many investigators today doubt the theory of maximum density, notably Professor Duff A. Abrams [1] who in 1918 advanced the Water-Ratio theory, urging that the strength of concrete of workable consistency is fixed by the amount of water used per bag of cement. The recommendations of the Joint Committee in 1916 were based upon the theory of maximum density; those of 1924 are that " the engineer shall determine by tests of the available aggregates in advance of use the proportions necessary to produce concrete of the desired strength. Where this is impracticable " the engineer is given as a guide a set of tables of required proportions of variously sized aggregates for various required strengths. These tables were prepared by Professor Abrams in accordance with his theory. Presumably the methods of test intended are also those of Professor Abrams. Several million cubic yards of concrete have been proportioned in accordance with the water-ratio theory during the past three years to the full satisfaction of the engineers, contractors and owners involved.

As has been indicated the water-ratio theory is rapidly winning wide adherence in this country. Recent simplifications in its application make it certain that there will be a great increase in its use. The days of carelessly made concrete are ending, for

[1] In charge of the Structural Materials Research Laboratory, Lewis Institute, Chicago. Professor Abrams' researches in concrete have been carried out through the coöperation of Lewis Institute and the Portland Cement Association.

concrete makers are learning how to make good concrete with economy.

Another theory worthy of note is that of Mr. L. N. Edwards who proportions the cement to the surface area of the aggregates. While this theory has not had wide application it seems more than probable that it will play its part in the final solution of the problem of proportioning.

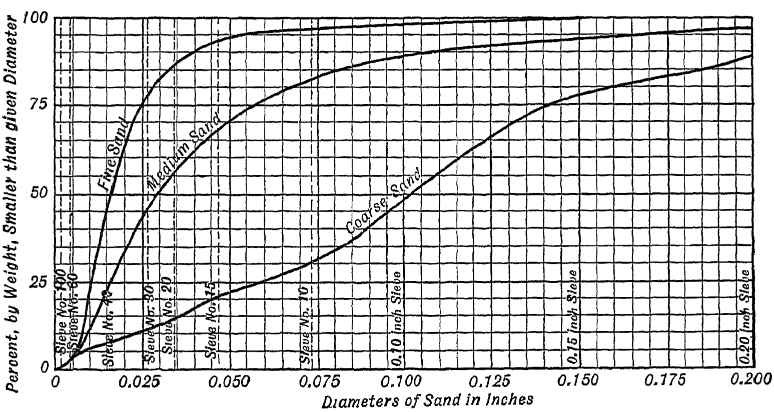
It cannot be urged too strongly that a brief outline of methods such as that given in this text gives a deceptive air of simplicity to a complicated problem. Extended reading such as is suggested by the references at the end of this chapter, and actual work with concrete in the laboratory are essential to any real comprehension of the subject.

15. Proportioning by Void Determination. For maximum density the interstices or voids between the stones forming the coarse aggregate should be completely filled by the fine aggregate, and all remaining voids by the cement paste, which also, if it is to perform its function as a glue binding the whole mass together, must coat completely every particle. Practically never, however, is the cement content determined by study of the voids in either the fine aggregate or in the combined aggregate. Instead the cement is made to bear that ratio to the total aggregate which experience has proved to be sufficient for ensuring the desired strength. The strongest possible concrete with the given material and assumed cement-aggregate ratio is then assured (assuming the theory and method to be correct) by taking slightly more than enough fine aggregate (5 per cent to 10 per cent excess) to fill the voids in the coarse aggregate, this excess being necessary because the measured voids are increased by the wedging apart of the stones by the mortar.

16. Arbitrary Proportions. Measurements show that much of the aggregate in everyday use contains approximately 50 per cent of voids. This early suggested the simple 1 to 2 ratio of fine and coarse aggregates which is so commonly used. The cement is combined in the proportions which tests have seemed to show necessary for obtaining the required strength, for example, 1 part cement to $4\frac{1}{2}$ parts of fine and coarse aggregates, measured separately, for a strong concrete for columns, a mix usually expressed as $1\text{–}1\frac{1}{2}\text{–}3$. Most of the concrete in this country has been mixed in such proportions as 1–2–4, 1–3–6, etc., as fixed by ordinary

practice and usage. Where good judgment has been shown in choice of aggregates and in workmanship the result has been good sound concrete. To speak of these proportions as Arbitrary is something of a misnomer. It would be more accurate to speak of proportioning by the Assumption of Average Void Conditions. Obviously when the aggregates vary much from the assumed average, as when they are poorly screened and there is considerable overlapping of sizes, the resulting mixes will be unsatisfactory. Where a job is large enough to support laboratory tests more careful proportioning will unquestionably result in stronger and more economical concrete. Furthermore, aggregates of unknown quality not only should be tested before being used as described above (Arts. 9–10) but also they should be subjected to a screen test before deciding on proportions for work of any importance.

17. Mechanical Analysis. In 1907 William B. Fuller and Sanford E. Thompson made public [1] a method of combining various aggregates to give the densest mixture by means of sieve analyses of the materials. The grading of any aggregate may be recorded graphically by a curve as in Fig. 1, the abscissa of any



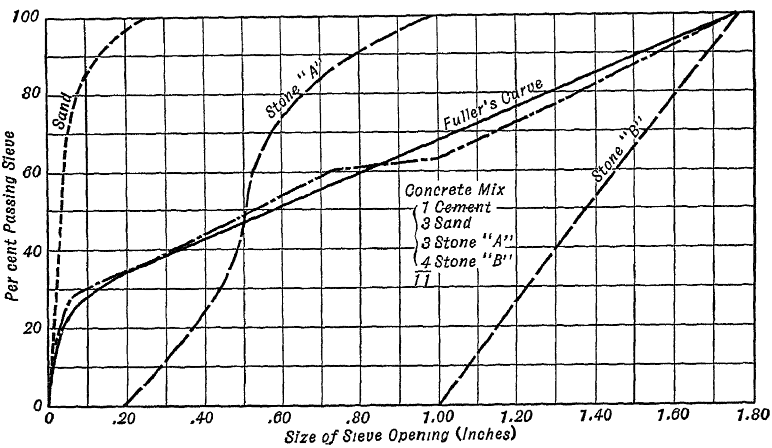
From *Concrete Plain and Reinforced.* 3rd Edition, Taylor & Thompson

Fig. 1

point indicating the size and the ordinate indicating the percentage of the material finer than that size. Messrs. Fuller and

[1] "The Laws of Proportioning Concrete," Transactions Am. Soc. C. E., Volume LIX, page 67, 1907. A full description of this method is given in "Concrete, Plain and Reinforced," 3rd Edition, Taylor & Thompson.

Thompson showed that when any concrete aggregates are combined so that the resulting mixture is the densest possible for that material, the grading curve for that mixture is, very closely, the combination of a straight line and an ellipse. Furthermore they showed "that a curve of substantially the same form would fit different materials" and gave data for constructing this maximum density curve for a variety of aggregates: crushed rock, gravel and sand. Knowing thus the ideal curve, any given aggregates may be analyzed, their grading curves plotted and, by cut and try methods, the proportions determined that will result in the curve most closely approximating the ideal. The accuracy with which this may be done depends upon the number of separate sizes into which the aggregates are divided. In this method the cement sometimes is considered as part of the sand; sometimes its curve is plotted and used in combination with those of the coarser materials. It is more important that the actual grading curve fit the ideal in the sand-cement portion than in that of the coarse aggregate. For best results the actual curve should intersect the theoretical approximately on the 40 per cent line.



From Mills' *Materials of Construction*

FIG. 2

Example 1. Reference to Fig. 2. Here the dot and dash line represents the combination of the materials in the proportions stated, the measurements being by weight. In this case it was decided to make the actual curve coincide with the ideal at the 0.25 in. opening, about 37 per cent. So all of stone B plus about $\frac{84}{100}$ of stone A makes up that part of the combination coarser than

this size, about 63 per cent of the total. The theoretical curve calls for 32 per cent of the mixture to be larger than 1 in. but this proportion makes the curve between 0.25 in. and 1 in. lie well above Fuller's curve. So a larger proportion of stone B, 37 per cent, was taken. Then

$$0.94\,A + B = 0.94\,A + 37 = 63 \text{ per cent}$$
$$A = 28 \text{ per cent}$$

and

$$\text{cement} + \text{sand} = 100 - (A + B) = 35 \text{ per cent.}$$

It was assumed that 1 part of cement to 10 parts of aggregate by weight, measured separately, would give the requisite strength. So

$$\text{cement} = \tfrac{1}{11} \text{ of total} = 9 \text{ per cent}$$

and

$$\text{sand} = 35 - 9 = 26 \text{ per cent}$$

giving for final proportions by weight 9–26–28–37 or closely 1–3–3–4. The following weights were assumed:

cement	94 lbs./cu. ft.	Stone A	105 lbs./cu. ft.
sand	108 lbs./cu. ft.	Stone B	104 lbs./cu. ft.

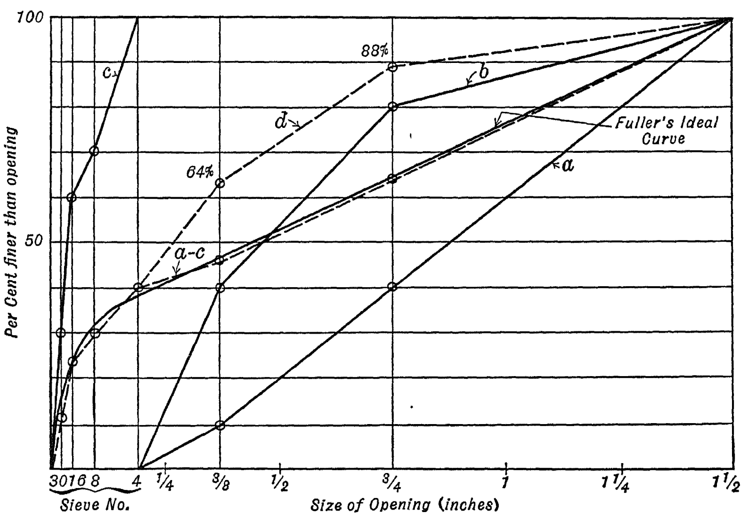
The proportions by volume are:

$$1 : 3 \times \frac{94}{108} : 3 \times \frac{94}{105} : 4 \times \frac{94}{104}$$

or

$$1\text{–}2.61\text{–}2.68\text{–}3.62.$$

Problem 1. (*a*) Plot the grading curve for a combination of aggregates *a* and *c* in proportions 1–3–6. (*b*) In what proportions must these two mate-



rials be combined so that the grading curve shall cut the Ideal Curve where it crosses the No. 4 sieve size? (*c*) What will be the proportions of each material for a 1–6 mix, the fine and coarse aggregates being measured separately?

Ans. (*a*) See plot. (*b*) 38 per cent of the total must be fine material. (*c*) 1–1.7–4.3.

Problem 2. Curve *d* gives the grading of a mixture of sand *c* and a coarse aggregate, all particles of which are coarser than the No. 4 sieve. Plot the grading curve for that coarse aggregate.

Ans. Curve *b* on plot.

18. Trial Mixes. The 1916 Joint Committee recommended that " the proportions should be carefully determined by density experiments, and the grading of the fine and coarse aggregates should be uniformly maintained, or the proportions changed to meet the varying sizes. For reinforced concrete construction, one part of cement to a total of six parts of fine and coarse aggregates, measured separately, should generally be used. For columns richer mixes are preferable. In massive masonry or rubble concrete a mixture of 1 : 9 or even 1 : 12 may be used. These proportions should be determined by the strength or other qualities required in the construction at the critical period of use."

Density experiments are easily made by determining the heaviest of a series of trial mixes of equal volume, made with varying proportions of the ingredients, the cement ratio alone being fixed. It is important that all of these trial batches be of the same working consistency and compacted in the container in a uniform manner. Sometimes dry aggregates alone are combined and studied in this way.

This is a very useful method of proportioning and one especially easy of application in checking the daily work in the field. It is generally used as a check on the method of Mechanical Analysis.

The 1916 Joint Committee Report gives the following table of the ultimate compressive strength that may be expected from different mixtures:

Compressive Strengths of Different Mixtures of Concrete

In lbs. per sq. in. at an age of 28 days, testing cylinders 8 in. in diameter and 16 in. long, made, stored, and tested under laboratory conditions.

Aggregate	1 : 3*	1 : 4½*	1 : 6*	1 : 7½*	1 : 9*
Granite, trap rock...	3300	2800	2200	1800	1400
Gravel, hard limestone and hard sandstone. .	3000	2500	2000	1600	1300
Soft limestone and sandstone.	2200	1800	1500	1200	1000
Cinders.	800	700	600	500	400

* Combined volume fine and coarse aggregate measured separately.

19. The Water-Cement Ratio Theory. Professor Abrams states that "with given concrete materials and conditions of test the quantity of mixing water used determines the strength of the concrete, so long as the mix is of a workable plasticity."[1] The equation expressing this relation he found to be for average conditions:

$$S = \frac{14{,}000}{7^x} \quad \text{(Curve } A \text{ in Fig. 3)}^2$$

where

S = compressive strength of concrete at 28 days

$$x = \text{water ratio} = \frac{\text{volume of mixing water}}{\text{volume of cement}} \quad \text{(an exponent here).}$$

If rigid control is lacking the concrete may be expected to be weaker as expressed by

$$S = \frac{14{,}000}{9^x} \quad \text{(Curve } B \text{ in Fig. 3).}$$

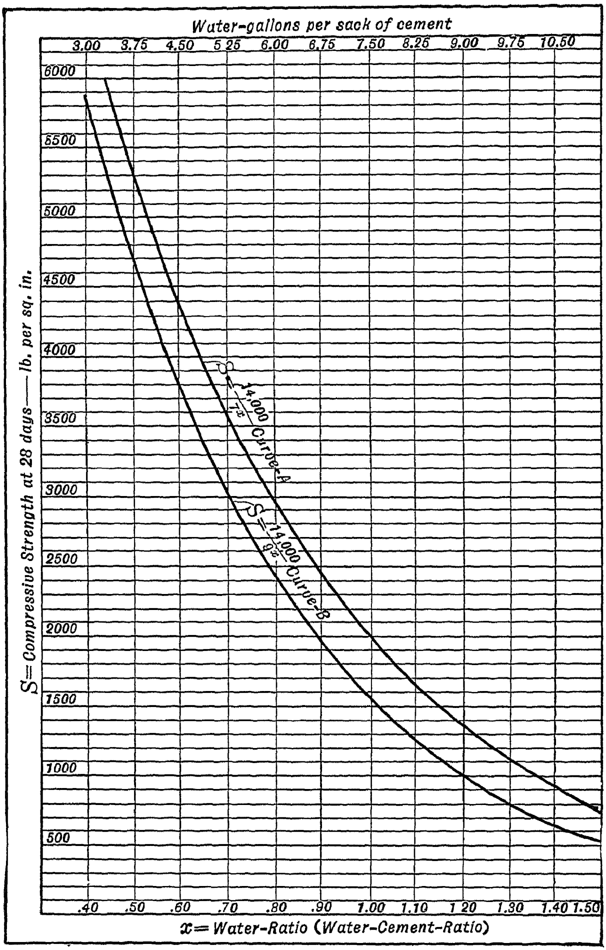
It must be kept in mind that these equations hold only for a workable mix. The limitations in the application of the water-ratio theory indicated in the following quotation are largely met by the requirement that the concrete must be of workable consistency. "So far as these tests indicate, it may be concluded that the expression 14,000 divided by 7 raised to a power equal to the water-cement ratio, is a fair measure of the strength of concrete, provided that at least one-third of the aggregate is sand (that is smaller than a No. 4 sieve) and that the quantity of coarse aggregate of any one size is not less than one-third as great as that of the next larger size. Other tests not included in the investigation indicate also that the sand should not be more than half the total aggregate in order to meet this criterion." (From Technical News Bulletin, U. S. Bureau of Standards, quoted in *Concrete*, June, 1925.)

Two and a half gallons of water are sufficient for the hydration of a sack of cement, usually taken to be 1 cubic foot in volume, a

[1] Design of Concrete Mixtures, Bulletin 1, Structural Materials Research Laboratory, Lewis Institute, Chicago.

[2] The diagrams and tables in this article and the next are from the pamphlet "Design and Control of Concrete Mixtures," published by the Portland Cement Association.

water ratio of one-third. Inspection of Fig. 3 reveals how quickly strength diminishes as the quantity of mixing water is increased. This long recognized fact first found quantitative expression in this work of Professor Abrams.



From *Design and Control of Concrete Mixtures.* Portland Cement Association

FIG. 3

The size and grading of the aggregates enter into the problem through the relation between these factors and the amount of water required to produce a workable mix. For the study of

aggregates Professor Abrams developed a measure of their size and grading which he named the Fineness Modulus. This modulus (or measure of fineness) is $\frac{1}{100}$ of the sum of the percentages of the material coarser than the opening of each of the following standard series of sieves: 100, 50, 30, 16, 8, 4, $\frac{3}{8}$, $\frac{3}{4}$, $1\frac{1}{2}$. Each sieve in this series has an opening twice the width of the preceding one. The method of calculating the fineness modulus is illustrated by the following table:

TYPICAL SIEVE ANALYSES OF AGGREGATES

Aggregate	Per cent coarser than each sieve									Fineness modulus	Range in size
	100	50	30	16	8	4	$\frac{3}{8}$	$\frac{3}{4}$	$1\frac{1}{2}$		
Sand	100	90	70	55	35	20	0	0	0	3 70	0–$\frac{3}{8}$ in.
Sand	100	85	65	40	20	0	0	0	0	3 10	0–4
Sand	95	75	60	30	0	0	0	0	0	2 60	0–8
Screenings .	85	80	75	35	25	0	0	0	0	3 00	0–4
Stone	100	100	100	100	100	100	100	40	0	7 40	$\frac{3}{8}$–$1\frac{1}{2}$
Pebbles	100	100	100	100	100	100	70	30	0	7 00	4–$1\frac{1}{2}$
Pebbles .	100	100	100	100	100	100	45	15	0	6 60	4–1*

* 1 and 2-inch sieves are used in determining size of aggregate but not used in calculating fineness modulus.

The fineness modulus increases with the coarseness of the aggregate and the same fineness modulus may be secured from an infinite number of different aggregates. The interesting fact here is that different aggregates with the same fineness modulus require equal amounts of mixing water for equal plasticity, and when so used produce concretes of equal strength. In practice, then, the problem is to find the most economical aggregate combination to use with the given water-cement ratio for the required strength.

Proportioning by the water-cement ratio theory is easy for one who has had sufficient experience in actual concrete making to be able to judge its workability and know what changes to make in the aggregate combination to improve quality. A good concrete foreman with no knowledge at all of the intricacies of fineness modulus and aggregate grading can be given all necessary instructions in a very few words. All he need be told is this: Use 7.5 gallons (or whatever the desired quantity may be, making

allowance for moisture contained in the aggregate) of water for each bag of cement and no more; make the aggregate proportions such that every batch is workable and will give a good dense concrete with a good surface without honeycombing (large voids). While this is sufficient for small jobs, larger operations, where an inspector is constantly employed, require more careful study for economy. Some who have done much work by this method state that the best aggregate combination is that which is the most dense when mixed dry, something very easily determined in the field by finding which one of several aggregate combinations gives the greatest weight in any given container, when compacted in a standard manner. More elaborate studies are desirable when the yardage is great enough to make it profitable, and the methods for this are treated in the next article.

19a. Proportioning by the Water-Cement Ratio Theory. Professor Abrams has systematized the data he has obtained by his study of aggregate in such a way that they can be used as a guide for proportioning. The Portland Cement Association, which joins with Lewis Institute in supporting Professor Abrams' research, is active in educating engineers in the use of this method, and consequently there is a rapidly increasing body of experience by which it may be judged. The material in this article is largely a rephrasing of their pamphlet "Design and Control of Concrete Mixtures" and is illustrated by cuts from that publication.

The recent work of John G. Ahlers and of Messrs. MacMillan & Walker[1] makes it plain that the somewhat complicated computations that follow are by no means an essential part of the water-cement ratio method. The engineer's concern is with the strength, durability and good appearance of the concrete. Accordingly his specifications for the guidance of the contractor need give accurate directions only for the securing of the water-cement ratio required for the desired strength, and for the proper workability, indicated largely by the slump test. The contractor desires to use the most economical combination of aggregates that produces concrete which can be easily placed, that is, workable concrete. The best proportions may be determined by studies such as are outlined in this article; nearly the same result may be reached by determining the combination of dry aggregates which is the densest, or by a series of trial batches.

[1] "New Experiences in Concrete Control," John G. Ahlers, Proceedings, American Concrete Institute, 1926. In the same volume F. R. MacMillan, a member of the Joint Committee, and Stanton Walker describe the application of this method to the making of the concrete for the new office building of the Portland Cement Association, Chicago. Both articles make practically the same simplification of the Abrams theory. Essentially the same method was proposed by the Committee on Field Methods of the American Concrete Institute in the 1924 Proceedings. Mr. MacMillan was a member of this committee.

MAXIMUM PERMISSIBLE VALUES OF FINENESS MODULUS OF AGGREGATES*

Real Mix	Size of Aggregates								
(Cement: Aggregate)	0–8	0–4	0–⅜	0–¾	0–1	0–1½	0–2	0–3	0–6
1 : 9	2.45	3.05	3.85	4.65	5.00	5.40	5.80	6.25	7.05
1 : 7	2.55	3.20	3.95	4.75	5.15	5.55	5.95	6.40	7.20
1 : 6	2.65	3.30	4.05	4.85	5.25	5.65	6.05	6.50	7.30
1 : 5	2.75	3.45	4.20	5.00	5.40	5.80	6.20	6.60	7.45
1 : 4	2.90	3.60	4.40	5.20	5.60	6.00	6.40	6.85	7.65
1 : 3	3.10	3.90	4.70	5.50	5.90	6.30	6.70	7.15	8.00
1 : 2	3.40	4.20	5.05	5.90	6.30	6.70	7.10	7.55	8.40
1 : 1	3.80	4.75	5.60	6.50	6.90	7.35	7.75	8.20	9.10

* For mixes other than those given in the above table, use values given for the next leaner mix. For maximum sizes of aggregates other than those given under "Size of Aggregates," use the values given under the next smaller size.

The table is based on the requirements for sand-and-pebble (gravel) aggregate composed of approximately spherical particles, in ordinary uses of concrete in reinforced concrete structures. For other materials and in other classes of work the maximum permissible value of the fineness modulus for an aggregate of a given size is subject to the following corrections:

1. For crushed stone or slag, reduce values given in table by 0.25.
2. For pebbles consisting of flat particles, reduce the values given by 0.25.
3. If stone screenings are used as the fine aggregate, reduce the values given by 0.25.
4. If top course of concrete roads is finished by hand, reduce the values given by 0.25.
5. If finishing of road is done by mechanical means no reduction should be made.
6. In work of massive proportions, such as where the smallest dimension is larger than 10 times the maximum size of the coarse aggregate, additions may be made to values given in the table as follows. for ¾-in. aggregate 0.10; for 1½-in. aggregate 0.20; for 3-in. aggregate 0.30; for 6-in. aggregate 0.40.

Sand having a fineness modulus lower than 1.50 is undesirable as a fine aggregate in ordinary concrete mixes. Natural sands of such fineness are seldom found.

Sand or screenings used for fine aggregate in concrete must not have a higher fineness modulus than that permitted for mortar concretes of the same mix.

Crushed stone mixed with both finer sand and coarser pebbles requires no reduction in fineness modulus provided the quantity of crushed stone is less than 30 per cent of the total volume of the aggregate.

The relation between fineness modulus and strength is shown by Fig. 4. Here the increase in the fineness modulus is accomplished by using more of the coarse material. Coarse aggregates require less water per sack of cement for a given consistency than fine aggregates, which is indicated by the rise of the curves to a maximum beyond which there is too much coarse material to make it possible to secure a workable mix with the given proportions of cement, the cement paste being insufficient to fill the voids. Increasing the amount of cement makes possible a working consistency with less water, resulting in higher strength. The maximum permissible values of the fineness modulus for various aggregates and mixes is given in the above table.

Consistency, or degree of plasticity, is measured by the slump test. The procedure is to fill with concrete an open-ended truncated cone, placed upright on a flat surface, and measure the height of the mass of concrete after the metal form is withdrawn. The difference between the original height of 12 inches and the measured height is the slump.

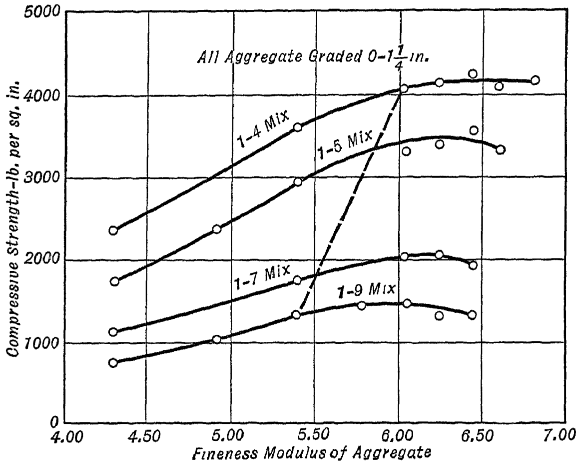


FIG. 4

The 1924 Joint Committee specifies "the quantity of water used shall be the minimum necessary to produce concrete of a workability required by the engineer. The consistency of the concrete shall be measured by the slump test as described in "Tentative Method of Test for Consistency of Portland-cement Concrete . ." (Serial Designation D138–22T) of the A.S.T.M. (Reprinted as Appendix XI of the 1924 Joint Committee Report.) The slump for the different types of concrete shall" preferably not be greater than indicated by the following table:

Types of concrete and mortar	Maximum slump in inches
Mass concrete	3
Reinforced concrete:	
Thin vertical sections and columns	6
Heavy sections	3
Thin confined horizontal sections	8
Roads and pavement:	
Hand-finished	3
Machine-finished	1
Mortar for floor finish	2

The relation between the fineness modulus, maximum size of aggregate, strength and mix is shown graphically in Fig. 5 which is based on curve *B* in Fig. 3. The proportions are here given in terms of the Real Mix, that is the volume of cement and the volume of the mixed aggregates, measured dry

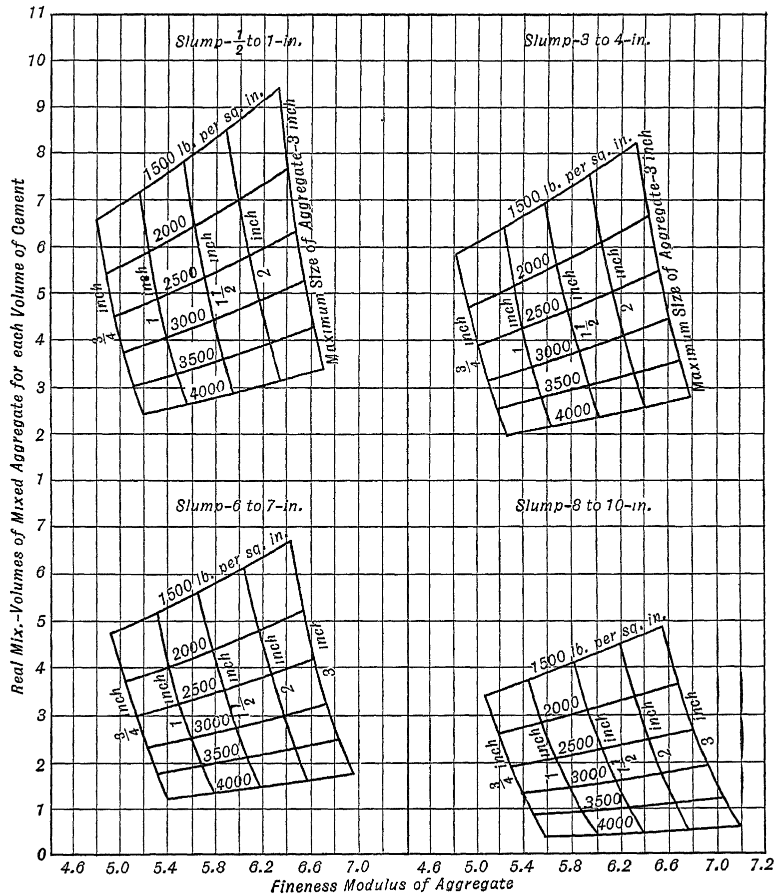


FIG. 5

and compacted in a standard manner. There is considerable variation of volume with varying degrees of compactness and the designation of a 1–2–4 or a 1–1.2–2.6 mix is a very indefinite mode of indicating concrete proportions unless information is added as to the manner in which the volumes are to be measured. The volume of the fine aggregate may vary widely with different amounts of contained moisture. A given volume of dry sand sometimes contains as much as 25 per cent to 30 per cent more material than the same

volume of damp sand (see Plate I).[1] For laboratory study the dry material is placed in a container and tamped (or "rodded") in a uniform manner[2] with the result that equal volumes of the same aggregate so measured contain equal amounts of material within about 1 per cent. Proportions such as 1–2–4 expressed in terms of volumes of dry material measured by this standard method are termed the Nominal Mix.

It is evident that neither the Real nor the Nominal Mix is convenient for use in the field, since the material in bin or stock pile contains more or less moisture and is measured in a loose condition. Proportions stated in terms of the aggregates in the condition found on the job and measured loose are called the Field Mix. It is necessary that the engineer be able to express the Real Mix found by laboratory studies as a Field Mix for use of the man on the job.

The tables prepared by Professor Abrams as a guide to the proportions of various aggregates required to give definite compressive strengths with concrete of specified consistencies were reprinted by the Joint Committee as an appendix to their report. One of these tables is printed in Appendix A. They give the Nominal Mix and correction must be made to obtain the Field Mix.

In important work it is not enough to take the mix from these tables. The aggregates should be tested and the proper proportions determined by means of the designing data given in this article, the results being checked by compression tests on cylinders.

The necessity of waiting 28 days for results is often a serious difficulty. Seven-day tests are coming into use as a guide to the 28-day strength. The relation between the 7-day and the 28-day strength (S_7 and S_{28}) is $S_{28} = S_7 + 30\sqrt{S_7}$ according to studies made by W. A. Slater.[3] It is possible that comparative tests between Portland cement concrete and that made with the new alumina cements will eventually be made, enabling 24-hour tests on aggregates with the quick-hardening cements to be used as a guide for designing Portland cement concrete mixes.

Parallel with more scientific methods of proportioning concrete there are being developed more exact methods for the field, notably more exact devices for measuring the materials. The uncertainty due to the bulking of loose moist sand is now often met by measuring the sand and water together by the so-called inundation method, taking advantage of the fact that when there is an excess of water over that required to fill the voids the sand volume is closely the same as when measured dry and loose. This is shown by Plate I where the bulking effects of varying percentages of water are recorded. The procedure in designing a mix is illustrated by the following example:

Example 2. It is desired to proportion the concrete for a reinforced concrete building. The city code places the 28-day compressive strength at

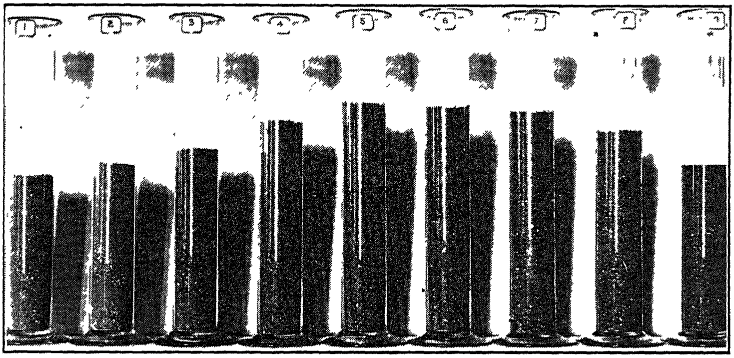
[1] "Effect of Moisture in Sands," R. R. Litehiser in "Concrete," January, 1925.

[2] "Standard Method of Test for Unit Weight of Aggregate for Concrete" (Serial Designation C29–21) A.S.T.M. Reprinted as Appendix XV of the 1924 Joint Committee Report.

[3] "Relation of Seven-Day to Twenty-eight-Day Compressive Strength of Mortar and Concrete," Proceedings of the American Concrete Institute, 1926.

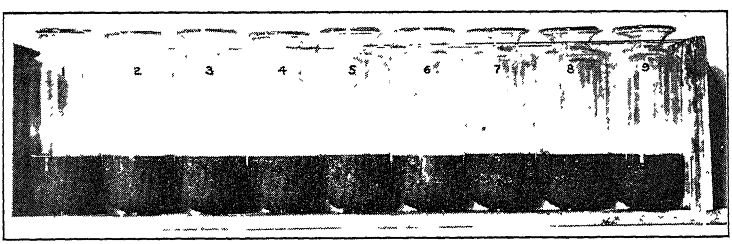
EFFECT OF MOISTURE UPON SANDS

No. 1 — Coarse Sand 0-#4
Fineness Modulus = 2.65



Tube	1	2	3	4	5	6	7	8	9
	Dry Rodded	Dry Loose	Loose	Loose	Loose	Loose	Loose	Loose	Loose
% Water (by wgt.)..	0	0	1	2½	5	7½	10	15	17½
% Bulking	0	8 5	18 2	30 3	41.8	35 2	33 9	26 6	0
% Loss per Cu. Ft..	0	7 8	15 5	23 2	29 3	26.0	25 3	21 0	0
Wgt. per Cu. Ft (Incl. water).	115 0 lb.	106.0	98 3	90.5	85 5	91 5	94 6	104 5	136 0
Wgt. Dry Sand in 1 Cu. Ft.......	115.0 lb.	106.0	97 3	88.4	81.4	85 1	86.0	90 9	115 0

All percentages based on Tube No. 1 — Dry, Rodded Sand.



INUNDATED SANDS
Note uniform level of sands regardless of original water content.

PLATE I[1]

[1] From a paper in "Concrete," Jan., 1925, by R. R. Litehiser, entitled "The Effect of Moisture in Sands in Proportioning Concrete Mixtures."

2000 lbs./sq. in. The concrete will be sufficiently plastic to flow into place around the reinforcing rods if it has a slump of 6 to 7 inches.[1]

Solution. The first step is to get together the necessary information regarding the aggregates, noting their quality and testing the fine aggregate for impurities as above described.

SUMMARY OF DATA REGARDING AGGREGATES

Aggregate	Per cent coarser than each sieve									Fineness modulus	Range in size
	100	50	30	16	8	4	$\frac{3}{8}$	$\frac{3}{4}$	$1\frac{1}{2}$		
Sand ..	85	70	55	25	15	0	0	0	0	2.50	0-4
Stone.	100	100	100	100	100	100	70	30	0	7.00	4–$1\frac{1}{2}$

Weight of 1 cubic foot, damp and loose as in stock pile........	Sand 90 lbs.	Stone 100 lbs.
Weight of same quantity of material when dried.	" 87	" 98
Weight of 1 cubic foot of dry material, rodded in standard manner..........................	" 108	" 105

Assuming that only ordinary care will be given to the making of this concrete, curve B in Fig. 3 is used for determining the water ratio, in this case 0.90 or $6\frac{3}{4}$ gallons of water per sack of cement. Referring to Fig. 5, slump 6–7 inches, the intersection of the 2000 lbs./sq. in. curve with that for $1\frac{1}{2}$ inch size shows the maximum permissible fineness modulus to be 5.8 and the Real Mix, 1–4.4.

It is evident that the fineness modulus of a mixture is the weighted mean of the moduli of the separate aggregates. Let x represent the proportion of the fine aggregate; then

$$2.5\,x + 7(1 - x) = 5.8$$

$$x = \frac{7 - 5.8}{7 - 2.5} = 0.27.$$

Accordingly, 27 per cent of the total volume of the aggregates measured separately must be fine aggregate and 73 per cent coarse. A sample of the aggregates mixed in these proportions is now prepared and its unit weight determined as 123 lbs./cu. ft.

The Real Mix, already found, is in terms of the volume of mixed aggregates, 4.4 cubic feet to each cubic foot of cement. In order to know how many cubic feet of aggregate measured separately are required to make 4.4 cubic feet of mixture the yield (or shrinkage factor) must be computed, that is the

[1] Examples 2 and 5 and Problem 3 are taken from the Portland Cement Association pamphlet, "Design and Control of Concrete Mixtures."

volume occupied by a total of 1 cubic foot of the separate materials when combined in the proportions found necessary. The yield is:

$$\frac{\text{Weight of 1 cu. ft. measured separately}}{\text{Unit weight of mixture}} = \frac{0.27 \times 108 + 0.73 \times 105}{123}$$
$$= 0.86 \text{ cu. ft.}$$

Since it requires 1 cubic foot of the separate materials to give 0.86 cubic foot of mixture, to give 4.4 cubic feet requires $4.4 \div 0.86 = 5.12$ cubic feet of material measured separately, 27 per cent being sand and 73 per cent stone. The nominal mix, therefore, is

$$1 - 5.12 \times 0.27 - 5.12 \times 0.73 = 1 - 1.38 - 3.74$$

based on dry rodded material.

To obtain the Field Mix it must be remembered that the Nominal Mix gives the amount of material needed per sack of cement. Expressing this nominal mix in terms of weight gives:

$$94 - 1.38 \times 108 - 3.74 \times 105 = 94 - 149 - 392.$$

Since 1 cubic foot of sand in the stock pile contains 87 pounds of sand and 1 cubic foot of stone 98 pounds, the Field Mix is

$$1 - \frac{149}{87} - \frac{392}{98} = 1 - 1.71 - 4.00.$$

The total amount of mixing water required per sack of cement is $0.90 \times 7.5 = 6.75$ U. S. gallons. Allowance should be made for the water contained in the aggregate and also for that which will be absorbed by it, average figures for which are as follows:

Average sand, pebbles, crushed limestone....	1 per cent by weight
Trap and granite..........	0.5 per cent by weight
Porous sandstone.................... ...	7 per cent by weight
Very light and porous aggregate may be as high as............................	25 per cent by weight

Assuming 1 per cent absorption, water must be added in amount $0.01\ (149 + 392) = 5.41$ lbs. or 0.65 gal. per sack of cement. From the moisture content already determined the total of contained water per sack of cement is $1.71 \times 3 + 4.00 \times 2 = 13.13$ lbs. or 1.58 gal. The total mixing water that must be added per sack of cement accordingly is $6.75 + 0.65 - 1.58 = 5.82$ gal.

It may be found that the mixture designed, when used with some aggregates, will be too rough or harsh to work easily, and a smoother mixture of the same strength may be desired. The usual custom in this case is to add sand. This may be done without affecting the strength so long as the water ratio is not changed. However, the addition of sand will cause the mix to become drier, and in order to increase the plasticity without affecting the strength more cement and water in the required ratio must be added.

The effect of adding a quantity of sand can be calculated by following the methods described. The present mix has a fineness modulus of 5.8. Increasing the sand to 35 per cent will reduce the stone to 65 per cent and change

this fineness modulus to 5.4. Then from Fig. 5, with a fineness modulus of 5.4 and a desired strength of 2000 lb. per sq. in. at 28 days, 6 to 7 in. slump, the new mix is found to be 1 : 4.0. With this new mix, the yield, Nominal Mix, Field Mix, etc., can be calculated in the manner previously outlined.

Example 3. It is desired to compute the quantities of materials required to make a cubic yard of the concrete proportioned in Ex. 2.

Solution. The quantity of cement is obtained from Fig. 6, the figure for a Real Mix of 4.4, being 5.8 sacks. Then since 1 sack is usually considered to have a volume of 1 cu. ft.,

$$\frac{5.8 \times 1.71}{27} = 0.37 \text{ cu. yd. of damp loose sand}$$

$$\frac{5.8 \times 4.00}{27} = 0.86 \text{ cu. yd. of damp loose stone}$$

the required quantities for making one cubic yard of concrete, no allowance being made for waste.

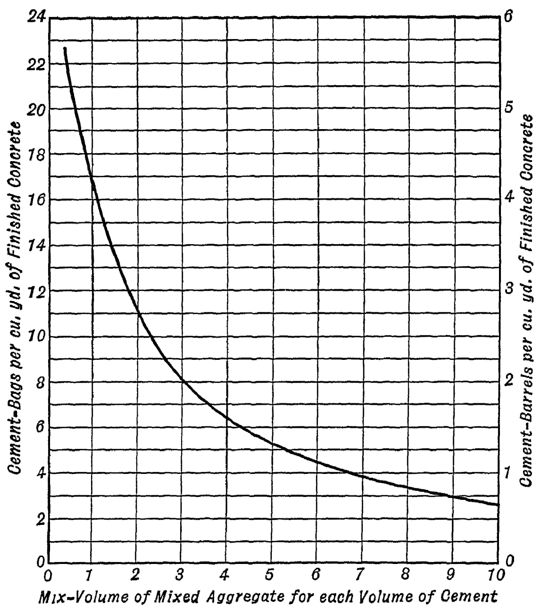


Fig. 6

Problem 3. A contract calls for a Field Mix of 1–2–4. What is the probable strength of the concrete with a slump of 6 to 7 inches?

Data. Study of the aggregate to be used gives the following information:

	Sand	Crushed Stone
Weight of 1 cu. ft. damp and loose	90	100
Weight of same material when dry	86	98
Weight of 1 cu. ft. of dry rodded material	110	108
Fineness modulus	3.2	6.8
Size	0–4	0–1½″
Weight of 1 cu. ft. of the dry materials mixed in proportions used, and rodded	124 lbs.	

Suggestion. Start with the Field Mix, converting the volumes to weights, and compute successively the Nominal Mix (dry, rodded, separate materials) and Real Mix (dry, rodded, combined materials).

Answer. 1800 lbs./sq. in. The Nominal Mix is found to be 1–1.56–3.63 and accordingly the sand is 30 per cent of the mix. The Real Mix is 1–4.5. The fineness modulus of the combined aggregates is 5.72.

20. A Comparison. The 1916 Joint Committee and many building codes place the average 28-day strength for ordinary 1–6 or 1–2–4 concrete at 2000 pounds per square inch, the measurements being made by loose volume. It seems certain the Joint Committee proportions refer to dry material although it is not so stated. Compacting the aggregates by rodding would reduce the proportions to about 1–1.9–3.8 or 1–5.7 Nominal Mix. The 1924 Joint Committee specifies a somewhat richer mix if this strength of 2000 pounds per square inch is desired. The table from that report (Appendix A) gives the average proportions of 1–1.7–3.4 or 1–5.1, for ordinary 2000 pounds per square inch concrete, 6–7 inch slump, made with the usual size materials for reinforced concrete construction.

To make this comparison on the basis of Field Mix account must be taken of the bulking effect of moisture on the fine aggregate as well as upon the difference between loose and rodded measurements. The moisture content of sand on the job is often 5 per cent, so for the coarse sand in Plate I, $115 \div 81.4 = 1.42$ cubic feet of damp loose sand must be taken to obtain as much sand as there is in 1 cubic foot dry and rodded; for the coarse aggregate of Ex. 2, $105 \div 98 = 1.07$ cubic feet. Assuming this amount of bulking to be possible in the field the 1916 Joint Committee nominal mix of 1–1.9–3.8 is equivalent to a field mix of

1–1.9 × 1.4–3.8 × 1.1 = 1–2.7–4.2 or 1–6.9. In the same way the 1924 Joint Committee nominal mix corresponds to a field mix of 1–1.7 × 1.4–3.4 × 1.1 = 1–2.4–3.7 = 1–6.1. For smaller sized aggregates the 1924 Joint Committee calls for a still richer mix.

The actual amount of bulking of wet sand depends on the manner of handling. The data in Plate I were obtained by pouring sand slowly into the several containers and the bulking is very much more than occurs when sand is shovelled from a pile into barrows. So the figures given above are useful only to emphasize by exaggeration the fact that applying the tabular proportions to field measurements is on the safe side.

21. Conclusion. It is not possible to say whether or no the generalization of the Water-Cement Ratio will stand finally as the unquestioned law of concrete strength but there is no doubt but that it marks a very long step ahead in our knowledge. In fact it is not an exaggeration to say that it is working a revolutionary change in concrete making with a great betterment of product. The theory of aggregate analysis with the Fineness Modulus as the standard is not at all a matter of the same order. While it has proved an extremely valuable method of analysis its use is not essential to the application of Professor Abrams' major theory. Unquestionably there remains much to be learned concerning the action of aggregates. To many the weak point in the new theory is the proviso that it holds only with workable mixes. Workability is a somewhat indefinite standard but one that implies to the practical man a rather narrow range of possible variation. There is argument as to whether the slump test really measures workability which is a function of both the wetness and the grading of the mix. The fact that quite generally it is necessary to add more sand than called for by the theory is a real difficulty. The leading investigators in this field agree in their recognition of the fundamental importance of the proportion of water used in concrete but in other points there are differences of opinion.

Probably the best place to turn to for information concerning progress is to the series of the Proceedings of the American Concrete Institute. Other rather random references are given in the list that follows.

REFERENCES DEALING WITH THE MAKING OF CONCRETE

Johnson's Materials of Construction.

Mills' Materials of Construction.

Proceedings of the American Concrete Institute. All recent issues, especially that of 1926.

Bulletins of the Structural Materials Research Laboratory, Lewis Institute, Chicago.

The New Methods of Proportioning Concrete in Theory and Practice. By R. B. Young. In Proc. Boston Soc. of Civil Engrs., March, 1921.

Proportioning Concrete on Job by Exact Methods. By R. B. Young and T. V. McCarthy. Engineering News-Record, March 17, 1921.

Placing 410,000 cu. yds. of Concrete on Ontario's Niagara Power Development. By A. C. D. Blanchard and R. B. Young. Engineering News-Record, April 6–13, 1922.

Report on Field Tests of Concrete used on Construction Work. By W. A. Slater and Stanton Walker. Proceedings of the American Society of Civil Engineers, Vol. LI, 1925.

Strength Specifications used for Large Concrete Bridge. Great Miami River Bridge of C. C. C. & St. L. Ry. at Sidney, Ohio. By J. B. Huntley. Engineering News-Record, October 11, 1923.

"Inundation used on Wacker Drive." Concrete, Sept., 1925.

"Specifying by Water-Ratio in Practice." Concrete, Jan., 1926.

Producing Concrete of Uniform Quality, By R. B. Young, and Water-Ratio Specification for Concrete, By F. R. McMillan and Stanton Walker. Proceedings Am. Soc. C. E., Sept., 1926.

CHAPTER IV

MANUFACTURE AND PROPERTIES OF CONCRETE

22. Concrete differs from other structural materials which come to the job as finished products in that it is manufactured where it is used. Good quality of concrete is the first essential for the permanence and solidity of the structure in which it is placed and accordingly its manufacture is a heavy responsibility upon the engineer. Structural steel is a standardized article of commerce, made under rigid supervision, and it can be bought in the open market with confidence that it will pass the rigid requirements of the American Society for Testing Materials. However on all important work involving large tonnage the engineer provides for careful inspection and tests of the steel. How much more essential is it that the engineer select with care the manufacturer of his concrete, the contractor, and hold him rigidly to the best methods of modern workmanship to ensure that the structural concrete be of the requisite strength and quality.

The preceding chapter outlined the best methods of proportioning concrete; the present chapter is concerned with the best methods of the actual manufacturing process itself. It consists largely of quotations from the 1924 Joint Committee report which give an excellent summary of the best practice. It should be realized that in some particulars these requirements are rather more strict than can be easily enforced on small jobs.

23. Mixing Concrete. (Arts. 31–34, Joint Committee.)

(*a*) "*Machine Mixing.* The mixing of concrete, unless otherwise authorized by the Engineer, shall be done in a batch mixer of approved type which will insure a uniform distribution of the materials throughout the mass so that the mixture is uniform in color and homogeneous. The mixer shall be equipped with suitable charging hopper, water storage, and a water-measuring device controlled from a case which can be kept locked and so constructed that the water can be discharged only while the mixer is being charged. It shall also be equipped with an attachment for automatically locking the discharge lever until the batch has been mixed the required time after all materials are in the mixer. The entire contents of the drum shall be discharged before recharging. The mixer shall be cleaned at frequent intervals while in use.

The volume of the mixed material per batch shall not exceed the manufacturer's rated capacity of the mixer."

" The mixing of each batch shall continue not less than one minute after all the materials are in the mixer, during which time the mixer shall rotate at a peripheral speed of about 200 ft. per min."

There is a considerable gain in strength if the mixing is carried on for a longer interval. The 1916 Joint Committee recommended $1\frac{1}{2}$ minutes.

(*b*) " *Hand-Mixing.* When hand-mixing is authorized by the Engineer it shall be done on a water-tight platform. The cement and fine aggregate shall first be mixed dry until the whole is of a uniform color. The water and coarse aggregate shall then be added and the entire mass turned at least three (3) times, or until a homogeneous mixture of the required consistency is obtained."

(*c*) " *Retempering.* The retempering of concrete or mortar which has partly hardened, that is, remixing with or without additional cement, aggregate, or water, will not be permitted."

24. Depositing Concrete. (Arts. 35–44, Joint Committee.)

(*a*) " *General.* Before beginning a run of concrete, hardened concrete and foreign materials shall be removed from the inner surfaces of the mixing and conveying equipment."

(*b*) " *Approval.* Before depositing concrete débris shall be removed from the space to be occupied by the concrete; forms shall be thoroughly wetted (except in freezing weather), or oiled. Reinforcement shall be thoroughly secured in position and approved by the Engineer."

(*c*) " *Handling.* Concrete shall be handled from the mixer to the place of final deposit as rapidly as practicable by methods which prevent the separation or loss of the ingredients. It shall be deposited in the forms as nearly as practicable in its final position to avoid rehandling. It shall be so deposited as to maintain until the completion of the unit, a plastic surface approximately horizontal. Forms for walls or other thin sections of considerable height shall be provided with openings, or other devices, that will permit the concrete to be placed in a manner that will avoid accumulations of hardened concrete on the forms or metal reinforcement. Under no circumstances shall concrete that has partly hardened be deposited in the work."

(*d*) " *Chuting.* When concrete is conveyed by chuting, the plant shall be of such size and design as to insure a practically continuous flow in the chute. The angle of the chute with the horizontal shall be such as to allow the concrete to flow without separation of the ingredients.[1] The delivery end of the chute shall be as close as possible to the point of deposit. When the operation is intermittent, the spout shall discharge into a hopper. The chute shall be thoroughly flushed with water before and after each run; the water used for this purpose shall be discharged outside the forms."

[1] " An angle of 27°, or one vertical to two horizontal, is the minimum slope which is considered permissible. Chuting through a vertical pipe is satisfactory when the lower end of the pipe is maintained as nearly as practicable, to the surface of deposit and the pipe full." (Joint Committee.)

(e) "*Compacting.* Concrete, during and immediately after depositing, shall be thoroughly compacted by means of suitable tools. For thin walls or inaccessible portions of the forms, where rodding or forking is impracticable, the concrete shall be assisted into place by tapping or hammering the forms opposite the freshly deposited concrete. The concrete shall be thoroughly worked around the reinforcement, and around embedded fixtures, and into the corners of the forms."

Experiment has shown that thorough rodding of laboratory specimens increases their strength as much as 100 per cent.

(f) "*Removal of Water.* Water shall be removed from excavations before concrete is deposited, unless otherwise directed by the Engineer. Any flow of water into the excavation shall be diverted through proper side-drains to a sump, or be removed by other approved methods which will avoid washing the freshly deposited concrete. Water vent pipes and drains shall be filled by grouting, or otherwise, after the concrete has thoroughly hardened."

(g) "*Protection.* Exposed surfaces of concrete shall be protected from premature drying for a period of at least seven (7) days after being deposited."

The proper protection of fresh concrete from drying out is second in importance only to its proportioning as to water content. This is especially a vital matter when concrete surfaces are exposed to wear.

(h) "*Temperature of Concrete.*[1] Concrete when deposited shall have a temperature of not less than 40° Fahr. nor more than 120° Fahr. In freezing weather suitable means shall be provided for maintaining the concrete, at a temperature of at least 50° Fahr. for not less than 72 hours after placing, or until the concrete has thoroughly hardened. The methods of heating the materials and protecting the concrete shall be approved by the Engineer. Salt, chemicals, or other foreign materials shall not be mixed with the concrete for the purpose of preventing freezing, unless approved by the Engineer."

Many disastrous failures have occurred through neglecting to protect fresh concrete from freezing. With proper care concrete construction may be carried on with perfect safety in the coldest weather if approved methods are followed. There are many reasons for requiring the engineer's approval, as in this specification. Overheating certain aggregates injures them. Neglect to provide moisture as well as heat in enclosed spaces weakens the concrete by too rapid drying. Fresh concrete that is set will not continue to harden if too cold. Ignorance of this fact may lead to too early removal of forms. The use of salt in the mixing water is apt to be injurious and is to be condemned. No chemical anti-freeze mixture should be used without the most searching investigation of its effects on all the important qualities of con-

[1] The Portland Cement Association publishes a very good pamphlet on "Concrete Work in Cold Weather."

crete. The advantages of the new alumina cements for cold weather concreting have already been pointed out.

At the present time there is a large amount of concrete work done in the winter, resulting in large savings of the waste hitherto inherent in the industry when carried on as a seasonal occupation.

(*i*) " *Depositing Continuously.* Concrete shall be deposited continuously and as rapidly as practicable until the unit of operation, approved by the Engineer, is completed . . . ."

(*j*) " *Bonding.* Before depositing new concrete on or against concrete which has set, the forms shall be retightened, the surface of the set concrete shall be roughened as required by the Engineer, thoroughly cleaned of foreign matter and laitance, and saturated with water. The new concrete placed in contact with hardened or partly hardened concrete, shall contain an excess of mortar to insure bond.[1] To insure this excess mortar at the juncture of the hardened and the newly deposited concrete, the cleaned and saturated surfaces of the hardened concrete, including vertical and inclined surfaces, shall first be slushed with a coating of neat cement grout against which the new concrete shall be placed before the grout has attained its initial set."

25. Depositing Under Water.[2] (Arts. 47–52, Joint Committee.)

(*a*) " *General.* The methods, equipment, and materials to be used shall be submitted to and be approved by the Engineer before the work is started. Concrete shall be deposited by a method that will prevent the washing of the cement from the mixture, minimize the formation of laitance, and avoid flow of water until the concrete has fully hardened. Concrete shall be placed so as to minimize segregation of materials. Concrete shall not be placed in water having a temperature below 35° Fahr."

(*b*) " *Proportions.* Concrete to be deposited under water shall contain $1\frac{3}{4}$ bbl. (7 bags) or more of Portland cement per cubic yard of concrete in place."

(*c*) " *Coffer-dams.* Coffer-dams shall be sufficiently tight to prevent flow of water through the space in which the concrete is to be deposited. Pumping will not be permitted while concrete is being deposited nor until it has fully hardened."

[1] Even so the joint is a plane of weakness, particularly as regards water-tightness.

[2] " Concrete should not be deposited under water if practicable to deposit in air. There is always uncertainty as to results obtained from placing concrete under water. Where conditions permit, the additional expense and delay of avoiding this method will be warranted. It is especially important that the aggregate be free from loam and other material which may cause laitance. Washed aggregates are preferable. Coarse aggregate consisting of washed gravel of a somewhat smaller size than that used in open-air concrete work will give best results. Concrete should never be deposited under water without experienced supervision. Many failures, especially of structures in sea water, can be traced directly to ignorance of proper methods or lack of expert supervision." (Joint Committee.)

(*d*) "*Depositing Continuously.* Concrete shall be deposited continuously, keeping the top surface as nearly level as possible, until it is brought above the water, or to the required height. The work shall be carried on with sufficient rapidity to prevent the formation of layers."

(*e*) "*Method.* The following methods" are used for depositing concrete under water:

"*Tremie.* The tremie shall be water-tight and sufficiently large to permit a free flow of concrete. It shall be kept filled[1] at all times during depositing. The concrete shall be discharged and spread by raising the tremie in such manner as to maintain as nearly as practicable a uniform flow and avoid dropping the concrete through water. If the charge is lost during depositing the tremie shall be withdrawn and refilled."

"*Drop-bottom Bucket.* The bucket shall be of a type that cannot be dumped until it rests on the surface upon which the concrete is to be deposited. The bottom doors when tripped shall open freely downward and outward. The top of the bucket shall be open. The bucket shall be completely filled, and slowly lowered to avoid back-wash. When discharged, the bucket shall be withdrawn slowly until well above the concrete."

"*Bags.* Bags of jute or other coarse cloth shall be filled about two-thirds ($\frac{2}{3}$) full of concrete and carefully placed by hand in a header-and-stretcher system so that the whole mass is interlocked."

(*f*) "*Laitance.* Great care shall be exercised to disturb the concrete as little as possible when it is being deposited in order to avoid the formation of laitance. On completing a section of concrete, the laitance shall be entirely removed before work is resumed."

26. Placing Reinforcement. It is of the greatest importance that reinforcing bars be accurately placed and held securely so that they will not be dislodged by the rough treatment necessarily incident to the placing of the concrete. An error of 1 inch in the vertical position of slab steel may easily double the stresses and result in serious cracking. Small errors in the location of beam reinforcement may cause large increases over the figured stresses.

Conservative engineers are more and more specifying the use of some approved type of bar support, several varieties of which are on the market. The best of these devices hold the steel securely at the proper distance from the forms with proper spacing and clearances. A common method for supporting slab steel is to place the bars on precast concrete blocks of the right height.

[1] "The tremie may be filled by one of the following methods: (1) Place the lower end in a box partly filled with concrete, so as to seal the bottom, then lower into position; (2) plug the tremie with cloth sacks or other material, which will be forced down as the pipe is filled with concrete; (3) plug the end of the tremie with cloth sacks filled with concrete." (Joint Committee.)

These blocks are not so good as some of the patented supports but are satisfactory for much work. Beam steel may be supported from the bottom of the forms or it may be hung in the stirrups by means of light bars, called loop bars, which run under the hooks of the stirrups and are supported from the slab forms.

Vertical steel in columns and walls usually requires temporary support at the top to keep it in correct alignment during pouring. In the design of such vertical steel consideration should be paid to its need for stiffness in supporting itself before and during the placing of the concrete.

Usually, where it is possible, reinforcement is made up into units before placing in the forms. This can almost always be done with column steel and usually with beam steel. Where a large amount of negative reinforcement (that is, reinforcement extending over the supports in the top of continuous beams) interlaces with column hooping it must be placed piece by piece. Slab steel can seldom be handled by units. Loose negative reinforcement, that is, straight top bars not connected with those in the bottom of the slab, must often be placed during pouring. However, this should be done only under careful and experienced supervision. Such top bars can usually be made up into units with the spacer bars (often called temperature reinforcement) and this is advisable because a unit is less likely to be pushed too deeply into the concrete than are separate rods.

Near cities and on large jobs in almost any location the bending of the reinforcement is usually done in the yard or shop of the reinforcing contractor and the steel is delivered on the job in bundles all bent and tagged. When it is bent on the job it should be done in a field shop and not in place.

27. Durability of Concrete and Reinforced Concrete.[1] The durability of concrete is manifestly of primary importance. Time has proved that when made from good materials, properly proportioned, mixed, placed and cured, concrete is well nigh everlasting, even under conditions of severe climatic exposure. Durability means, however, not only inherent permanence and resistance to weathering but also resistance to wear and to the chemical attack of substances that may in any manner be brought in contact with the concrete.

[1] See paper by Prof. A. H. White, "Cause of Disintegration of Concrete" in the *Canadian Engineer*, Oct. 6, 1925.

If a surface is to be exposed to wear the concrete must be of hard aggregate, carefully proportioned for great density and strength, and kept warm and moist for a curing period of from 10 days to three weeks. Certain applied concrete hardeners may be advantageous here. They will accomplish little that may not be done without them but their use may prove economical.

Good Portland cement concrete exposed to the action of sea water in general remains unaffected when protected from abrasion. Permeable concrete may disintegrate badly, especially in the parts lying between low and high water, this action being much more rapid when frost action aids the destruction. Waters charged with alkalis of the sulphate type, such as are common in many regions of Western North America, cause serious disintegration of Portland cement concrete even when it is of the best quality. Alkalis of the carbonate and chloride types are far less destructive. Accordingly alkali ground waters must be kept from contact with concrete if it is to endure. Thorough drainage and the use of waterproof coatings are beneficial here. Rich mixes should be used; the Joint Committee specifies that for Portland cement concrete exposed to sea water between tide levels or exposed to alkali water, a minimum of 7 bags of cement be used for each cubic yard of concrete. Concretes made with the new quick-hardening high alumina cements are not attacked by sea water or alkalis.

Concrete reinforced with steel is far more susceptible to deterioration by weather and the action of sea water and alkali than plain concrete because of the destructive effects that follow the corrosion of improperly protected reinforcement. Steel increases in volume when it corrodes and cracks or splits off the concrete covering with consequent increase in the rate of destruction. If the steel is embedded a sufficient distance in concrete of good quality, as provided by the usual rules, it is secure against corrosion. For structural members in sea water, the reinforcement is required by the Joint Committee to be at least 3 inches from the surface except at corners where 4 inches is prescribed. Somewhat less protection is required for alkali exposure. For ordinary construction the Joint Committee specifies:

"Metal reinforcement in wall footings and column footings shall have a minimum covering of 3 in. of concrete. At surfaces of concrete exposed to the weather, metal reinforcement shall be protected by not less than 2 in. of concrete." (Art. 67.)

Concrete is often used where it is exposed to the action of oils, of sewage or of various chemicals. The effect of all such liquids must be carefully considered and suitable protective measures adopted when needed.

Concrete made from most of the usual types of aggregates is an excellent fire-resistive and fireproofing material, the rate of thermal conductivity being so low that properly covered steel is thoroughly protected. A certain amount of spalling and surface calcination is inevitable in a severe fire but properly made with proper materials a reinforced concrete structure should stand an ordinary fire without material structural damage. For fireproofing cover the Joint Committee specifies:

"Metal reinforcement in fire-resistive construction shall be protected by not less than 1 in. of concrete in slabs and walls, and not less than 2 in. in beams, girders, and columns, provided aggregate showing an expansion not materially greater than that of limestone or trap-rock is used; when impracticable to obtain aggregate of this grade the protective covering shall be 1 in. thicker and shall be reinforced with metal mesh, having openings not exceeding 3 in., placed 1 in. from the finished surface."

"In structures where the fire hazard is limited, the metal reinforcement shall not be placed nearer the exposed surface than $\frac{3}{4}$ in. in slabs and walls, or $1\frac{1}{2}$ in. in beams, girders, and columns." (Art. 68.)

It should be noted that common practice is that indicated by the second paragraph quoted.

The corrosion of steel reinforcement by electrolytic action occasionally takes place but in general no special precautions need to be taken on this account.[1] It is worthy of note that the use of salt in the mixing water increases the conductivity of concrete and enormously increases electrolytic action when it takes place. Stray electric currents are rarely a source of trouble except in structures in contact with sea water.

Another source of damage to concrete is the percolation of water. Accordingly means should be taken to prevent this by proper drainage and waterproofing, being careful to avoid any arrangement that permits accumulation of water.

28. Waterproofing. It is perfectly possible to make concrete water-tight up to heads of 70 feet or more provided cracks do not develop due to stress, temperature or to the opening up of construction joints. Structures below ground are relatively little

[1] Technologic Papers of the Bureau of Standards, No. 18, "Electrolysis in Concrete."

affected by temperature but those exposed to the weather are very likely to develop leaks through construction joints no matter how well these joints are made. Continuous metal dams across such joints are probably the best means for minimizing this difficulty. Stress cracks can be lessened by reducing the stresses. In reinforcing water-tight work it is particularly important that steel be used wherever bending would tend to cause tension and hence cracks, whether the steel is needed for stability or not.

The following directions for obtaining water-tight concrete are given by the Portland Cement Association:

1. All portions of the structure should be strong enough to resist the head of water, either internal or external, to which the concrete may be subjected.
2. Use clean, well graded aggregates.
3. Use a relatively rich mixture, such as a 1 : 2 : 3, or 1 : $1\frac{1}{2}$: 3.
4. Use the minimum amount of mixing water that will give a workable, plastic consistency; not over 6 gallons per sack of cement.
5. Mix the concrete at least $1\frac{1}{2}$ minutes after all materials are in the mixer.
6. Place the concrete carefully in layers 6 to 12 inches deep, spading or rodding it thoroughly to prevent the formation of stone pockets or voids.
7. If possible place the concrete in one continuous operation to avoid construction joints. If placing is interrupted, be sure to get a good bond between the fresh concrete and that placed previously.
8. Keep the concrete warm and damp for the first ten days.

"Concrete Data for Engineers and Architects."

However, waterproof concrete is more expensive to manufacture and place than ordinary commercial concrete and it is often cheaper to apply a waterproofing to ordinary concrete than to make better concrete.

There are two main types or methods for waterproofing concrete: integral and applied. Integral waterproofing consists in introducing some substance into the mix to make the concrete denser with fewer voids for the percolation of water, or to incorporate some material which is water-repelling. For example, clay makes a lean concrete less permeable by reducing the voids but it does not add to strength. It is doubtful if any product will reduce the voids of a rich mix. There is doubt concerning the permanency of many of the water-repelling substances. Some of them reduce strength. In many cases additional cement is the best and cheapest integral waterproofing excepting perhaps hydrated lime which makes the mix "fatter," that is, lubricates it so that it is more readily compacted into a dense mass.

The other type, applied waterproofing, is made up of pore fillers, plasters and membranes. The pore fillers are less used than formerly except for damp-proofing. Some of the floor hardeners and certain proprietary paints and compounds are undoubtedly effective for this purpose.

The plaster type of waterproofing is somewhat of a cross between the integral and the applied types. A cement plaster made with an integral waterproofing agent is applied to the inside surface of walls and floors in thicknesses usually from $\frac{5}{8}$ inch to 1 inch, forming a satisfactory finish for plastered walls or wearing surface for floors. It is largely and effectively used in basements and pits.

All of the foregoing methods are ineffective however in preventing leakage through cracks. For the standpipe type of structure or any other which is liable to cracks there remains only the membrane waterproofing. This is built up of fabric and asphalt or tar in manner similar to roofing and is somewhat flexible. The membrane must be protected from abrasion by a heavy concrete or brick covering and so it is very difficult to repair. It is much more expensive than any other mode of waterproofing. While a plaster surface cracks with the concrete it is very easy to repair and the leaks are of course very easily found.

Whatever the method of waterproofing used the protection must form a continuous surface around pits under boilers and so on, and the concrete must be structurally sufficient to resist all stresses due to water pressure.

29. Strength of Concrete. The strength of concrete is very variable, depending on many factors, and it requires skilled workmanship and control to secure even reasonable uniformity in quality. Average values for the 28-day compressive strength for various mixes are given in Article 18. This table gives results to be expected with specimens made and stored in accordance with standard laboratory practice. A test piece obtained by drilling a core from the concrete in place on the job should not be expected to give as great strength. In comparing test results it is important to note whether the test specimen was a cube or a cylinder since cubes show materially higher strength than cylinders on account of the lack of opportunity for free fracture along the inclined planes of maximum shear.[1]

[1] See Bulletin 16, Structural Materials Research Laboratory, Lewis Institute, Chicago.

The tensile strength of concrete is small, averaging about 10 per cent of the compressive strength. It is common to neglect it in design. The shearing strength is approximately 60 per cent to 80 per cent of the compressive strength.

Working stresses for design are generally expressed as percentages of the 28-day compressive strength of cylinders. At this time the concrete has attained approximately two-thirds of the strength it will have at six months and about one-half of that at three years. Working stresses 25 per cent higher than those used in design for concrete 28 days old are usual for computing the safe loads for old structures and in designing for future additions. The average strength to be expected from various mixes and the classes of construction where they are employed may be roughly summarized as follows:

1 : 1 : 2	3000–3300#/sq. in.	Reinforced columns.
1 : 1½ : 3	2500–2800#/sq. in.	Reinforced columns and highly stressed members.
1 : 2 : 4	2000–2200#/sq. in.	The standard mixture for reinforced concrete slabs, beams, columns, bridges, arches, and ordinary watertight work.
1 : 2½ : 5	1600–1800#/sq. in.	Foundation walls, plain concrete, retaining walls, piers, abutments, machine foundations.
1 : 3 : 6	1300–1400#/sq. in.	Unimportant mass work.

30. Elastic Properties of Concrete and Steel. Concrete is not an elastic material, strain increasing faster than stress from the very first and permanent deformation occurring under low stress. On the average the stress-strain curve in compression may be taken as approximately a parabola with vertex at the point of ultimate strength and axis vertical, stress being plotted vertically and strain horizontally. Within the range of working stresses this curve does not deviate greatly from a straight line and it is universally the custom to assume that the modulus of elasticity is a constant for working stress conditions. The values of the

modulus given by different authorities differ greatly, 3,000,000 pounds per square inch being perhaps the most accepted average for ordinary 1–2–4 concrete at 28 days. As the concrete ages it gets harder and stiffer and the modulus increases. In design computations a lower value than the actual is used for reasons that will appear in the discussion of the mechanics of reinforced concrete. The Joint Committee recommendations for the design modulus are given in Appendix B. Alumina cement concrete has a considerably higher modulus than Portland cement concrete of the same proportions.

Strictly speaking there is no elastic limit for concrete. The term is often used inexactly however to indicate the limit of stress that may be applied repeatedly without causing increase in permanent deformation. This limit is about 50 per cent of the ultimate; loads beyond this limit and below the static ultimate when applied repeatedly, cause continually increasing deformation, and, finally, rupture.

The modulus of elasticity of steel is taken as 30,000,000 pounds per square inch for all grades of steel in all computations of reinforced concrete. This value is slightly higher than the average given by tests.

Problem 4. (*a*) A certain concrete has an ultimate compressive strength of 2200 lbs./sq. in. with a corresponding strain of 0.0018. Compute the value of the modulus of elasticity for a stress of 700 lbs./sq. in. This modulus is the slope of the chord connecting the given stress with the origin and is called the secant modulus.

(*b*) Compute the value of the initial modulus (that is, the slope of the tangent to the curve at the origin or point of zero stress) for this concrete.

Answers. (*a*) 2,260,000 lbs./sq. in. (*b*) 2,440,000 lbs./sq. in. *Note:* Facility in the use of the parabola is best gained by discarding the classic equation $y^2 = 4\,px$ and substituting (see Fig. 7),

$$O_1 : O_2 = D_1^2 : D_2^2$$

which may be better expressed in words, using the terminology of the surveyor: "offsets vary as the squares of the distances."

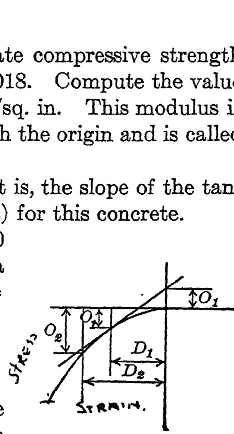


FIG. 7

31. Contraction and Expansion of Reinforced Concrete. On hardening in air concrete shrinks about 0.0005 of its length due apparently to the drying out. Hardened in water it expands slightly but if later removed from water it shrinks nearly the same

as though originally hardened in the air. In consequence of this shrinking compressive stresses are set up in any embedded steel which are generally ignored so far as the principal reinforcement is concerned. When any mass of concrete is not free to contract on shrinking or with temperature fall it becomes necessary to provide reinforcement to compel the formation of many small cracks in place of a few large ones. For this purpose small deformed rods of high carbon steel with a cross-sectional area of 0.002 to 0.005 of that of the concrete are conservatively used. "Expansion from a rise of temperature rarely causes trouble except at angles where the lengthening of the surface may produce a buckling." The coefficient of expansion for concrete has an average value of about 0.000006 per degree Fahrenheit. The 1924 Joint Committee recommends the use of 0.5 per cent of steel to prevent the opening of construction joints. (Article 72.)

32. Coefficient of Expansion. Steel and concrete change in length with temperature variations very nearly the same amount so that there is practically no stress set up on this account tending to break the bond between the two materials. The coefficient of expansion for steel is 0.0000065 per degree Fahrenheit.

33. Bond between Concrete and Steel. It would be impossible to reinforce concrete with steel rods but for the adhesion of the concrete to the steel so that there is no slipping between the two materials as the combined member deforms under load. In design care must be taken that there is no excessive tendency for the steel to slip from the grip of the surrounding concrete since in general a small movement will result directly or indirectly in the destruction or serious damage of the piece. This bond, or resistance to sliding, is of two kinds: an adhesion between the two materials and a sliding resistance that develops after the adhesion is broken and movement begins. Tests made at the Structural Materials Research Laboratory[1] by pulling out ordinary plain round rods from 8 inch by 8 inch concrete cylinders of different ages, where the only resistance to the pull was the force developed by bond on the surface of the rod, showed that there was no slip until the bond stress reached an average value of about 10 per cent to 15 per cent of the compressive strength of the concrete, and that the maximum bond resistance, reached when the slip

[1] Reported in Bulletin 17, "Studies of Bond between Concrete and Steel," by Duff A. Abrams.

was about 0.01 inch, equalled approximately 24 per cent of the concrete strength. Earlier tests made at the University of Illinois[1] showed that square bars give results about 75 per cent of those obtained with plain round bars. The same series of tests proved that deformed bars begin to slip at about the same bond stress as plain rounds and that the resistance to sliding offered by the bearing of the projecting lugs on the concrete, while considerably larger than that for the plain bars, does not become effective until a considerable slip has occurred. "The large slip and the high bearing stresses developed in the later stages of the tests show the absurdity of seriously considering the extremely high values that are usually reported to be the true bond resistance of many types of deformed bars." The reputed value of square twisted bars as offering high bond resistance was also denied by these tests, these bars showing lower bond stresses at small slips than the rounds and developing high resistance only at extremely large slips. "The use of deformed bars of proper design may be expected to guard against local deficiencies in bond resistance due to poor workmanship and their presence may properly be considered as an additional safeguard against ultimate failure by bond. However it does not seem wise to place the working bond stress for deformed bars higher than that used for plain bars."

34. Weight of Reinforced Concrete. It is usually specified that the weight of reinforced concrete shall be assumed at 150 pounds per cubic foot. Several important building codes allow the more convenient figure of 144 pounds per cubic foot which is perhaps somewhat lighter than the average reinforced concrete.

[1] Bulletin 71, "Tests of Bond between Concrete and Steel," also by Duff A. Abrams.

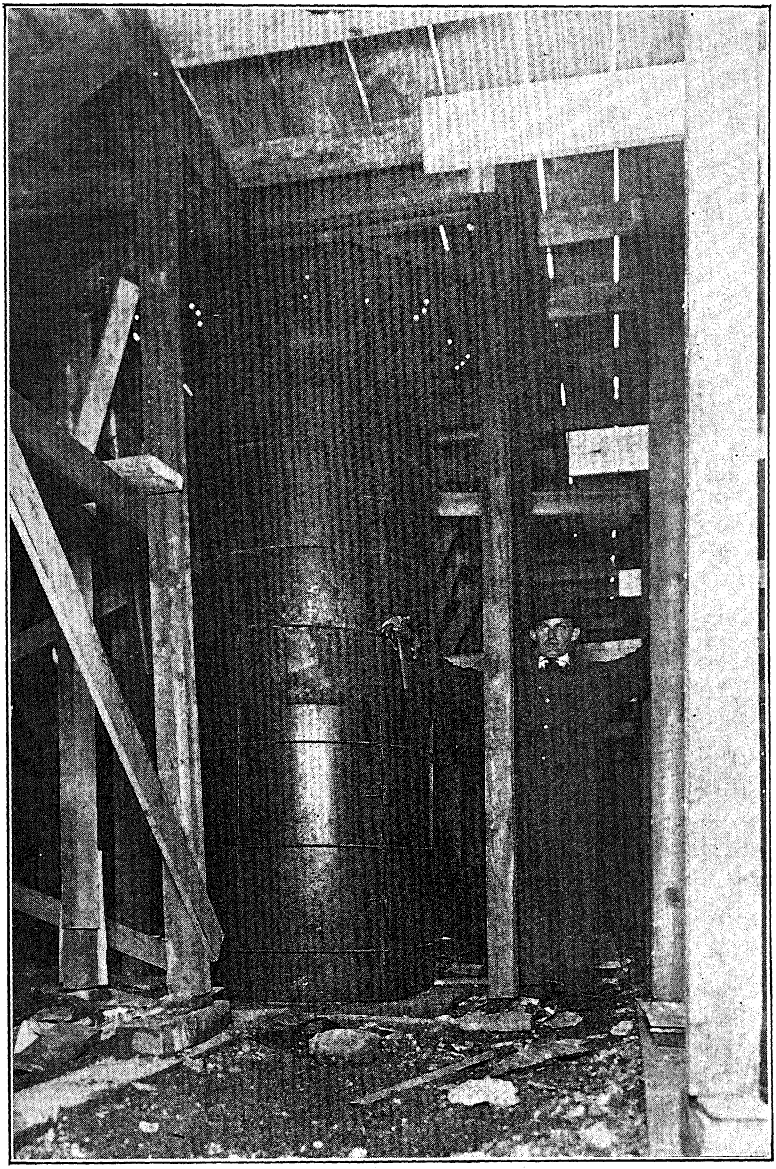
CHAPTER V

FORMS

35. Since concrete is manufactured in a plastic or semi-fluid state, it must of necessity be confined in a mould or form until it has set or hardened sufficiently to hold its shape and in many cases support its own weight. Under certain circumstances forms may be omitted. Such cases are the bottoms of foundations and ground floors and sometimes the sides of footings or walls. This ordinarily means that an earth surface acts as a form. Forms represent 10 to 25 per cent of the final cost of a concrete structure. This cost may vary considerably with design, and therefore forms must be considered by the designer although their detailed layout is commonly left to the contractor or field engineer.

36. Requirements. The first essential of forms is that they must be carefully built to the required dimensions and made of sufficient strength to hold their shape and alignment under the load of the wet concrete and any construction loads which may come upon them. They must also be sufficiently tight to prevent the escape of water, for escaping water carries with it much of the finest and most effective cement. A second essential is that they be designed to facilitate as far as possible easy removal.

The requirements of the finished concrete surface must often be considered. Forms for footings or other substructure work may be of the roughest construction. Forms for ordinary building work must be of dressed stock to give a satisfactory appearance. For ornamental work, cornices, etc., the forms must be built with great care and the outlines should be designed with a view to reasonable construction. Offsets in mouldings should be $\frac{7}{8}$ inch, $1\frac{3}{4}$ inch or some other stock lumber size, no offsets less than $\frac{7}{8}$ inch being practical in ordinary commercial construction. It should be remembered that sharp corners are always liable to spalling when forms are stripped. This is one of the reasons why triangular fillets are usually used at all corners of beams and columns. Construction joints are seldom completely obliterated, so they should be made where they will show the least, or located with a view to symmetry.



Courtesy Blaw-Knox Co., Pittsburgh, Pa.

PLATE II

STEEL COLUMN FORM

This illustration shows a steel form for a typical round column as used in flat slab construction. It also shows posts for slab forms with levelling wedges at the bottom.

37. Materials. Wood is still the most common form material, spruce and pine being most used. (See Plates II–V.) Certain woods, notably hemlock, are unsuitable for nice work because they stain the concrete. Partially seasoned stock is usually used, for fully dried lumber swells too much when wet. For all except substructure work lumber should always be planed on at least one face and one edge, and usually it is dressed on all four sides.

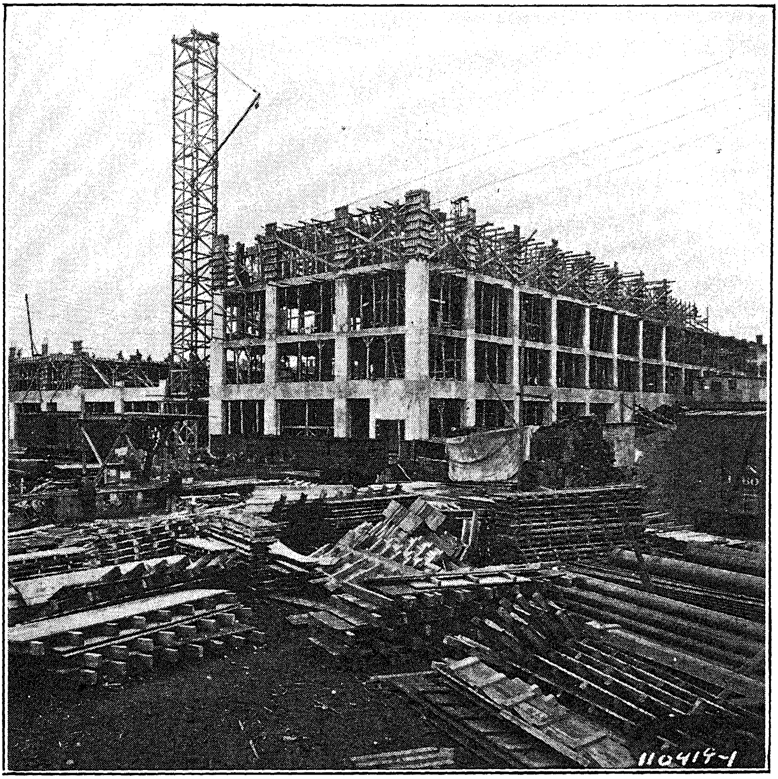
Steel forms for concrete have a considerable and perhaps growing use. They are more expensive in first cost than wood, but more substantial for rehandling, so that if steel forms can be used enough times, they are cheaper than wood. Steel gives a smoother concrete surface than wood, without board marks, but the joints of the panels show in the finished work. Steel is used generally for concrete chimney forms, for circular columns and flat slab column capitals. It is also used for slab-forms both in the shape of domes and pans for ribbed floors and in panels, sometimes reinforced with wood ribs, for plain slabs and walls.

Plaster and glue forms are used for ornamental work, usually the province of the architect rather than the engineer. Outlines so elaborate as to require such special forms should be designed in consultation with someone experienced in special form work.

38. Design. Concrete forms are usually built by rule of thumb, although experience has shown that careful designing saves money on a good sized job. In computing the size of form members required for strength, account must be taken of the fact that concrete will exert a hydrostatic head of about 125 pounds a cubic foot until set. The capacity of the concrete plant must be known therefore to design forms, since they must be figured for a head equal to the depth which will be poured during the time of initial set. It is not common practice to design ordinary concrete members for the use of stock widths of lumber although there would seem to be economy in so doing for many jobs.

39. Construction. Building a form for concrete is just the opposite of the older carpentry problems, for it is building outside a surface instead of inside. The ordinary concrete form is a surface of boards, plank, or steel plate supported by joists and posts with the necessary braces.

Posts and braces are usually adjusted to length with a pair of wedges. (See Plate II.) This allows the strain to be taken off for removal. There are three steps in the use and cost of a



Courtesy Aberthaw Construction Co., Boston, Mass.

PLATE III

WOOD FORMS

In the lower portion of this plate are shown form panels as made up in the shop on the job ready for erection. In the background some forms are in place. Note the posts left in place under the beams of floors which have already been stripped.

concrete form: making, erecting and removing. The making, including material, is the most expensive step; consequently, economy is effected by reusing as many times as possible. To facilitate this, forms are usually made up in panels of convenient sizes. Beams and column sides and beam bottoms are usually single panels, and slab forms are divided into panels of a convenient size for handling. (See Plates III and IV.) Where there is little chance for reuse and large plane surfaces are to be formed, it is sometimes cheaper to build forms like an ordinary joist floor.

To improve surfaces, prevent absorption and facilitate removal, a coating of mineral oil is sometimes used on form surfaces of the better grade that are to be reused. Column and wall forms should have clean-out doors in the base so that all shavings and other débris in the bottom of the forms may be removed before pouring concrete.

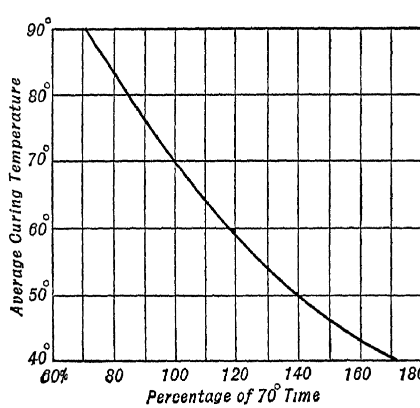
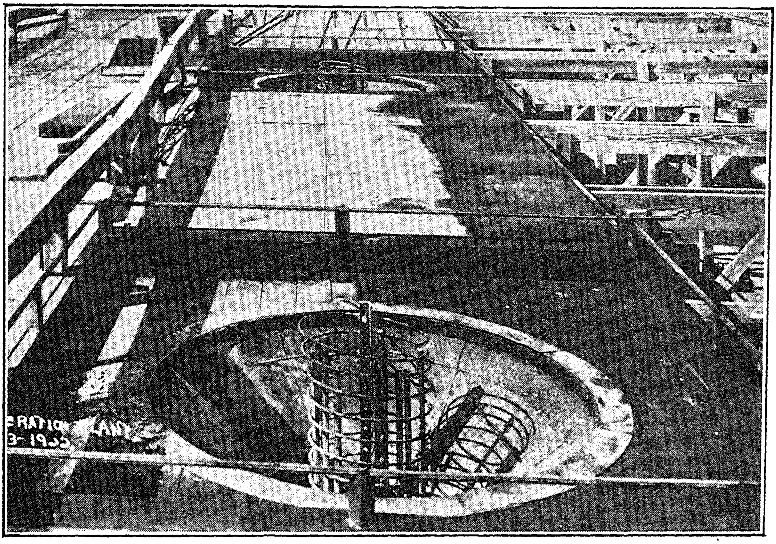


FIG. 8

The time that forms must remain in place will vary with the kind of the members, the weather and the character of the concrete. The minimum time will vary from 48 hours for walls and columns to 7 days for slabs and 10 days for beams at a standard 70 degree average setting temperature. This time should be increased if members are carrying load beside their own weight. Lower temperature delays setting. Figure 8 gives an idea of the time of setting at different temperatures compared with the time required at 70 degrees. The time forms remain in place should be at least that required for normal 70 degree set increased by the ratio of setting time at the average temperature to the setting time at 70 degrees, the time when the temperature is below 40 degrees not being considered. It is sometimes desirable to leave forms in place to prevent premature drying of the concrete.

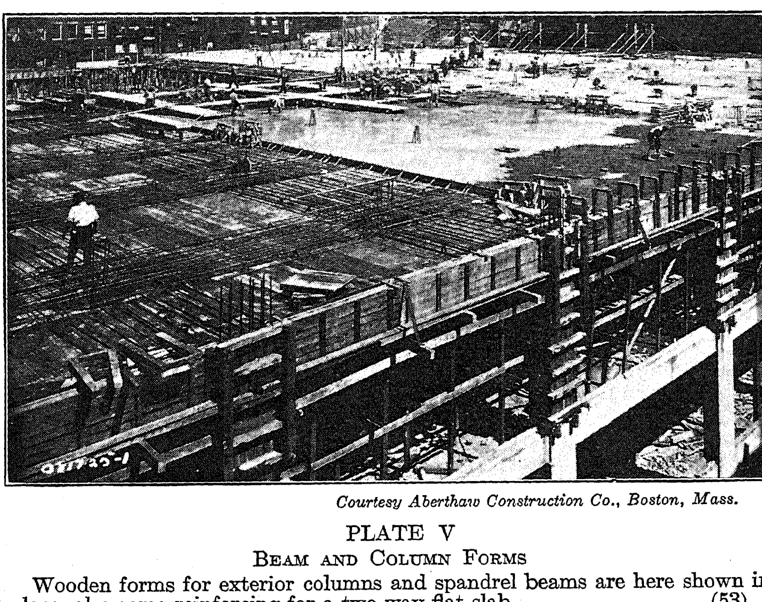


Courtesy Blaw-Knox Co., Pittsburgh, Pa.

PLATE IV

Slab Forms

In the foreground is the top of a column form like that in Plate II. The slab forms shown are of steel plate except those for the dropped panel around the column, which are wood. The wooden rail at the left is to support a runway on which to distribute concrete on the floor.



Courtesy Aberthaw Construction Co., Boston, Mass.

PLATE V

Beam and Column Forms

Wooden forms for exterior columns and spandrel beams are here shown in place, also some reinforcing for a two-way flat slab.

CHAPTER VI

BEAMS

40. The ordinary formulas used for steel and timber beams apply only to members of homogeneous material and accordingly are not directly applicable to composite beams of steel and concrete. The special formulas which have been devised for reinforced concrete are numerous and somewhat complicated but simple of solution with the help of the charts and tables in common use. Unfortunately the beginner finds them a serious obstacle as he attempts to get an understanding of the few basic principles of reinforced concrete design and he often falls into the fatal habit of using them blindly. More unfortunately still, he often becomes dependent upon formulas and incapable of solving problems without a list of them at hand. It is to his advantage, therefore, to master the fundamental principles of composite beams before attempting to use, or even to derive these special formulas, with their involved notation, and before attacking problems that bring in the confusing details of actual design. This is easily done since nearly all problems of stress in reinforced concrete members may be solved by the ordinary methods made familiar to the student by his study of structural pieces of homogeneous material, methods and formulas being made applicable by transforming the steel-concrete section into its equivalent in the one material, concrete. Anyone who has mastered the method of the "transformed section" not only understands the derivation and use of the standard reinforced concrete formulas but also is independent of them and their necessary accompaniments, tables and plots, an independence conducive to self-confidence and often very desirable in emergencies. This process is fully illustrated in the succeeding articles. In the examples given it should be noted that all practical considerations, such as choice and spacing of bars, are ignored in order that attention may be centered on the principles involved. Experience has shown that it is important that the student avoid raising such questions during this preliminary study.

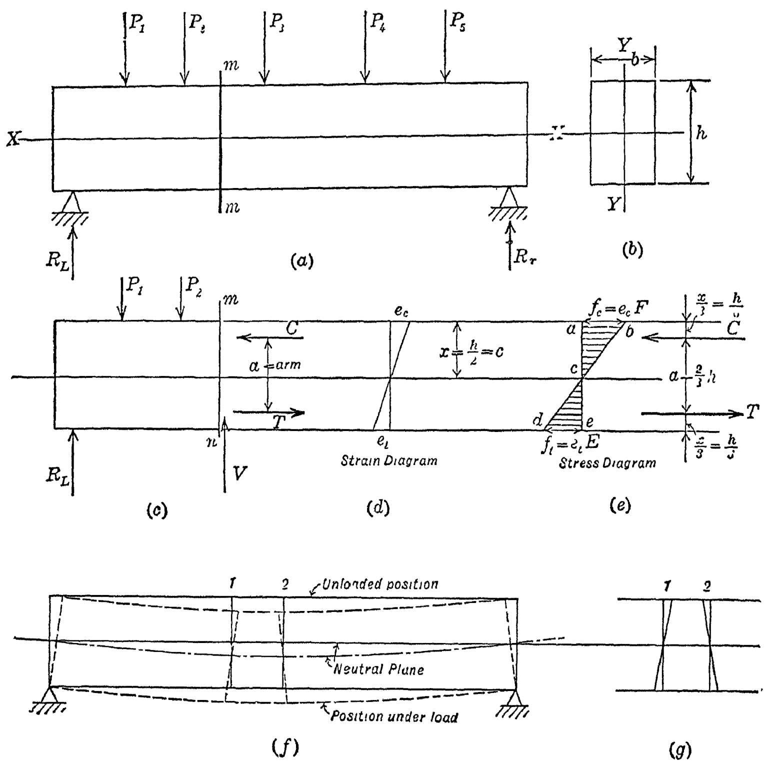
41. Kinds of Reinforced Concrete Members. Every structural member of reinforced concrete belongs to one of three classes: beams,[1] subject only to bending, caused by loads that act perpendicular to the longitudinal axis, or by applied couples or by both transverse loading and applied couples; compression members, carrying loads whose lines of action coincide with the longitudinal axis and which cause uniform compression on any section normal to that axis; members subject to both direct compression and bending. Since the concrete of a purely tension member does not assist in carrying the load such a piece cannot logically be said to be one of reinforced concrete.

42. Beams of Homogeneous Material. The relation that exists between the internal fiber stresses of a homogeneous rectangular beam and the external forces may be found in the following manner.[2] Consider the end supported beam AB (Fig. 9*a*–*b*) shown for convenience in a horizontal position, with the known external forces acting perpendicular to the longitudinal axis and in the plane of the vertical axis, thus ensuring bending alone, without direct stress or torsion. The beam as a whole is at rest under the action of the outer forces. Therefore the portion of the beam to the left of the plane section mn (Fig. 9*c*) is also in equilibrium, the balanced system of forces acting thereon being the external forces applied to that part of the beam and the internal fiber stresses exerted on it by the part of the beam to the right of mn. Considering each of these forces to be represented by its resultant, this force system is co-planar and the conditions of equilibrium for such a system give certain information regarding the unknown fiber stresses. The condition $\Sigma Y = 0$ shows that the total shearing stress along mn (V in Fig. 9*c*) equals the resultant of the external forces to the left of the section, that is, the internal shear equals the external shear. The condition $\Sigma M = 0$ (taking moments about any point in the plane mn) shows that

[1] The strict limitation of the term "beam" adopted in this text is not in accordance with everyday usage which applies the word loosely to any member whose loading is either largely or entirely transverse. The strict usage is desirable for clearness of analysis.

[2] For a general and rigorous derivation of the common beam theory the reader is referred to the standard texts on strength of materials. The purpose here is to review important principles and emphasize the main points of the argument.

there must be a counter-clockwise moment acting on this part of the beam to counterbalance the clockwise moment of the external



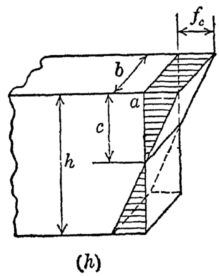


FIG. 9

forces. Since only the horizontal (or X) components of fiber stress can have moment about a point in plane mn this counter-clockwise moment must be that of a compressive and a tensile force, represented by C and T respectively in Fig. 9c. Since $\Sigma X = 0$ these two forces must be equal ($C = T$) and the moment of the internal

fiber stresses is, accordingly, a couple, equal to $C \cdot a$ or $T \cdot a$ where a is the arm of the couple, the distance between the resultant compression and resultant tension. This couple is the **resisting moment** (MR), equal in value and opposite in direction to the external **bending moment** (BM), at the section.

Having gained all the information possible from the principles of statics, resource next must be had to direct observation of the actual beam under load, the loaded and unloaded positions being represented in Fig. 9*f*, with the deflection greatly exaggerated. Careful measurements show that any normal section, such as 1 or 2, is practically a plane section in the loaded as well as in the unloaded beam. Examination of the positions taken by these two planes in the bent beam shows that one horizontal fiber extending from section to section remains unchanged in length and that the fibers above are all shortened and those below lengthened. Therefore the amount of the deformation in any fiber varies directly with its vertical distance from the fiber which remains unchanged in length (Fig. 9*g*). This makes possible the drawing of the strain diagram for the fibers at the section mn (Fig. 9*d*) which will be a continuous straight line crossing the section at an unknown distance (x) from the top.

Experiment has also shown that within the elastic limit of the material there is a fixed relation between strain and stress, that $\dfrac{\text{stress (pounds per square inch)}}{\text{strain (inches per inch)}}$ = a constant named the modulus of elasticity (pounds per square inch) or in the usual notation $E = \dfrac{f}{e}$. It is also true that for timber and steel the modulus of elasticity in compression may be taken as equal to that in tension. It is now possible to draw the stress diagram (Fig. 9*e*) each abscissa of the strain curve being multiplied by E to obtain that of the stress diagram with the result that it also is a straight line (bcd). The compressive fiber stress on the section, therefore, is represented by the solid seen in side elevation as abc (compare Fig. 9*h*) and the tensile force by that projected as cde. Since the total compression equals the total tension, area abc = area dce. Since the angles at c are equal $ab = de$ and $x = ac = ce = \dfrac{h}{2}$; or, in words, the maximum unit compressive stress equals the maximum unit tensile stress and the neutral fiber is at mid-

depth. This is a very important fact to keep in mind; that the **neutral axis** (which is the trace of the neutral plane with the plane of a right section) **passes through the center of gravity** (or **centroid**) **of the cross-section.** The resultant compression and the resultant tension evidently act through the centroids of the triangles by which they are respectively represented and the lever arm of the resisting moment couple equals

$$a = \tfrac{2}{3}\,h.$$

The total compression equals the average compressive unit stress multiplied by the area over which the compression acts; thus

$$C = T = \tfrac{1}{2} f \times b \times \tfrac{1}{2}\,h$$

and the resisting moment equals

$$MR = C \cdot a = T \cdot a = \tfrac{1}{2} f \times b \times \tfrac{1}{2}\,h \times \frac{2\,h}{3} = \tfrac{1}{6} fbh^2,$$

which is the familiar expression derived by substituting the values for a rectangular cross-section in the general form of the relation, $M = \dfrac{fI}{c}$.

The moment of resistance (the couple formed by the internal fiber stresses) developed at any section of a beam equals the bending moment (the moment of the external forces acting on the beam to the right or to the left of the section) at that section, and so the expression

$$BM = MR = \tfrac{1}{6} fbh^2$$

gives a direct relation between the maximum fiber stress in a beam and the external loads, making it possible to proportion and investigate rectangular homogeneous beams so far as normal stress is concerned. The problem of shearing stress will be studied later.

43. Beams of Reinforced Concrete. A beam of plain concrete breaks under very small load on account of the weakness of the concrete in tension. Reinforced with steel rods as shown in Fig. 10 it will carry much more, the concrete cracking at the same load as though unreinforced but failure being prevented by the steel. These cracks usually appear somewhat as shown in the figure, inclined more and more toward the end of the beam.

Each crack is approximately normal to the lines of maximum tension in its portion of the beam, and it is this inclined tension that causes the crack. A large increase of load is possible if rods are placed crossing those sections where a failure might occur, as sketched in Fig. 27*b*. The problem of designing such web reinforcement is considered in a later section: at present the discussion is limited to the main tension steel.

An expression for the moment of resistance of a rectangular reinforced concrete beam can be derived following the general argument of the last article. It is customary to make two preliminary assumptions: that initial stresses set up by the shrinkage of the concrete are negligible and that the concrete carries no

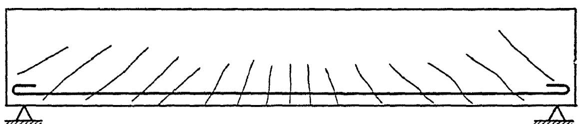


FIG. 10

tension. The latter is not strictly true as the concrete plainly will resist tension for a small distance below the neutral plane, where the elongation is not so large as to cause cracking, but it is on the side of safety and simplicity to make the assumption.

Consider the portion of a rectangular reinforced concrete beam to the left of any section mn such as that shown for a homogeneous beam in Fig. 9*c*. The resisting moment is considered to be supplied by the compression in the concrete and the tension in the steel since the tension in the concrete is neglected. As a plane section before bending remains plane after bending, the strain diagram is a straight line as before (Fig. 11*c*), the neutral fiber being an unknown distance x from the compressive face of the beam. To obtain the stress diagram (Fig. 11*d*) each abscissa of the strain curve above the neutral axis is multiplied by E_c, the modulus of elasticity for the concrete (assumed to be constant), giving the line ab, and the strain at the level of the steel, e_t, is multiplied by E_s, the modulus for steel, giving the abscissa $ec = f_s$, the thickness of the layer of steel being neglected. As in the case of the homogeneous beam, the total compression may be indicated as $C = \frac{1}{2} f_c bx$ acting at a distance $\frac{x}{3}$ from the compression face.

The total tension is $f_s A_s$ where f_s is the unit steel stress and A_s the steel area, and the arm of the couple is $a = d - \frac{x}{3}$. To evaluate x, the unknown distance of the neutral axis from the compressive face of the beam, the steel concrete section is transformed into one entirely of concrete, as shown in Fig. 11*b*, by replacing the steel with concrete or rather with a hypothetical

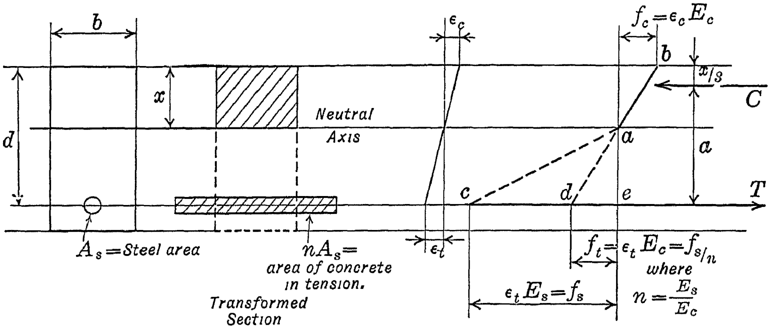


FIG. 11

concrete having the same modulus of elasticity as the concrete in the compression portion but differing from it in its assumed ability to carry tension. The area (nA_s) of this tension concrete is determined by the following considerations. This transformed beam is the elastic equivalent of the steel-concrete beam and therefore the strain curve is unchanged by the transformation. The stress curve is changed only below the neutral plane since the modulus of elasticity of the assumed tension-carrying concrete differs from that of steel and equals that of the compression part. Therefore the abscissa *ec* is replaced by $ed = E_c e_t$. The points *a–b–d* all lie on one straight line. The total tension equals $f_t n A_s$, where nA_s is the unknown amount of concrete used in substitution for the steel. The value of n is determined by making use of the fact that the total tension is the same for the steel-concrete beam and for the transformed homogeneous beam; that is, $f_s A_s = f_t n A_s$ or, putting stress in terms of strain, $E_s e_t A_s = E_c e_t n A_s$; whence $n = \frac{E_s}{E_c}$. Or, more briefly, since the deformation in the steel equals that in the tension concrete, the unit

stresses in the concrete and the steel bear to each other the ratio of their moduli; whence the tension areas must be to each other inversely as their moduli, since the total tension is the same in both cases.

Since the neutral axis of a homogeneous beam passes through the center of gravity of the cross-section, that of any given reinforced concrete beam, the steel area of which is known, may be located by determining the centroid of the transformed section, the shaded portions of Fig. 11*b*. Since the concrete of the original beam carries no tension its outline is shown by dotted lines below the neutral axis, where it serves simply to transmit shearing stresses. Knowing the value of x the rest of the problem follows directly.

Although no formulas have been written, all the means are now at hand for the design and investigation of reinforced concrete beams of any shape provided the section is symmetrical about the plane of bending. It is much easier to solve problems by the simple methods that follow logically and simply from the outline just given than it is to use formulas, unless plots and tables are at hand to use with the formulas.

44. Rectangular Beams with Tension Reinforcement. Three problems arise regarding rectangular concrete beams reinforced in tension only: (1) the design of a beam to carry a stated bending moment at given stresses; (2) investigation of the maximum stresses at a given section subjected to a known bending moment; (3) investigation of the maximum permissible bending moment for a given beam with certain limiting stresses given. It is better to consider the cases of investigation first. The problem of design is by far the most common in practice.

Example 5. The beam here shown (Fig. 12*a*) carries a bending moment of 40,000 ft.-lbs.; E_s = 30,000,000 lbs./sq. in.; E_c = 2,000,000 lbs./sq. in. What are the maximum fiber stresses?

Solution. Following the argument of the last article the transformed section is that here given (Fig. 12*b*). The neutral axis is located by noting that the statical moment of the compression area about that axis (which passes through the center of gravity) equals the statical moment of the tension concrete area about the same axis, giving the expression

$$\frac{x}{2}(10\,x) = 30\,(20 - x)$$

$$x^2 + 6\,x = 120$$

completing the square[1]

$$x^2 + 6x + 9 = 129$$
$$x = 11.36 - 3 = 8.4 \text{ in.}$$

Consequently the lever arm of the couple is 17.2 in. and as $MR = BM$

$$C = T = \frac{40{,}000 \times 12}{17.2} = 27{,}900 \text{ lbs.}$$

The maximum fiber stresses then are obtained as follows:

$$C = \tfrac{1}{2} f_c \times 10 \times 8.4 = 27{,}900 \text{ lbs.} \qquad f_c = 665 \text{ lbs./sq. in.}$$
$$T = f_s A_s = 2 f_s = 27{,}900 \text{ lbs.} \qquad f_s = 13{,}950 \text{ lbs./sq. in.}$$

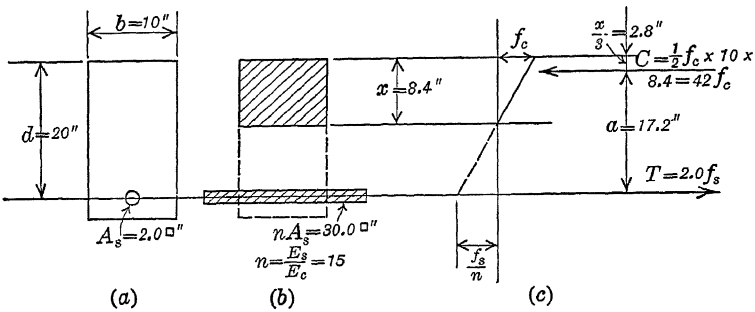


FIG. 12

Example 6. What is the maximum moment that can be carried by the beam of Example 5, the limiting fiber stresses being $f_s = 16{,}000$ lbs./sq. in. and $f_c = 650$ lbs./sq. in.? $n = E_s/E_c = 15$.[2]

Solution. As before, transform the section (Fig. 12*b*), locate the neutral axis and determine the arm of the resisting couple, $a = 17.2$ in. If f_c has its maximum value of 650 lbs./sq. in., C equals $\frac{1}{2} \times 650 \times 10 \times 8.4 = 27{,}300$ lbs.; if f_s has its maximum value T equals $16{,}000 \times 2 = 32{,}000$ lbs. If this last value is attained C also equals 32,000 lbs. and f_c exceeds the limit of 650 lbs./sq. in. It is necessary to limit C and T to 27,300 lbs. each, thus using the steel at a lower stress than the permissible, and the limiting moment accordingly is

$$27{,}300 \times 17.2 \times \tfrac{1}{12} = 39{,}100 \text{ ft.-lbs.}$$

Another, but slightly longer, method of carrying through this problem is to complete the stress diagram by assuming either unit stress as realized

[1] This method of solving a quadratic equation is logical and easily remembered, while solving by use of a memorized formula is a dangerous and foolish strain on that useful mental function, the memory.

[2] The standard notation uses the letter n to designate the ratio of the moduli and it will be employed for that purpose henceforth.

and determining by proportion the simultaneous value of the other. For example, assuming f_s at 16,000 lbs./sq. in. gives $\frac{16{,}000}{15} = 1067$ as the tension stress in the transformed section, the neutral axis of which has been located. Then[1]

$$f_c = 1067 \times \frac{8.4}{11.6} = 770 \text{ lbs./sq. in.}$$

Since this is greater than the allowable 650 the beam is limited by the strength of the concrete. It is not necessary to determine the stress in the steel since this is less than the allowable.

Example 7. Design a beam to carry a total moment of 40,000 ft.-lbs. with stresses of $f_s = 16{,}000$ lbs./sq. in., $f_c = 650$ lbs./sq. in., $n = E_s/E_c = 15$.

Solution. Since the most economical beam results when both materials are stressed to the limit, the problem is to determine the breadth, b, the depth to the steel, d, and the steel area, A_s, such that the given stresses

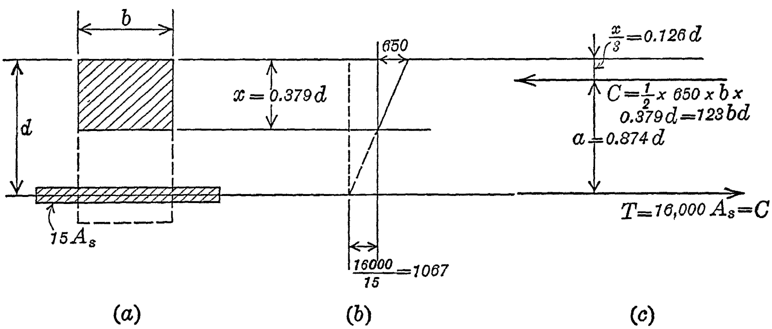


FIG. 13

will be realized simultaneously when the moment equals 40,000 ft.-lbs. The section and stress diagram then appear as shown in Fig. 13*a–b*. In order that these stresses be realized it is necessary that

$$x = d\left(\frac{650}{1067 + 650}\right) = 0.379\ d.^2$$

[1] Many students have difficulty in writing simple proportions dealing with similar triangles. They will fare better if they observe the rule of taking the unknown as the first term and the corresponding quantity in the *other* triangle as the second. This permits the equation above to be set down at once, whereas any other order is usually more difficult.

[2] The best usage requires that the decimal point be preceded by a cipher. See Proceedings American Concrete Institute, 1923, page 289, Art. 7*d*. There is less chance of error thus.

There results $a = 0.874\ d$ and the moment of resistance in terms of the concrete stress is

$$MR = \tfrac{1}{2} \times 650 \times b \times 0.379\ d \times 0.874\ d = 108\ bd^2$$

which must equal the bending moment. Therefore

$$bd^2 = \frac{40{,}000 \times 12}{108} = 4440$$

which condition is satisfied by $b = 10$ in., and $d = 21.1$ in.[1]

To obtain the steel area

$$C = \tfrac{1}{2} \times 650 \times 10 \times 0.379 \times 21.1 = 26{,}000 \text{ lbs.} = T = 16{,}000\ A_s$$
$$A_s = 1.63 \text{ sq. in.}$$

In practice the depth would commonly be made an integral number of inches. The following two problems are given therefore to illustrate the principles involved by this change from the theoretical dimensions.

Example 7a. What is the steel area required for the beam of Example 7 if d is made 22 in. and $b = 10$ in.?

Solution. Since the beam is deeper than required, the compression area furnished is in excess of that needed. The maximum concrete stress can be made lower than the limit and that of the steel equal to the limiting value by using the proper steel area as determined below. The stress

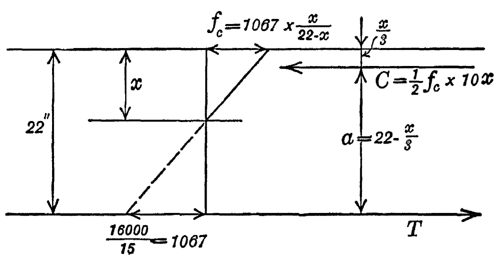


FIG. 14

diagram then is shown in Fig. 14. To determine x proceed as before, writing an expression for MR thus:

$$C \cdot a = \left[\tfrac{1}{2}\,(1067)\left(\frac{x}{22 - x}\right)(10\,x)\right]\left(22 - \frac{x}{3}\right) = 40{,}000 \times 12$$

$$x = 8.1 \text{ in.}$$

whence $$a = 19.3 \text{ in.}$$

and $$f_c = 620 \text{ lbs./sq. in.}$$

Then

$$C = T = \tfrac{1}{2} \times 620 \times 10 \times 8.1 = 25{,}100 \text{ lbs.}$$

[1] In small rectangular beams b is commonly made $\frac{1}{2}$ to $\frac{3}{4}$ of d, $\frac{1}{4}$ to $\frac{1}{3}$ in large beams.

and the steel area required

$$A_s = 25{,}100 \div 16{,}000 = 1.57 \text{ sq. in.}$$

Approximate Solution. The above exact method is laborious and in general unnecessary. Assuming that the lever arms of the couples vary as the depths (d), the steel area required for $d = 22$ in. is

$$A_s = 1.63 \times \frac{21.1}{22.0} = 1.57 \text{ sq. in.}$$

Example 7b. What is the steel area required for the beam of Example 7 if d is made 20 in. and $b = 10$ in.?

Solution. Since the beam is smaller than the theoretical, if enough steel is used to make the ratio of fiber stresses the same as before, the given stresses, 16,000–650, will be exceeded if the required moment of resistance is realized. The lever arm of the resisting couple is smaller than previously and in order to secure the necessary increase in the compres-

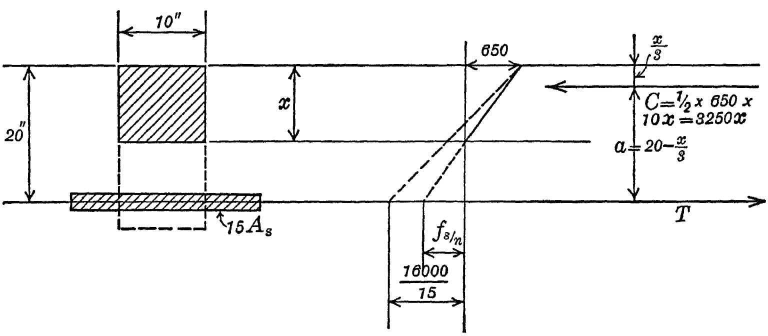


FIG. 15

sive force of the resisting couple the compression area must be made larger, the extreme fiber stress being limited. The lowering of the neutral axis to give this increased area is accomplished by making the steel area larger, for this lowers the center of gravity of the section. The steel stress will be less than the limiting value as is indicated by the stress curve of Fig. 15. The value of x is arrived at as before:

$$C = \tfrac{1}{2} \times 650 \times 10\,x = 3250\,x$$

$$C \cdot a = (3250\,x)\left(20 - \frac{x}{3}\right) = 40{,}000 \times 12$$

$$x = 8.6 \text{ in. and } a = 17.1 \text{ in.}$$

$$C = T = 40{,}000 \times 12 \div 17.1 = 28{,}100 \text{ lbs.}$$

$$f_s = 15\left(650 \times \frac{11.4}{8.6}\right) = 12{,}940 \text{ lbs./sq. in.}$$

$$A_s = 28{,}100 \div 12{,}940 = 2.17 \text{ sq. in.}$$

It is less labor to determine the steel area required by means of the known position of the neutral axis. Since this is the center of gravity of the cross-section the moment of the compression *area* about this unknown axis equals that of the tension *area*.

$$10 \times 8.6 \times 4.3 = 11.4 \times 15 A_s$$
$$A_s = 2.16 \text{ sq. in.}$$

When the depth is smaller than the theoretical there is too much error in the assumption of proportionality between depth, d, and lever arm, a, to permit the approximate solution previously used. For example, this assumption would give $A = 1.63 \times \frac{21.1}{20.0} = 1.72$ sq. in., where 2.16 sq. in. are actually required.

The several examples of this article have demonstrated these important facts:

(1) **In any given beam** $\left(b,\ d,\ A_s,\ n = \frac{E_s}{E_c} \text{ known}\right)$ **the neutral axis has a fixed position (through the centroid of the transformed section) and therefore the ratio of the maximum unit stresses in steel and concrete is constant.** It is evident this ratio is independent of any stresses that may be set as limits in future investigations of the beam.

(2) **An increase of tension steel area lowers the neutral (gravity) axis.**

(3) **A lower value of** $n = \frac{E_s}{E_c}$, **such as results from the increase of the concrete modulus with age, raises the neutral axis,** since the tension area of the transformed section is thereby decreased.

45. The Ratio $\frac{E_s}{E_c}$ in Beams. It will be noted that the previous examples have used a value of 15 for the ratio $\frac{E_s}{E_c}$ whereas Art. 30 would indicate that 10 is the correct value. This higher value is used in compensation for the assumption that the concrete can carry no tension. The actual stress-carrying cross-section of a beam would appear as in Fig. 16, the cross-hatched portions representing those subjected to normal stress. To locate the neutral axis correctly the omission of the tension concrete area is compensated for by using a larger tension steel area, that is, by mul-

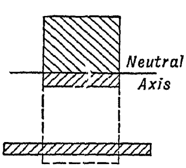


Fig. 16

tiplying the actual steel area by a larger value of the ratio of moduli. Tests show that the use of this higher value is proper and also that the common theory of design gives conservative results.

It should be kept in mind that, since the modulus of elasticity for the concrete has been assumed constant, it is not possible to compute the breaking strength or the effect of loads that cause high concrete stresses by the methods outlined here. For such problems it becomes necessary to consider the actual shape of the stress-strain curve for concrete.

46. Tee Beams. In practice tee beams usually form part of a floor system and act integrally with the slab on either side, which forms a flange giving added strength in the compressive part. When beams are widely spaced the compressive stresses are not uniform in intensity across the whole width of slab. In order to investigate or design the usual tee beam it is necessary to make some assumption regarding the width of slab that may reasonably be considered to act with the stem and be uniformly stressed over the whole width. If this width is assumed too large, not only will the total compression be given an exaggerated value provided enough steel is used to develop it, but also excessive shearing stresses will be induced at the junctions of the flange and stem. There is more or less disagreement among design specifications regarding the proper limit for flange width and the Joint Committee rule of 16 times the flange thickness plus the stem width is rather higher than is customary.

There are two approaches to the problem of designing a tee beam with the flange provided by a floor slab. One method considers the full width of flange available to be in action. Usually the compressive stress in the concrete is found by this assumption to be low in value. The other method assumes that the limiting working stresses are realized and that the width of slab called into play is only that necessary. Usually this width is less than the limit set by the codes. The neutral axis and the lever arm of the resisting couple have different values by these two methods. Of course the actual values are uncertain and neither assumption is more than a convenience which gives satisfactory results.

The design of T-shaped beams consists in proportioning the stem or web (the portion below the slab), and determining the tension steel area. Since proportioning the stem requires con-

sideration of the shearing stresses, that part of the problem is deferred until later. The following examples illustrate the determination of the required steel area in a given tee beam and the investigation of a given beam. These examples are carried through with greater precision than is necessary in order to illustrate clearly the method of analysis.

The design of an independent tee beam not connected with a floor slab is considered in Art. 48 and the design of tee beam stems in Chapter XII.

Example 8. Locate the neutral axis of this tee beam (Fig. 17), $E_s/E_c = n = 15$.

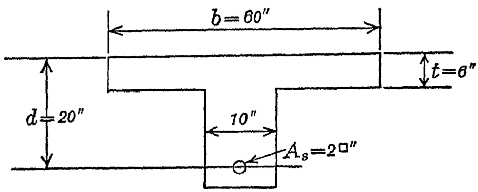


FIG. 17

Solution. The thickness of the flange leads one to suspect that the neutral axis falls within it, and accordingly the investigation proceeds as for a rectangular beam. Taking moments of the areas of the *transformed* section about the neutral axis:

$$\tfrac{1}{2}\,x(60\,x) = (15 \times 2)(20 - x)$$

whence

$$x = 4.0 \text{ in.}$$

It is plain that this is simply a rectangular beam with a portion of the concrete below the neutral axis removed, a change that does not affect the moment of resistance. The methods of Art. 44 suffice for this beam. Had the axis fallen in the stem the procedure would have been that of the succeeding examples.

Example 9. With a total moment of 40,000 ft.-lbs., what are the maximum fiber stresses in the tee beam of Fig. 17, the flange thickness being made $3\frac{1}{2}$ in. instead of 6 in.? $n = 15$.

Solution; Exact Method. (Considering the compression in the stem.) Sketch the transformed section (Fig. 18), and locate the center of gravity in order to find the neutral axis, dividing the compression area into the two parts indicated; taking moments about the unknown neutral axis:

$$(10\,x)(\tfrac{1}{2}\,x) + (60 - 10)(3.5)(x - 1\tfrac{3}{4}) = 30\,(20 - x)$$
$$x = 4.0 \text{ in.}$$

The line of action of the resultant compression may be located by taking moments about the top fiber, the total stress being equal to the compression that would exist did the slab have a thickness of 4 in., less the stress on the area below the actual slab.

$$
\begin{array}{rcr@{\;:\;}r@{\;:\;}r}
 & & \text{Compression} & \text{C} & \text{arm : moment} \\
(\tfrac{1}{2} f_c)(60 \times 4.0) & = & 120.0\, f_c & \tfrac{4}{3}\text{ in.} & 160.0\, f_c \\
(\tfrac{1}{2} \times \tfrac{1}{8} f_c)(50 \times 0.5) & = & \underline{1.6\, f_c} & 3\tfrac{2}{3}\text{ in.} & \underline{5.9\, f_c} \\
C = \text{total compression} & = & 118.4\, f_c\ ) & & \underline{154.1\, f_c} \\
\text{Distance of } C \text{ from top fiber} & & & & 1.3\text{ in.}
\end{array}
$$

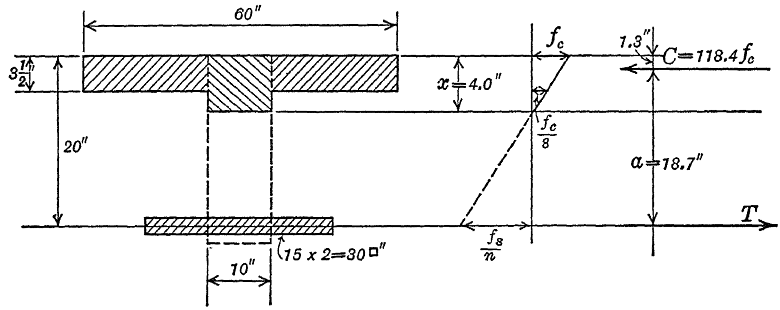


FIG. 18

The lever arm, a, equals $20 - 1.3 = 18.7$ in.

$$C = T = 40{,}000 \times 12 \div 18.7 = 25{,}700 \text{ lbs.}$$

The maximum unit stresses are

$$f_c = 25{,}700 \div 118.4 = 220 \text{ lbs./sq. in.}^{1}$$

$$f_s = \frac{25{,}700}{2.0} = 12{,}850 \text{ lbs./sq. in.}$$

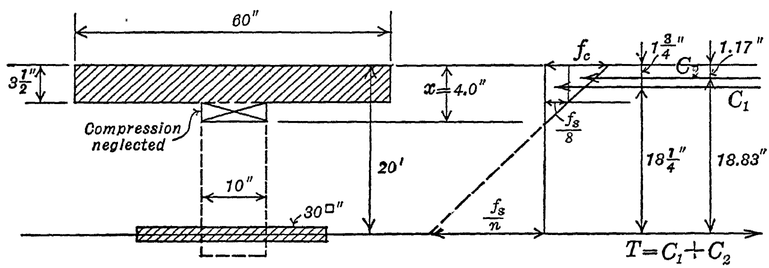


FIG. 19

Solution: Approximate Method. (Neglecting compression in the stem.) (Fig. 19.) By far the greater number of beams met with in practice are

[1] More precisely 217 lbs./sq. in. The third figure has no precision value.

of such proportions that the amount of compression in the stem below the flange is small compared with that in the flange and can be neglected with small error. On this basis the following expression gives the location of the neutral axis, taking moments about that unknown axis:

$$(60 \times 3.5)(x - 1.75) = 30\,(20 - x)$$
$$x = 4.0 \text{ in.}$$

Drawing the stress diagram, the intensity of the compression on the under edge of the slab is seen to be $f_c \times 0.5 \div 4 = f_c/8$. The total compression is the sum of two forces: C_1, acting with a uniform intensity of $f_c/8$ over the flange, and C_2, acting with an intensity varying from 0 to $\frac{7}{8} f_c$. Force C_1 acts at the center of the flange, 18.25 in. from the tension steel, and force C_2 acts at a distance of $(20 - 3.5/3) = 18.83$ in. from the steel. Instead of computing the lever arm of the resultant as in the previous example, some prefer to consider the moment of resistance as consisting of two couples, 18.25 C_1 and 18.83 C_2. The total moment of resistance in terms of the unknown f_c then is:

Compression:		C: arm : moment
$C_1 = (f_c/8)(60 \times 3.5)$	=	$26.2\,f_c$: 18.25: $478\,f_c$
$C_2 = (\frac{1}{2} \times 7\,f_c/8)(60 \times 3.5)$	=	$91.8\,f_c$: 18.83: $1730\,f_c$
$C_1 + C_2 = C$	=	$118.0\,f_c$ $MR = 2208\,f_c$

Then $2208\,f_c = 40{,}000 \times 12$ and $f_c = 220$ lbs./sq. in. (217 lbs./sq. in.).

$$C = 118 \times 217 = 25{,}600 \text{ lbs.} \quad \text{and} \quad f_s = 12{,}800 \text{ lbs./sq. in.}$$

It is equally simple to determine the lever arm of the resultant compression, $2208\,f_c \div 118\,f_c = 18.7$ in., and proceed as illustrated above in the exact solution.

Example 10. What is the maximum moment that can be carried by the beam of Example 9? (Fig. 17 with $t = 3\frac{1}{2}$ in.) Limiting fiber stresses are $f_s = 16{,}000$ lbs./sq. in., $f_c = 650$ lbs./sq. in. $n = 15$.

Solution. The exact method is so seldom used further illustration of it will not be made. As in the preceding problem, determine the location of the neutral axis and the lever arm of the resisting couple. The limiting value of C is $118 \times 650 = 76{,}600$ lbs.; that of T is $16{,}000 \times 2 = 32{,}000$ lbs., which limits. Accordingly the maximum moment of resistance is $32{,}000 \times 18.7 \div 12 = 49{,}800$ ft.-lbs.

Example 11. A beam of the same dimensions as that of Example 9 (Fig. 17 with $t = 3\frac{1}{2}$ in.) carries a moment of 100,000 ft.-lbs. The limiting fibre stresses are $f_s = 16{,}000$ lbs./sq. in., $f_c = 650$ lbs./sq. in. $n = 15$. What is the steel area required?

Solution. Any other than a cut-and-try solution would be exceedingly

cumbersome. It is simple to assume the value of the lever arm of the couple and determine the area of steel required for a fiber stress of 16,000 lbs./sq. in. The results may be checked by either the exact or the approximate method as desired and the steel area corrected if necessary.

In a beam where the compression in the stem is so small that it may be neglected, the lower the neutral axis lies, the nearer the line of action of the resultant compression approaches the center of the flange. If the compression is assumed to act at this point the corresponding lever arm will be smaller than can ever be realized actually, and the values of C and T larger than the actual. Making this conservative and very common assumption (*i.e.*, that the lever arm equals the depth to the steel less half the flange thickness),

$$T = C = 100{,}000 \times 12 \div 18.25 = 65{,}700 \text{ lbs.}$$
$$\text{Steel area } A_s = 65{,}700 \div 16{,}000 = 4.11 \text{ sq. in.}$$

The average stress in the concrete is $65{,}700 \div (60 \times 3.5) = 310$ lbs./sq. in. A brief mental calculation shows that the neutral axis is below the flange[1] and so the maximum concrete stress is less than twice the average, and less than 650 lbs./sq. in., the exact value being a matter of indifference.

The results thus obtained are safe but not economical. A check by the approximate method as in Example 9 will indicate what change to make. A better approximation for the lever arm is 0.9 d, 18 in. in this problem.

All of the examples so far have dealt with beams of fixed dimensions and correspond to the situation when the width of slab acting as flange is definitely assumed. The following examples illustrate the second method with working stress limits assumed to be realized.

Example 10a. (Same as Ex. 10 except that width of flange is not given.) What is the maximum moment that can be carried by the beam shown in Fig. 20 with limiting fiber stresses of $f_s = 16{,}000$ lbs./sq. in. and $f_c = 650$ lbs./sq. in.? $n = 15$.

Solution. As shown in Fig. 20 the stress diagram is drawn with extreme stresses taken as equal to the given limits, which locates the neutral axis. Following the procedure of Ex. 9, considering the compression acting on 1 in. width of flange:

[1] Taking moments about the bottom of the flange, assuming it as 4 inches thick for convenience and the steel area as 4 square inches

$$60 \times 4 \times 2 \text{ is less than } 15 \times 4 \times 16$$

showing the centroid to be more than 4 inches from the top.

Compression (C)	Arm	Moment about top
$3.5 \times 350 = 1225$	$3.5 \times \frac{1}{2} = 1.75$ in.	2140
$\frac{1}{2} \times 3.5 \times 300 = 525$	$3.5 \times \frac{1}{3} = 1.17$	615
1750 lbs./in.	)	2755
Distance of C from top of beam		1.6 in.

Lever arm of resisting couple $= a = 20 - 1.6 =$ 18.4 in.

Maximum moment of resistance $= T \cdot a = 2 \times 16{,}000 \times 18.4/12$
$= 49{,}100$ ft.-lbs.

Check of compressive stress; width of flange limit $= 16\,t + b' = 66$ in.

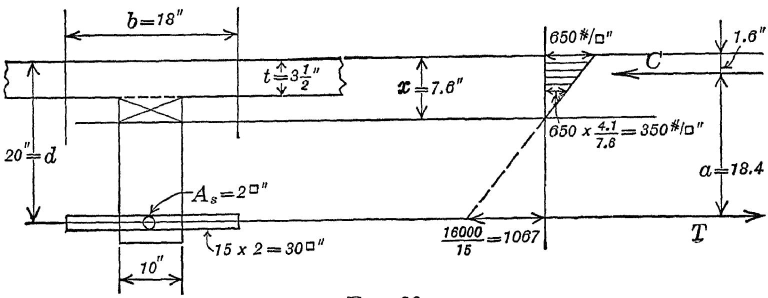


Fig. 20

Width of flange required:

$$C = T = 32{,}000 \text{ lbs.} = \tfrac{1}{2}\,(650 + 350) \times 3.5\,b$$
$$b = 18 \text{ in.}$$

which is less than the limit. The same result follows by using the fact that the neutral axis passes through the center of gravity of the cross-section.

Example 11a. (Same as Ex. 11 except that the width of flange is not given.) A beam of the same dimensions as that shown in Fig. 20 carries a moment of 100,000 ft.-lbs. The limiting fiber stresses are $f_s = 16{,}000$ lbs./sq. in., and $f_c = 650$ lbs./sq. in. $n = 15$. What steel area is required?

Solution. As in the previous example the stress diagram is constructed, the neutral axis and the lever arm of the resisting couple calculated. Then

$$T = 100{,}000 \times 12 \div 18.4 = 65{,}200 \text{ lbs.}$$
$$\text{Steel area} = 65{,}200 \div 16{,}000 = 4.07 \text{ sq. in.}$$

Check on width of flange:

$$C = \tfrac{1}{2}\,(650 + 350) \times 3.5\,b = 65{,}200$$
$$b = 37 \text{ in.}$$

which is less than the limit allowed.

47. Beams Reinforced for Both Tension and Compression. A concrete compression member is both stiffened and strengthened by longitudinal steel reinforcement, provided that this reinforcement is properly restrained from buckling. The two materials will deform equally, the stress being transmitted to the steel by the bond between the steel and the concrete. Accordingly the unit stress in the compression steel equals that in the concrete at that point multiplied by the value of n. The steel in the compression part of a beam acts in the same way. However such reinforcement is not economical since the ratio of the cost per unit volume of the steel as compared with the concrete is always greater than the value of n, the ratio of the stresses in the two materials at the same point.

Example 12. What are the maximum fibre stresses in this double-reinforced beam? (Fig. 21.) The bending moment equals 40,000 ft.-lbs. $n = 15$.

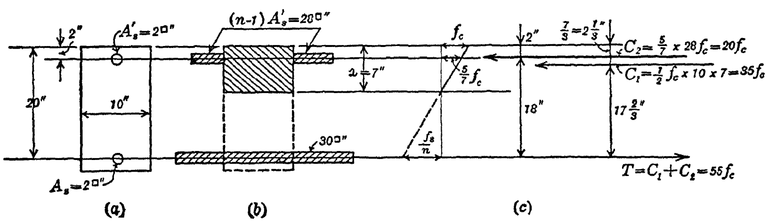


FIG. 21

Solution. To transform the steel-concrete section into its equivalent in concrete each area of steel is replaced by $15 \times 2 = 30$ sq. in. of concrete at the same level in the beam as the steel it replaces. On removing the compression steel a hole of 2 sq. in. is left in the concrete which requires an equal amount of the added concrete to fill, leaving 28 sq. in. in the compression wings. The neutral axis is found as usual

$$\left(\frac{x}{2}\right)(10\,x) + 28\,(x - 2) = 30\,(20 - x)$$

$$x = 7.0 \text{ in.}$$

The total compression is considered to be the sum of two elements, C_1 the compression on the rectangular 70 sq. in., and C_2 that on the 28 sq. in. of the wings, where the stress intensity is $\frac{5}{7} f_c$ (Fig. 21c). The lever arm of the couple is the distance of this resultant compression from the tension steel; (see Fig. 21 for values used):

$$
\begin{aligned}
C_2 \times 18 &= 20\,f_c \times 18 &&= 360\,f_c \\
C_1 \times 17\tfrac{2}{3} &= 35\,f_c \times 17\tfrac{2}{3} &&= 618\,f_c \\
& 55\,f_c \quad) && 978\,f_c \\
& a = && 17.8 \text{ in.}
\end{aligned}
$$

$$
\begin{aligned}
T &= C = 40{,}000 \times 12 \div 17.8 = 27{,}000 \text{ lbs.} \\
f_c &= 27{,}000 \div 55 = 490 \text{ lbs./sq. in.} \\
f_s &= 27{,}000 \div 2 = 13{,}500 \text{ lbs./sq. in.}
\end{aligned}
$$

In replacing the compression steel with concrete the usual practice[1] is to neglect the hole left by removal of the steel and assume the area added in the wings as n times the steel area. This is not unreasonable considering the uncertainty that exists regarding the true value of the modulus of elasticity of the concrete. However, this approximation is never made in column design.

Example 13. If the limiting fiber stresses are $f_s = 16{,}000$ lbs./sq. in., $f_c = 650$ lbs./sq. in., what is the maximum moment of resistance of the beam of Example 12? (Fig. 21.) $n = 15$.

Solution. The transformed section is sketched and the neutral axis located as in Example 12. The procedure is the same as in Example 6.

Answer. 47,400 ft.-lbs.

The design of a double-reinforced beam to meet given conditions is considered in the next article.

48. Beams of Limited Size. A beam is said to be of balanced design when both the tension steel and the concrete are stressed to their working limits simultaneously. It often happens that a beam must not exceed certain dimensions which are inadequate to provide the desired strength with a balanced design, using tension reinforcement only. Three different methods of strengthening such a beam follow:

Example 14. The beam shown in Fig. 22 carries a moment of 60,000 ft.-lbs. Limiting stresses are $f_s = 16{,}000$ lbs./sq. in., $f_c = 650$ lbs./sq. in. $n = 15$. What area of tension steel is required?

Solution. The first problem is to determine whether the beam is larger or smaller than that theoretically needed to carry the given bending moment with the given fiber stresses, which is easily done as in Example 7 (page 63), where for the same limiting stresses the moment of resist-

[1] The formulas presented by the Joint Committee are on this approximate basis, Art. 104*b*, Appendix B.

ance was shown to be 108 bd^2. Applying this information here this problem gives:

$$\text{Maximum } MR = 108 \times 10 \times 20^2 \div 12 = 36{,}000 \text{ ft.-lbs.}$$

Knowing the beam to be smaller than the theoretical and proceeding as in Example 7b (page 65), the result is 13.6 sq. in. of steel required, the steel stress coming out 3500 lbs./sq. in. Whether or not it is possible to provide this amount of steel in the beam is not a matter of concern here.

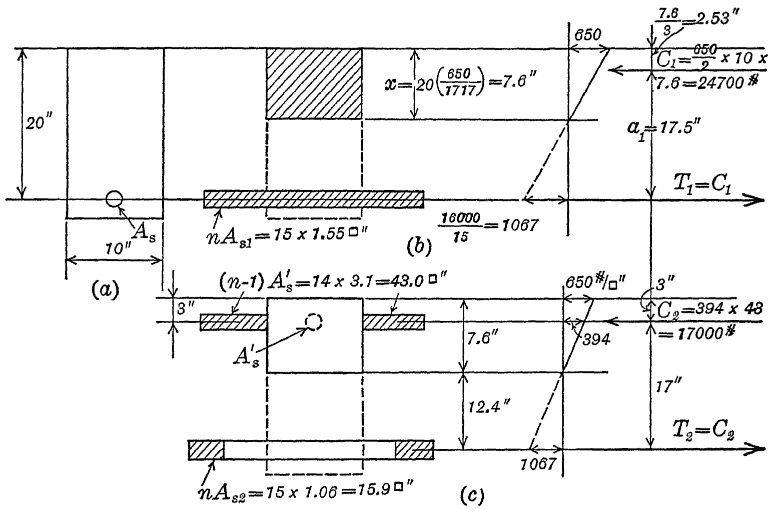


FIG. 22

Example 15. Data of Example 14. How much steel is required using both tension and compression reinforcement, the latter being placed 3 in. from the compression face?

Solution. The most economical design will employ both materials up to their working limit and the stress diagram, accordingly, will be as shown in Fig. 22*b–c*, with neutral axis 7.6 in. from the top. As in the beam discussed in Article 47 the moment of resistance is considered to consist of two couples, one in terms of the compression on the rectangular part of the transformed section (see Fig. 21) and the other in terms of the compression on the projecting wings. The first step is to determine the maximum moment the beam can carry without compression reinforcement. From the data of Fig. 22*a–b* it is possible to write at once:

$$MR = C_1 \cdot a_1 = 24{,}700 \times 17.5 \div 12 = 36{,}000 \text{ ft.-lbs.}$$

The tension steel area required for this moment is

$$A_{s1} = 24{,}700 \div 16{,}000 = 1.55 \text{ sq. in.}$$

The difference between 60,000 ft.-lbs. and 36,000 ft.-lbs., 24,000 ft.-lbs., must be provided for by adding A_s' sq. in. of compression steel, whose effect on the transformed section is to add reinforcing wings of area $(n - 1)\ A_s'$ sq. in. (Compare Fig. 21*b* and the first paragraph of Example 12.)

As shown in Fig. 22*c*, these concrete wings are 17 in. from the tension steel and at a level where the compressive stress is 394 lbs./sq. in. The total compression acting on these wings is $24{,}000 \times 12 \div 17 = 17{,}000$ lbs. The compression steel area required is

$$14\ A_s' = 17{,}000 \div 394 = 43.0 \text{ sq. in.}$$
$$A_s' = 3.1 \text{ sq. in.}$$

The effect of adding this steel alone would be to shift the neutral axis, which is fixed by the conditions as noted. Since the neutral axis passes through the center of gravity of the transformed section, the addition of 43 sq. in. at 4.6 in. from that axis must be balanced by adding $43 \times 4.6 \div 12.4 = 15.9$ sq. in. $= nA_{s2}'$ sq. in. on the other side, at the level of the tension steel. Whence $A_2 = 1.06$ sq. in. of steel, making the total tension steel area

$$A_{s1} + A_{s2} = A_s = (1.55 + 1.06) = 2.61 \text{ sq. in.}$$

The tension steel area can be found more easily thus:

$$A_s = \frac{T_1 + T_2}{f_s} = (24{,}700 + 17{,}000) \div 16{,}000 = 2.61 \text{ sq. in.}$$

The total steel area required by this method of reinforcement is 5.7 sq. in. as against 13.6 sq. in. needed if only tension steel is used. Evidently only a relatively small excess moment can be cared for economically with tensile reinforcement only.

Example 16. Data of Example 14 except that there is no limit to width. Design a tee beam to satisfy the requirements.

Solution. Fig. 23. Assume that the stem has been designed and a width of 10 in. determined upon. As in the previous example, the rectangular portion of the beam, 10 in. wide, can carry a moment of 36,000 ft.-lbs. with the given limiting stresses, steel area = 1.55 sq. in., leaving 24,000 ft.-lbs. to be carried by the compression in the flanges and the tension in the extra tension steel. Assume the flange 7.6 in. deep, thus keeping the beam as narrow as possible. Let its width equal x in. Since the arms of the tee are here brought down to the neutral axis the total width of the flange, x, is best found by one operation:

$$60{,}000 \times 12 \div 17.5 = \tfrac{1}{2} \times 650 \times 7.6\, x$$
$$x = 16.7 \text{ in.}$$

Then

$$C = \tfrac{1}{2} \times 650 \times 7.6 \times 16.7 = 41{,}200 \text{ lbs.}$$

Had these arms been made thinner the procedure of the previous example would have offered a quick solution. The total tension steel equals 41,200 ÷ 16,000 = 2.58 sq. in.

In practice the limiting depth of a beam given is the over-all concrete dimension and it is necessary for the designer to choose the depth to the steel such that it will conform to the proper

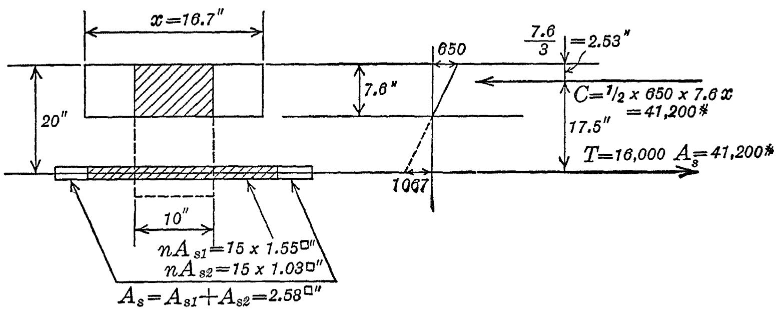


FIG. 23

placing of the reinforcing rods used. This is one of the inevitable cut-and-try problems that is solved easily only with experience.

49. Bending of Longitudinal Tension Steel. Beams of reinforced concrete are usually of uniform section over their whole length and accordingly the area of longitudinal steel required at the section of maximum bending moment is greater than that required elsewhere. When this maximum area is supplied by two or more bars it is possible to dispense with some of them when not needed for main tension reinforcement. This is done either by bending the surplus bars up into the web to act as reinforcement there, or by cutting them off at some point beyond where they are needed, as cover plates are cut on plate girders. The ends of the cut bars should be bent up to the neutral axis or hooked to give anchorage.[1] Many designers require that this be done in such manner as to keep the beam symmetrical about the vertical axis at all sections.

The point where a bar becomes unnecessary may be located by computing the moment of resistance of the section with that bar omitted and finding where the bending moment equals that moment of resistance. This method is cumbersome, however, and is

[1] See Art. 95, p. 203.

not used. As has already been pointed out the moment of resistance equals the total tension times the lever arm of the resisting couple; or

$$BM = MR = T \cdot a = f_s \cdot A_s \cdot a$$

using the notation already familiar. The arm, a, varies so little for varying tension steel areas that it may be assumed constant without serious error. With this assumption it becomes possible to lay down the principle that **the area of steel required varies directly with the bending moment and the same curve by proper choice of scale may serve both as moment curve and area-required curve.** Usually practical considerations as to commercial size of bars result in the maximum area furnished being larger than that required. It is common to neglect this difference and compute bar lengths as though the maximum area furnished equalled that required.

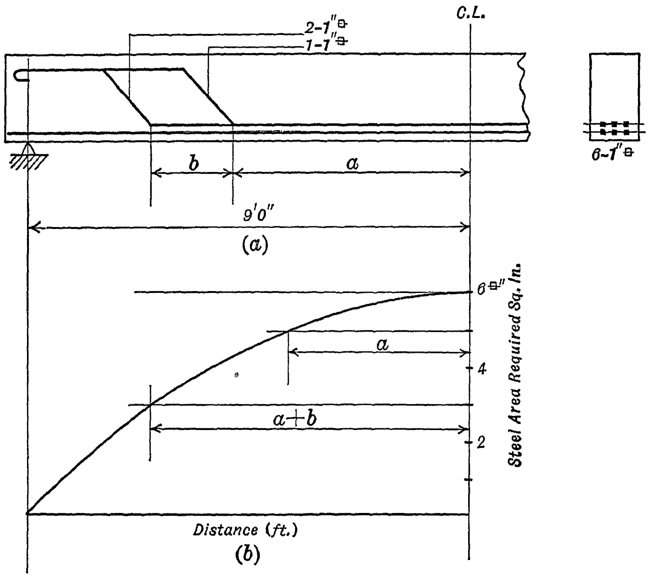


FIG. 24

Example 17. This reinforced concrete beam (Fig. 24) carries a uniform load. What are the minimum possible values of the dimensions a and b?

Solution. The bending moment curve for this loading is a parabola with maximum ordinate at the center and the area-required curve ac-

cordingly is the same. The parabola of Fig. 24*b* is drawn with the center ordinate representing 6 sq. in. One bar may be bent up when 5 sq. in. only are required; so[1]

$$a = 9\sqrt{\tfrac{1}{6}} = 3.7 \text{ ft.}$$

Similarly

$$a + b = 9\sqrt{\tfrac{3}{6}} = 6.4 \text{ ft.}$$

50. Shearing Stresses in Homogeneous Beams. A brief review of the shearing stresses in homogeneous beams is desirable in order that a clear picture may be obtained of the web stresses in beams of all kinds. For rigorous demonstration of these matters the reader should consult the standard treatises on the strength of materials.

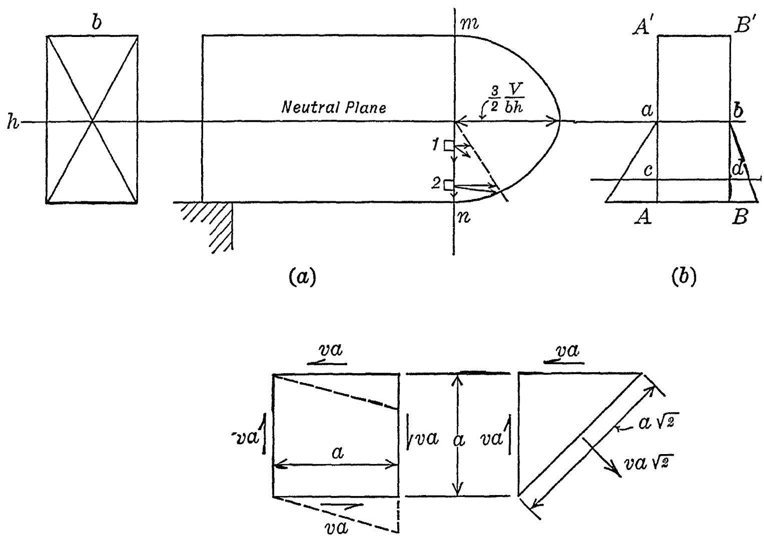


Fig. 25

In a previous article the law governing the variation of normal stress intensity on any section was discussed and the present problem is the variation of the intensity of the tangential or shearing stress, the total value of which at any section equals the external shear (V). It is easily proved by study of the shearing stress on an elementary prism that the intensity of the vertical shear (see Fig. 25*c*) and that of the horizontal shear at any point

[1] For method of dealing with the parabola see note to Problem 4, page 45.

are equal. A knowledge of the variation of horizontal shear intensity, therefore, gives also that of vertical shear. In Fig. 25*b* is shown a portion of a rectangular beam lying between any two sections, AA' and BB'. The variation of normal tension intensity at each section is indicated by the partial stress diagrams, the abscissas on AA' being shown larger than those on BB' on the assumption that the moment at AA' is the larger. Considering the stability of the small piece of beam $cdBA$, the pull on the cA face is larger than that on the dB face and the only force available to balance the difference is the horizontal shear on the plane cd. A brief consideration of the problem shows that the nearer the cd plane is to the neutral plane ab, the larger is the difference between the two tensions, and the larger the horizontal shear. Therefore the horizontal shear, and accordingly also the vertical shear, increase in intensity at a decreasing rate from zero at the extreme fiber to a maximum at the neutral plane. For a rectangular section the law of this variation is a parabola (Fig. 25*a*) with a maximum intensity of $\frac{3}{2}\frac{V}{bh}$.

The resultant intensity of stress at any point away from the extreme fibers, as, for example, on the vertical faces of the elementary prisms 1 and 2, Fig. 25*a*, must be inclined in direction, acting somewhat as shown. Referring again to the elementary prism shown in Fig. 25*c*–*d*, the shearing forces there shown may be resolved into components along the diagonals, and these components may be combined to give inclined tensile and compressive forces acting at 45 degrees (Fig. 25*d*) with an intensity (v) equal to that of the shear. This illustrates the case when the prism lies at the neutral plane where there is no direct stress. When it lies in the face of the beam there are no horizontal nor vertical shearing stresses and the resultant tension is horizontal, being that given by the usual formula for fiber stress.

A more detailed study of the state of stress at any point in this cross-section would show that passing through it are two inclined planes, 90 degrees apart, on which there is no shear, the resultant stress being compression on one and tension on the other, of an intensity greater than on any other plane through the point. These stresses are called the principal stresses at the point. Midway between these planes are those of maximum shear intensity.

In the web of a plate girder the action of the inclined tension

is easily resisted by the steel but the diagonal compression tends to cause buckling and it is necessary to limit the minimum thickness of the web or to provide suitably spaced stiffeners, or both. In a concrete beam, on the other hand, the material easily resists the diagonal compression but is weak in tension. The wooden beam resists both.

51. Shearing Stresses in Reinforced Concrete Beams. The variation of shear in a rectangular reinforced concrete beam may be ascertained by considering a small portion of such a beam between any two sections a small distance, ds, apart, as shown in Fig. 26a, the breadth of the beam being taken as b inches. The forces acting on this bit of beam consist of the normal stresses (C and T) and the shear (V), it being assumed that the moment at BB' is larger than that at AA', and that the sections are so close together that the two shears may be considered equal. These forces are in equilibrium, and applying the condition $\Sigma M = 0$, there results:

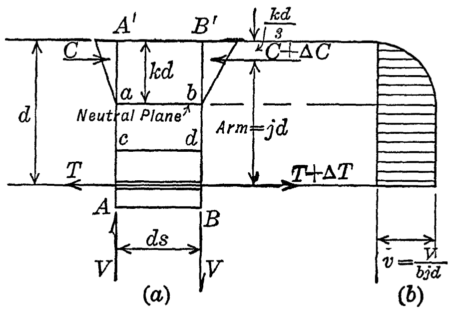


FIG. 26

$$\Delta T \times jd = V \times ds.$$

The tendency of the small portion of the beam $cdBA$ to be pulled to the right is resisted by the horizontal shear on the cd plane which may be expressed as the intensity of shear on that plane (v), assumed to be uniform, multiplied by the area bds. Then

$$v \cdot b \cdot ds = \Delta T.$$

Combining these two equations gives:

$$v \cdot b \cdot ds \cdot jd = Vds$$

and

$$v = \frac{V}{bjd}. \tag{1}$$

As the concrete is assumed to take no tension, the shear intensity is constant between the neutral plane and the steel while above that plane it varies as in a homogeneous rectangular beam

(Fig. 26*b*). Accordingly Equation (1) gives the maximum intensity of horizontal, and likewise of vertical, shear (as explained in Art. 50) at any section of a rectangular reinforced concrete beam.

This demonstration applies equally well in essential details to a rectangular concrete beam reinforced for both tension and compression and to a reinforced concrete tee beam. Tests confirm the conclusion that in the matter of shear a tee beam may be considered as equivalent to a rectangular beam of the same depth, with a width equal to that of the stem of the tee beam. The standard notation for this tee beam stem width is b' and so for tee beams the formula is written

$$v = \frac{V}{b'jd}. \tag{1a}$$

The value of j does not vary greatly for a wide range of conditions and an average value of $\frac{7}{8}$ or 0.86 is usually taken for all shear computations. Since all computations in which the value of the shear is used are highly approximate greater precision than that obtained by the average value is unnecessary.

52. Diagonal Tension in Reinforced Concrete Beams. The concrete in a reinforced beam is no stronger in itself than when unreinforced and it cracks in any loaded beam when the tensile limit is exceeded, the line of cracking being indicated in a general way in Fig. 10, sloping more steeply toward the ends of the beam, tending to lie at right angles to the inclined web stress. The function of the reinforcement is not to prevent cracking, that being impossible, but to keep any one crack from opening up widely, thus compelling the formation of many minute cracks in place of a single large one which would cause failure.

It is plain that so long as the cracks are vertical the horizontal bars are effective reinforcement, but where they are inclined horizontal bars are very ineffective, there being nothing but concrete to carry the vertical component of the inclined tension. When a beam is reinforced for normal stress only, failure occurs under small load somewhat as pictured in Fig. 27*a*, the part of the beam toward the center dropping below the end portion. To be accurate the sketch should show only gradual curves in the steel. There is insufficient strength in the concrete below the rods to the left of the rupture to resist the pressure brought upon it, and it spalls off in such a failure.

A beam is made secure against diagonal tension failure by supplying it with a sufficient amount of reinforcement, so placed as to cross a sufficient number of the inclined lines of potential failure. The more nearly perpendicular to the cracks the more effective are the rods. In practice use is made of stirrups (Fig. 27*b*), generally vertical, looped about the main steel, and of main longitudinal rods bent up at an angle across the region of diagonal

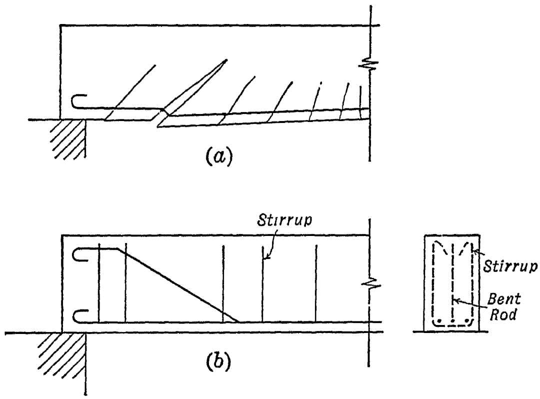


Fig. 27

tension stress in those portions of the beam where they are no longer needed to resist the normal tension. In order to proportion such reinforcement knowledge must be had of the amount of the diagonal tension. Unfortunately this cannot be computed accurately in a reinforced concrete beam since the concrete cracks irregularly and just how much tension is taken by the steel it is impossible to say. If there were no normal tension on any section below the neutral axis the maximum diagonal tension would act at 45 degrees and have an intensity equal to that of the shear at the section. This is always the assumption made in design.

In all discussions of diagonal tension these words from the 1916 Report of the Joint Committee should be kept in mind:

"In designing, resource is had to the use of calculated vertical shearing stresses as a means of comparing or measuring the diagonal tension stresses developed, it being understood that the vertical shearing stress is not the numerical equivalent of the diagonal tensile stress, and that there is not even a constant ratio between them. ... It does not seem feasible to make a complete analysis of the action of web reinforcement and more or less empirical methods of calculation are therefore employed."

Study of tests indicate that the concrete is effective in resisting small amounts of diagonal tension and may be counted on with safety to perform this duty unaided when the shearing stress is less than about 2 per cent of the ultimate compressive strength of the concrete, about 40 pounds per square inch for ordinary 1–2–4 mixes. When the shearing stress exceeds this limit, the concrete is ordinarily still counted on as carrying a portion of the diagonal tension.

The use of the shear as a measure of the diagonal tension accounts for the fact that diagonal tension failure and diagonal tension reinforcement are very commonly, and erroneously, spoken of as shear failure and shear reinforcement. It is hardly worth while to quarrel with this usage so long as it is held clearly in mind exactly what the terms refer to.

53. Stresses in Diagonal Tension Reinforcement. No entirely satisfactory and consistent theory of the action of web reinforcement of concrete beams has yet been devised and very likely never will be. The usual methods as here presented are frankly approximate, little more than empirical rules that experience shows give safe and reasonably economical results.

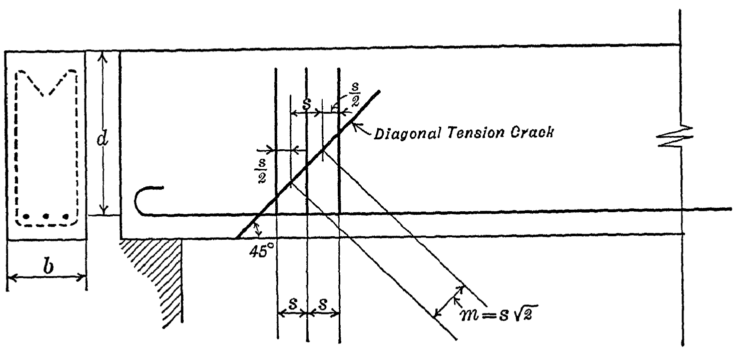


FIG. 28

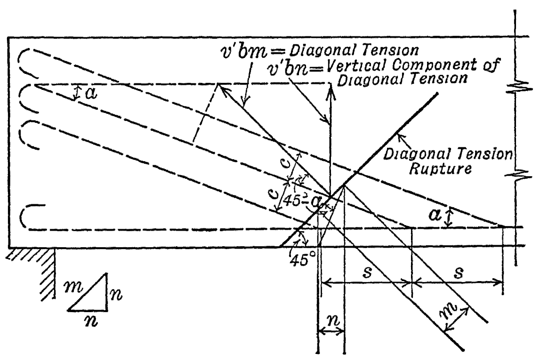
(a) *Vertical Stirrups.* The center stirrup of the three shown in Fig. 28 may be assumed to carry all, or part, of the vertical component of diagonal tension acting in the distance m along the 45 degree line of potential rupture. The horizontal opening of the crack is prevented by the longitudinal steel which may be considered, accordingly, to carry the horizontal component of the inclined tension. To rate the stirrup as carrying all of the ver-

tical component of diagonal tension, however, would be to underestimate the strength of the concrete. Tests show that it is proper to consider that the concrete assists the stirrups by carrying either a fixed amount or a given proportion of the diagonal tension, one-third being the fraction recommended by the 1916 Joint Committee.

The total amount of diagonal tension in the distance m is vbm, v being the average intensity of shear and also of diagonal tension in the distance s, and the vertical component of diagonal tension is vbs. The stress (S) in the stirrup then is:

$$S = v'bs \tag{2}$$

where v' is the amount of shear measuring the share of the diagonal tension carried by the steel, $\frac{2\,v}{3}$ by the 1916 Joint Committee ruling.



From *Journal of Boston Society of Civil Engineers*, Feb., 1925

$$c = s \sin a \qquad\qquad n = m \cos 45°$$

$$m = c \div \cos(45° - a) \qquad\qquad = \frac{s \sin a \cos 45°}{\cos 45° \cos a + \sin 45° \sin a}$$

$$= \frac{s \sin a}{\cos(45° - a)} \qquad\qquad = \frac{s \sin a}{\cos a + \sin a}$$

FIG. 29

The expressions, vbs and $v'bs$, should be translated into words and remembered, not arbitrarily, but as the statement of an easily visualized relation: **the vertical component of diagonal tension in any distance along a reinforced concrete beam is taken as equal to the horizontal shear in that distance.**

(*b*) *Inclined Rods.* The middle inclined rod of the three shown in Fig. 29 is affected by that portion of the diagonal tension that acts in the distance m along the 45 degree line of rupture. The total amount of this diagonal tension is vbm and the amount causing stress in the bar is $v'bm$. There are two common ways of determining the stress caused in the inclined rod by this diagonal tension: one method of analysis asserts that the vertical component of diagonal tension is the vertical component of the stress in the bar; the other that the stress in the bar is that component of the diagonal tension that is parallel to the steel, the other component being perpendicular to it. By the first method the stress, S, in the rod is (see Fig. 29):

$$S = \frac{v'bn}{\sin a} = \frac{v'bs}{(\sin a + \cos a)}. \tag{3}$$

By the second method

$$S = v'bm \cdot \cos (45^\circ - a) = v'bs \cdot \sin a. \tag{4}$$

For a 45 degree slope the two methods give identical results. The second method is equivalent to saying that the vertical component of diagonal tension in a distance along the beam equal to the longitudinal spacing (s) of the bars is resolved into two components (see Fig. 30), one normal to the steel, resisted by the diagonal compression in the concrete, the other along the bent rod and carried in part by it ($v'bs$ sin a) and in part by the concrete.

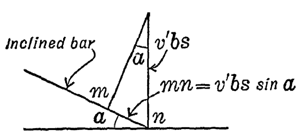


FIG. 30

It is plain that the flatter the slope of the bar the less effective the 45° diagonal compression in resisting the load thrown on it. Accordingly this method of analysis does not lend itself to approval.

By substituting the value of $v'b = \frac{V'}{jd}$ (see Art. 51, Eq. 1) in Equations (3) and (4) they take the form given them in most textbooks and directly or indirectly in the various Joint Committee reports:

$$S = \frac{v'bs}{(\sin a + \cos a)} = \frac{V's}{jd(\sin a + \cos a)} \tag{5}$$

which is the expression indicated in the 1924 Joint Committee report for values of a between 15 degrees and 45 degrees.

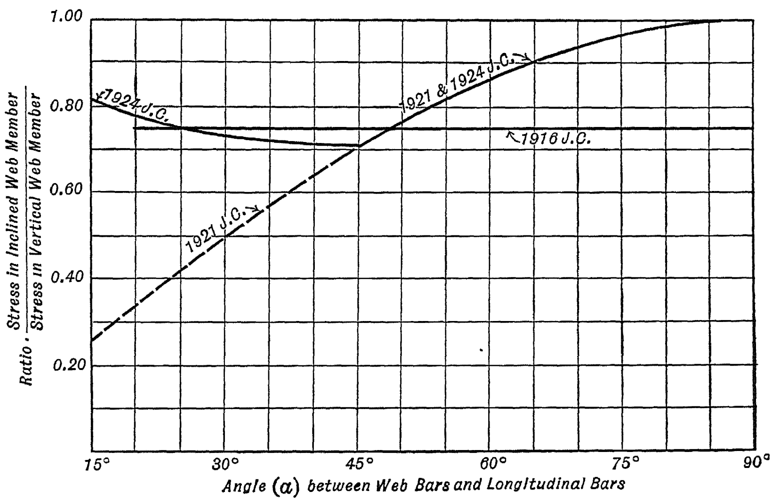
For (4)

$$S = v'bs \cdot \sin a = \frac{V's}{jd}(\sin a) \tag{6}$$

the expression indicated in the 1924 report for values of a between 45 degrees and 90 degrees, and in the 1921 report between 20 degrees and 90 degrees. The 1916 Joint Committee recommends a single value for all slopes

$$S = \frac{3\,V's}{4\,jd} = \tfrac{3}{4}\,v'bs. \tag{7}$$

The variations of these several expressions with changing slope of bars are shown in Fig. 31.



From *Journal of Boston Society of Civil Engineers*, Feb., 1925

1916 Joint Committee (a = 20°–90°) $\quad S = \dfrac{3}{4}\dfrac{V's}{jd}$

1921 Joint Committee (a = 20°–90°)
1924 Joint Committee (a = 45°–90°) $\quad S = \dfrac{V's}{jd}\sin a$

1924 Joint Committee (a = 15°–45°) $\quad S = \dfrac{V's}{jd\,(\sin a + \cos a)}$

Fig. 31

In practice the angle of slope is nearly always 45 degrees or less. Inspection of Fig. 31 shows that for these angles the 1916 rule gives results about the average of those of the preferred formulas. In view of the uncertainties of the problem it would

seem to be assuming an unwarranted degree of precision to attempt to compute the stress in inclined web reinforcement any more exactly than recommended by the 1916 Joint Committee.

The preceding paragraphs have dealt with the ordinary arrangements of web reinforcement; vertical stirrups, one or more sets of bent-up longitudinal bars, one or more bars in each set, combinations of stirrups and bent-up bars. (See Appendix C, Figs. 6, 9 and 11.) In the 1921 Joint Committee Report another method of diagonal tension reinforcement was provided for; that of bending up one or more longitudinal rods at the point where reinforcement becomes unnecessary and carrying them through on a slope to the center or top of the beam at the edge of the support. (See Appendix C, Fig. 10.) It was specified that the vertical component of stress in the bar should be taken as that part of the maximum end shear of the beam causing stress in the web reinforcement. The 1924 Joint Committee computes the stress in this sort of reinforcement by Equation (5).

Example 18. The beam shown in Fig. 32 carries an end shear of 17,500 lbs. What is the unit stress in the end stirrup assuming

(*a*) that the concrete carries $\frac{1}{3}$ and the stirrups $\frac{2}{3}$ of the diagonal tension;

(*b*) that the concrete carries diagonal tension to the amount measured by a unit shear of 40 lbs./sq. in., and the stirrups the remainder?

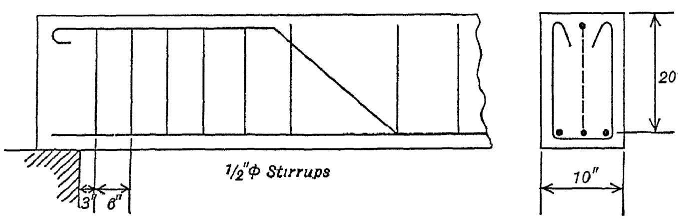


FIG. 32

Solution. (*a*) The unit end shear is

$$v = \frac{17{,}500}{10 \times \frac{7}{8} \times 20} = 100 \text{ lbs./sq. in.}$$

The end stirrup cares for 6 in. of distance along the beam and in that length two-thirds of the vertical component of diagonal tension equals two-thirds of the horizontal shear, equals (neglecting variation of shear intensity in the end 6 in.)

$$\tfrac{2}{3} \times 100 \times 10 \times 6 = 4000 \text{ lbs.}$$

which is the load on the stirrup. Dividing this by the area of the two legs gives the unit stress

$$f_s = \frac{4000}{2 \times 0.196} = 10{,}200 \text{ lbs./sq. in.}$$

(*b*) As the concrete carries diagonal tension to the amount of 40 lbs. of shear there is left 60 lbs. for the stirrup, making

$$f_s = \frac{60 \times 10 \times 6}{2 \times 0.196} = 9200 \text{ lbs./sq. in.}$$

Example 19. What is the theoretical end spacing of the stirrups in the beam of Example 18 for a unit stress in the steel of 16,000 lbs./sq. in., making the assumptions noted in that problem?

Solution. (*a*) One stirrup can carry a load of $16{,}000 \times 2 \times 0.196 =$ 6270 lbs. **The end spacing equals the load that one stirrup can carry divided by the load per inch, that is, by the horizontal shear per inch,** equals:

$$s = \frac{6270}{\frac{2}{3} \times 100 \times 10} = 9.4 \text{ in.}$$

(*b*) End spacing equals:

$$s = \frac{6270}{60 \times 10} = 10.4 \text{ in.}$$

If stirrups are too far apart there will be opportunity for inclined cracks to open between them and the limiting spacing is often taken as one-half the depth, that is 10 inches in this case. Another point that should be watched is the anchorage of the ends of the stirrups. They may be stressed to 16,000 pounds per square inch at the neutral plane, say approximately at a depth of 0.4 *d*. If the ends are anchored by a hook as shown there is no difficulty.[1]

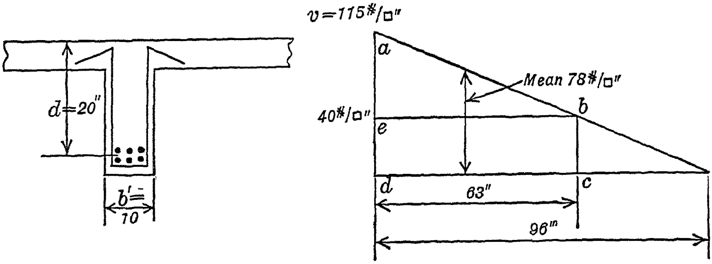
Example 20. The beam shown in Fig. 33*a* carries a total load of 2500 lbs./lin. ft. on a span of 16′ (clear). Design the diagonal tension reinforcement using vertical stirrups, making the assumptions noted in Ex. 18; $f_s = 16{,}000$ lbs/sq. in.

Solution. (*a*) Draw the curve of unit shear variation. The end shear $= \frac{1}{2} \times 16 \times 2500 = 20{,}000$ lbs. The unit end shear is $v =$ $20{,}000 \div (10 \times \frac{7}{8} \times 20) = 115$ lbs./sq. in. It will be assumed that no stirrups are required where the shear is less than 40 lbs./sq. in. which gives a length of 63 in. to be reinforced. A single loop stirrup of $\frac{3}{8}$ in. round material can carry a load of $16{,}000 \times 2 \times 0.11 = 3520$ lbs.

[1] However the 1924 Joint Committee is more rigorous. See Appendix B, Art. 141.

The number of stirrups required in one end of the beam on the first assumption equals two-thirds of the total vertical component of diagonal tension in 63 in. divided by the allowable load on one stirrup; that is,

$$\frac{\frac{2}{3} \times 78 \times 10 \times 63}{3520} = 10.$$



(a) (b)

FIG. 33

Noting that spacing varies inversely as the shear gives: average spacing at center of 63 in. distance, $s = 6.3$ in.; at end of beam, $s = 6.3 \times \frac{78}{115} =$ 4.3 in.; at inner end of 63 in. distance, $s = 6.3 \times \frac{78}{40} = 12.3$ in., which is greater than the 10 in. $= \frac{d}{2}$ limit. A spacing of 2–4–4–6–6–6–6–6–10–10 is satisfactory, the first dimension being from the edge of the support.[1]

[1] Since many designers desire an exact and easy method of spacing stirrups for varying shear the following slide-rule solution is presented but its use is not recommended.

The area under the shear curve is proportional to the vertical component of diagonal tension and so the problem is to divide the triangle *abc*, Fig. 34, into as many equal parts as there are to be stirrups and place a stirrup midway (approximately) of each area. The graphical method of doing this is illustrated in the figure and from the geometrical relations that obtain it can be shown that the division points are located as dimensioned, N being the number of stirrups. To solve by the ordinary Mannheim slide rule set the runner to L on the D-scale and bring N on the B-scale to the cross line thus dividing L by $\sqrt{N}$. Then place the runner successively at N–1, N–2, etc., on the B-scale, noting results on the D-scale, which, subtracted from L, etc., give the division lengths which are closely the spacings required.

Solving Ex. 20*b* gives the following readings on the D-scale: 63–58–53–48–41–34–24; and this spacing: 2–5–5–5–7–7–16, the last space being to the third point of the 24-inch division which is a triangle. The method can be applied to Ex. 20*a* by computing the number of stirrups required in 96 inches and omitting those not needed.

In the previous solution the total load on the stirrups could be represented by two-thirds of the area *abcd* (Fig. 33). Allowing the concrete to carry diagonal tension up to 40 lbs. of shear, the load is proportional to the area *abe* and the number of stirrups equals

$$\frac{\frac{1}{2}(115 - 40)\ (10 \times 63)}{3520} = 7,$$

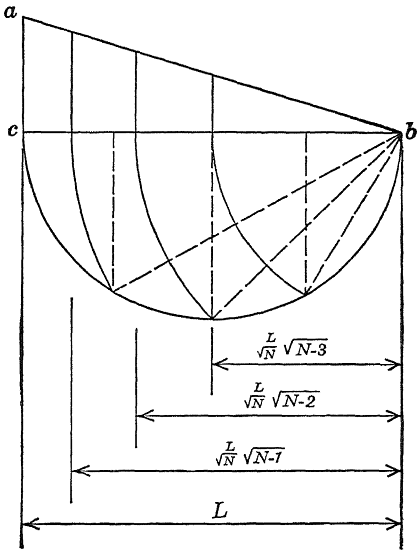


FIG. 34

the average spacing being 9 in. and the end spacing $\frac{1}{2} \times 9 = 4.5$ in. since so far as stirrups are concerned the shear at the end is twice that at the center of *eb*. At the inner end of the 63 in. distance, *eb*, the spacing is infinite. The spacing 2–5–5–8–10–10–10–10 is suggested. Careful designers would employ more stirrups than required in order to keep the spacing down to the maximum allowable.

In choosing stirrups for a beam a convenient practice is to compute the number required in each end and space them approximately, the spacing being given in multiples of 2 in. or 3 in. Any attempt at greater exactness is to expect greater precision in placing steel than is usual on even the most carefully supervised work and also errs in giving rather too much weight to the theory. Some designers prefer to calculate the spacing required at several points and place the stirrups with this guidance.

Example 21. Are stirrups needed in addition to the bent-up rods in the end of this beam (Fig. 35): (*a*) assuming that the concrete carries one-third the diagonal tension; (*b*) assuming the concrete carries diagonal

tension to the amount measured by a shear of 40 lbs./sq. in.? $f_s = 16{,}000$ lbs./sq. in. (The beam shown is that considered in Example 20.)

Note. The horizontal spacing of the bent rods here shown is $\frac{3}{4}$ of the depth of the beam, a common maximum imposed so as not to give opportunity for dangerous cracks to open between rods.

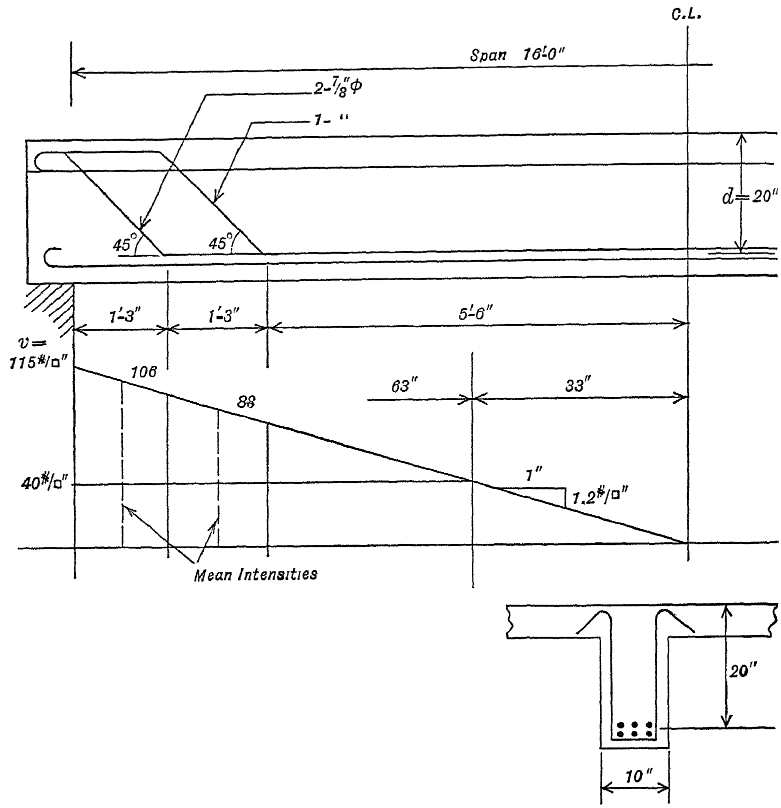


FIG. 35

Solution. (*a*) Draw the curve of shear variation and compute the mean intensities of shear in the 15 in. horizontal projections of each set of bent bars. (See Ex. 20.) The vertical component of diagonal tension in the end 15 inches is $\frac{2}{3} \times 106 \times 10 \times 15 = 10{,}600$ lbs. and the component of this component parallel to the steel equals $10{,}600 \div \sqrt{2} = 7500$ lbs. (Same result by either equation (3) or (4), the latter being easier to use.) Or it may be taken as $\frac{3}{4} \times 10{,}600 = 8000$ lbs. The stress in the steel then is equal to, or less than, $8000 \div (2 \times 0.60) = 6700$ lbs./sq. in.,

which is less than the 16,000 lbs./sq. in. limit. No stirrups, plainly, are required to assist either this or the adjoining set of bent bars.

(*b*) The vertical component of diagonal tension in the end 15 in. is $(106 - 40) \times 10 \times 15 = 9900$ lbs., less than the 10,600 lbs. above. No stirrups are needed in the end 30 in.

As the shearing stress at the inner point of bend exceeds 40 lbs./sq. in., which may be assumed as the limit of shear for concrete unreinforced for diagonal tension on either assumption (*a*) or (*b*), stirrups are required from that point to where the shear becomes equal to 40 lbs./sq. in.

54. Bond Stress and Anchorage. Undesirable cracking and even failure of reinforced concrete structures result if there is slipping between the steel and the concrete. Two general considerations respecting bond strength must be kept in mind: the anchorage or length of embedment of rods and the rate at which stress passes from the concrete to the rod.

A simple case of anchorage is that of the reinforcement of the cantilever beam shown in Fig. 36 projecting from a supporting column. The length of embedment (L) must be such that the resistance to pulling out, developed with the allowable bond stress, equals or exceeds the total stress (P) in the rod at the face of the column. Then if u = allowable unit stress in bond, Σo = total perimeter of the stressed bars, D = diameter of round or side of square bar, and f_s = unit tension in steel, there results as a general expression

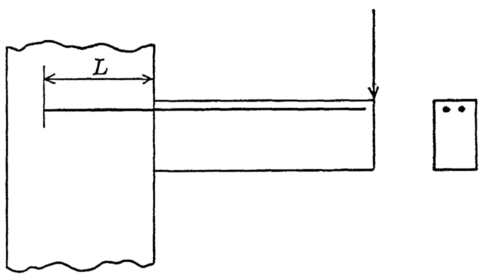


FIG. 36

$$u \cdot \Sigma o \cdot L = P \tag{8}$$

and

$$u \times 4\,D \times L = f_s \times D^2$$

for a single square bar, and

$$u \times \pi D \times L = f_s \times \tfrac{1}{4}\,\pi D^2$$

for a round bar, giving for both rounds and squares:

$$L = \frac{f_s}{4\,u} D \tag{8a}$$

It is suggested that the student note that the relation is the same for both round and square bars and make no attempt to learn the formula, working it out for a square bar of size D on each occasion as needed.

Another way to secure anchorage is to hook the end of the bar. The 1921 Joint Committee Report recommended a total length of bend of 16 times the bar diameter, and a radius of the semicircular hook of 4 bar diameters. These proportions are fixed on to ensure the elastic limit of the steel being developed without bringing excessive compression on the concrete under the hook. The Committee also allowed the use of mechanical devices such as nuts and washers, but these come into action only after slipping has started elsewhere along the bar. In this country short right-angle bends are much used but they are relatively ineffective as the concrete may be crushed and split by the excessive bearing The most effective hook is useless, however, if the mass of concrete in which it is embedded is too small to resist the stresses brought upon it. The question of proper length of embedment arises wherever there is stressed steel in concrete. Whether the stress be tension or compression a rod must extend beyond any stated point of stress a distance sufficient to develop in bond the total stress there existing. The bearing of the end of a rod on concrete is usually considered to be negligible.

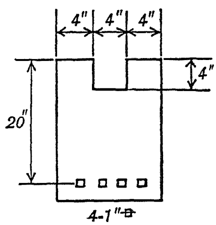
The question of the rate of transfer of stress from concrete to steel arises chiefly with regard to the tension steel in beams. Inspection of Fig. 26*a* shows that the bond stress, the tendency of the rods to slip, equals the horizontal shear and that the unit bond stress may be obtained by dividing the horizontal shear by the sum of the rod perimeters (Σo). Expressed as a formula this becomes

$$u = vb/\Sigma o. \tag{9}$$

Study of test results shows that this theoretical relation does not cover all the facts of the case. "The usual method of computing the bond stress in a reinforced concrete beam does not take into account all the phenomena of bond action" which "may be expected to greatly modify the distribution of bond stress over the length of the bar and otherwise to affect resistance to beam bond stress. However the nominal values for bond resistance, computed by the usual formula, form a useful basis for comparison in beams in which the dimensions and general make-up are similar." (Bull. 71, Univ. of Ill.)

It is not often that the bond stress in compression steel needs investigation. Usually such steel is nearer the neutral axis than the tension bars and is made up of rods of about the same diameter. The rate of stress transfer to the compression steel will be to that of the tension steel as their respective unit stresses, which bear the ratio of their respective distances from the neutral axis. If the rods are of the same size the unit rate of stress transfer (or the horizontal shear on the bar perimeters) for compression steel is therefore generally less than that of the tension steel.

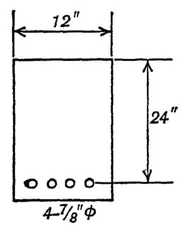
PROBLEMS



5. Locate the neutral axis of this beam $n = 15$.

Ans. 10.7 in. from top.

6. (*a*) Determine the position of the neutral axis of this beam for $n = 15$ and for $n = 12$.



(*b*) Compute the moment of resistance for $f_s = 15{,}000$ lbs./sq. in. $f_c = 500$ lbs./sq. in. $n = 15$.

(*c*) Determine the fiber stresses for a bending moment of 60,000 ft.-lbs. $n = 15$.

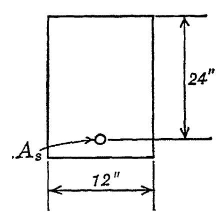
Ans. (*a*) 9.4 in. and 8.6 in. from top.

(*b*) $MR = 48{,}800$ ft.-lbs.

(*c*) $f_s = 14{,}400$ lbs./sq. in. $f_c = 615$ lbs./sq. in.

7. Determine the size of rectangular beam required for a span of 20 feet and a uniform *live* load of 1000 lbs. per foot. $f_s = 18{,}000$ lbs./sq. in. $f_c = 800$ lbs./sq. in. $n = 15$.

Ans. Breadth of 12 in. and depth to steel of 21.7 in. assuming dead weight at 300 lbs./ft. $A_s = 2.31$ sq. in.



8. The actual fiber stresses in this beam are $f_c = 500$ lbs./sq. in., and $f_s = 18{,}000$ lbs./sq. in. What are the steel area and the bending moment? $n = 12$.

Ans. $A_s = 1.0$ sq. in. $BM = 33{,}000$ ft.-lbs.

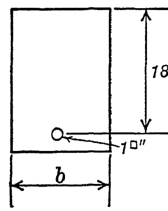
9. Compression steel to the amount of 2 sq. in. is placed 2 in. from the top of the beam of Prob. 8. What is the bending moment pro-

vided for, the fiber stresses and n being as before? What is the tension steel area?

Ans. BM = 46,400 ft. lbs. A_s = 1.41 sq. in.

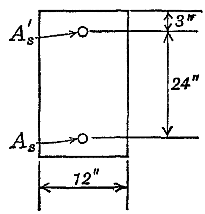
10. The lever arm of the resisting moment of this reinforced concrete beam is 16 in. What is the breadth of the beam? What are the extreme fiber stresses when the bending moment equals 24,000 ft.-lbs.? $n = 15$.

Ans. b = 10 in.
f_s = 18,000 lbs./sq. in.
f_c = 600 lbs./sq. in.



11. Determine the areas of tensile and compressive steel for this beam for limiting fiber stresses of f_c = 500 lbs./sq. in. and f_s = 15,000 lbs./sq. in. and a bending moment of 90,000 ft.-lbs. $n = 15$.

Ans. A_s = 3 sq. in. A_s' = 3.86 sq. in.

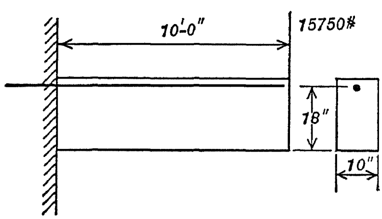


12. Solve the problem of Example 12 (page 73) by the approximate method, that is, taking the area of the compression wings of the transformed section at 30 sq. in. instead of 28 sq. in.

Ans. The unit stress in the steel together with the neutral axis and the lever arm of the resisting couple remains unchanged. f_c = 480 lbs./sq. in.

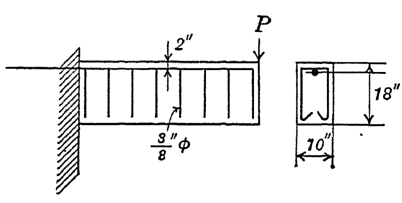
13. If one stirrup can carry 4000 lbs. how many will be required for this beam, assuming that the concrete can carry one-third the diagonal tension? Neglect the weight of the beam itself. $n = 15$.

Ans. 20 stirrups.



14. The stirrups in this beam are spaced 4 in. apart and are stressed to 10,000 lbs./sq. in. on the assumptions that they carry *all* the diagonal tension and that the weight of the beam may be disregarded. What is the amount of the load P?

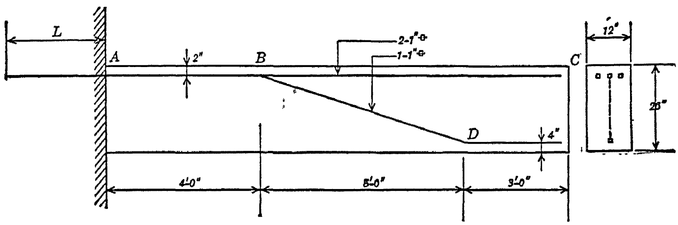
Ans. 7700 lbs.



15. This reinforced concrete cantilever beam projects from a concrete pier and carries a uniformly distributed load extending from A to C.

(*a*) Compute the intensity of bond and shear at A, assuming the total load at 12,000 lbs.

(*b*) Determine the required length of embedment (L) of the three bars, assuming them to be stressed to 16,000 lbs./sq. in. and the allowable bond stress to be 80 lbs./sq. in.



(*c*) Locate the theoretical point for bending down the one bent rod, assuming the steel to be stressed to the working limit at the support.

(*d*) What is the unit tensile stress caused by diagonal tension in the bent portion (BD) of the bent rod under a 12,000 lb. total load? Assume that the rod is effective over 20 in., and that the concrete carries one-fourth of the diagonal tension.

Ans. (*a*) v = 48 lbs./sq. in. u = 48 lbs./sq. in.
(*b*) L = 50 inches.
(*c*) 2.2 ft. from A.
(*d*) 1630 lbs./sq. in. (Equation **4**).
4080 lbs./sq. in. (Equation **3**).

CHAPTER VII

COMPRESSION MEMBERS

55. The common type of reinforced concrete compression member has a circular or rectangular concrete section with a row of rods, parallel to the longitudinal axis of the piece, set about $2\frac{1}{2}$ inches back from the surface all around the perimeter. These main reinforcing bars are held in place either by being wired to a series of encircling hoops or ties (made of $\frac{1}{4}$ inch or $\frac{3}{8}$ inch round material, spaced 8 inches to 12 inches apart), or to a closely spaced spiral (properly a helix) of steel wire.

The vertical reinforcement with either ties or spirals acts in exactly the same manner, deforming the same as the surrounding concrete as the column shortens under load. The action of the ties is to bind the rods to each other and into the mass of concrete in such a way that they will not buckle and cause the failure of the column. Owing to the shrinkage of the concrete in setting the longitudinal reinforcement has a heavy initial stress before any load comes on the column and the function of the ties is therefore very important. This task is more efficiently performed by the spiral which also serves to restrain the lateral deformation of the enclosed concrete core that follows upon its shortening. In consequence the spiral column is a much tougher and more dependable member than the tied column. However the spiral does not come into active service until the load on the column passes the elastic limit, so most authorities consider it improper to count directly upon the increased strength that its use affords. The spiral greatly increases the ultimate compressive strength and the resistance to shear.

Most reinforced concrete columns are so short that their ultimate strength is not limited by any tendency toward bending or buckling, their length being ordinarily less than 12 times their least lateral dimension. When they are more slender the working stresses must be reduced from those allowable on short columns of the same cross-section.[1]

[1] See Appendix B, Art. 170.

56. Design of Columns. The fundamental principles of column design are illustrated by the following examples.

Example 22. What are the fiber stresses in this column (Fig. 37), under an axial load of 200,000 lbs.? $n = 15$.

Solution. The steel in the column is equivalent to $15 \times 4 = 60$ sq. in. of concrete which makes a net increase of 56 sq. in. (Compare Ex. 12, Art. 47.) The stress in the concrete equals $200{,}000 \div 456 = 440$ lbs./sq. in., and that in the steel $15 \times 440 = 6600$ lbs./sq. in.

Example 23. What is the allowable load on the column of Example 22 (Fig. 37), if $f_c = 400$ lbs./sq. in. and $n = 12$?

Solution. Transformed area equals $400 + (12 - 1)(4) = 444$ sq. in. Allowable load then is $444 \times 400 = 177{,}600$ lbs.

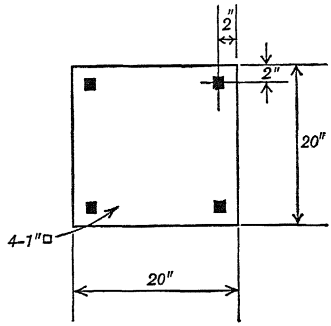


FIG. 37

Example 24. Design a column to carry a load of 100,000 lbs. $f_c = 450$ lbs./sq. in. $n = 15$.

Solution. The total area of concrete required for the transformed section is $100{,}000 \div 450 = 222$ sq. in. A 14×14 section furnishes 196 sq. in., leaving 26 sq. in. as the excess area furnished by transforming the steel area to equivalent concrete, that is $(n - 1)$ or 14 times the steel area equals 26 sq. in. The reinforcement required, accordingly, is $26 \div 14 = 1.9$ sq. in. If the cross-section and steel first chosen are unsatisfactory for any reason further trials must be made. There is no direct road to a final design except by the use of tables or diagrams.

In columns exposed to fire hazard the steel must be protected by at least two inches of concrete and the greater portion of this protective cover is not usually counted on in computing the strength of the column. The above examples have considered the gross area to be effective which is permitted by the Joint Committee (Art. 165, Appendix B). Computations of spiral-reinforced columns would differ from the preceding only in the detail that the effective area is that of the concrete inside the spiral, which is itself covered by two inches of concrete.

The value of n is taken the same for columns as for beams, somewhat higher than the actual given by test specimens at low working load. This takes account of the fact that the value of n increases with increasing stress, thus throwing a larger and larger proportion of the load on the steel.

57. Considère's Theory of Spiral Columns. In 1903 Considère published tests of columns reinforced with longitudinal steel and spirals, on the basis of which he concluded that the steel in the spiral is 2.4 times as effective in increasing the strength of the column as that placed vertically. Certain cities of the United States, notably New York and Chicago, have adopted in their building codes spiral column formulas based upon this supposed effectiveness of the closely spaced hooping. Conservative practice is represented by the Joint Committee recommendations (Art. 162, Appendix B), which takes no direct account of the spirals.

Example 25. What is the allowable load on the column section shown in Fig. 40 according to Considère's theory? The spiral reinforcement consists of $\frac{3}{8}$-in. wire with a pitch of 2 in. Allowable $f_c = 700$ lbs./sq. in.

Solution. In 1 ft. of length of this column there are $\frac{12}{2} = 6$ turns of wire, making a total length of $6 \times \pi \times 20 \times \frac{1}{12} = 31.4$ ft. If this length were used as vertical reinforcement it would provide 31.4 bars, each with an area of 0.11 sq. in., making a total of 3.45 sq. in. This spiral area is assumed to be 2.4 times as effective as an equal amount of vertical steel, so it is equivalent to $2.4 \times 3.45 = 8.28$ sq. in., making the total area of reinforcement $8.28 + 6 = 14.28$ sq. in. The transformed area is $\frac{1}{4} \times \pi \times 20^2 + (15 - 1)(14.28) = 514$ sq. in. The allowable load is $700 \times 514 = 359{,}800$ lbs.

In contrast it is interesting to note that the Joint Committee allows a load of 335,000 lbs. on this section.

58. Members Carrying Compression and Bending. When the load, P, on a homogeneous column is applied eccentrically on one of the axes of symmetry at a distance of e inches from the other axis, as shown in Fig. 38, it is equivalent to the same load applied axially, together with a moment of Pe inch-pounds, and the stress is found by use of the familiar relation

$$f = P/A \pm My/I = P/A \pm Pey/I.$$

The application of this formula to the transformed section of a reinforced concrete column presents no difficulties if there is compression over the whole section. Since concrete cannot carry high tensile stresses, where the eccentricity of the load is sufficient to cause a tension in excess of about 50 pounds per square inch a modification of this method must be employed which is described in Ex. 28.

Example 26. This column (Fig. 39) carries a load, parallel to the column axis, of 150,000 lbs., which may be considered as applied at point indicated. What are the fiber stresses? $n = 15$.

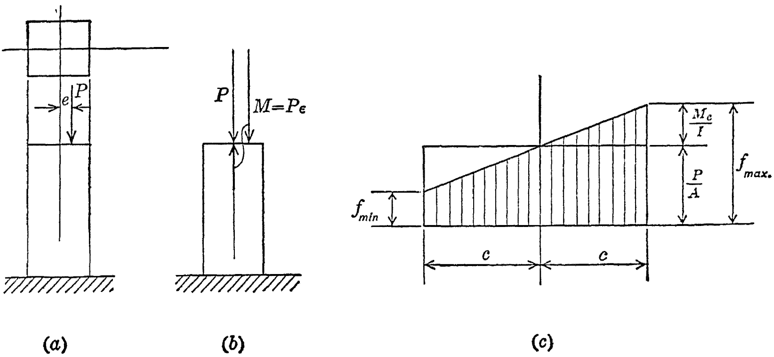


FIG. 38

Solution. The steel-concrete section may be replaced by its equivalent in concrete, as shown in Fig. 39*b*, the concrete in the wings acting at the same distance from the axis in the direction of the bending as the steel

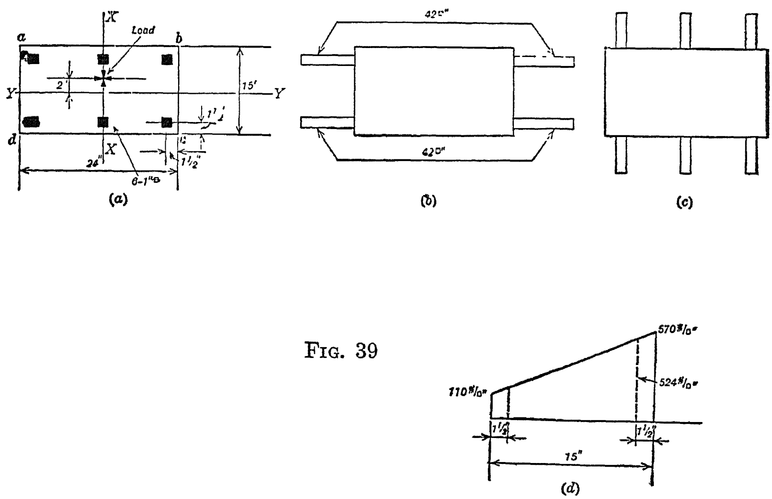


FIG. 39

it replaces. Were the arrangement of Fig. 39*c* adopted the transformed section would not be equivalent to the original so far as resisting the given bending is concerned since here the added wings have a greater leverage than the steel.

The transformed section and moment of inertia are calculated in the usual fashion, disregarding the small moment of inertia of the wings about their own axis.

$$
\begin{array}{rlcr}
24 \times 15 = & 360 & \times & 15^2/12 = 6750''^4 \\
14 \times 6 = & \underline{84} & \times & 6^2 = \underline{3020} \\
\text{Transformed area} = & 444 \text{ sq. in.} & & 9770''^4
\end{array}
$$

Whence

$$
\begin{aligned}
f_c &= \frac{150{,}000}{444} \pm \frac{(150{,}000 \times 2)(7.5)}{9770} = 340 \pm 230 \\
&= 570 \text{ lbs./sq. in. for maximum stress} \\
&= 110 \text{ lbs./sq. in. for minimum stress,}
\end{aligned}
$$

giving the stress diagram shown in Fig. 39*d*. The maximum steel stress equals $15 \times 524 = 7860$ lbs./sq. in.

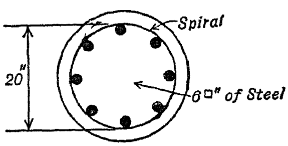


FIG. 40

Example 27. What are the maximum stresses in this column (Fig. 40), under a direct load of 150,000 lbs. and a moment of 150,000 in.-lbs.? $n = 15$.

Solution. The area of the transformed section is 314[1] $+ 14 \times 6 = 398$ sq. in. In computing the moment of inertia, consider the concrete that replaces the steel as a ring of 19 in. mean diameter;

$$
\begin{array}{lll}
\text{20 in. circle.} & I = \dfrac{1}{64}\pi D^4 = \dfrac{1}{4}\pi D^2 \times \dfrac{D^2}{16} = \text{Area} \times \dfrac{D^2}{16}; & \\
& \quad i.e., \quad \dfrac{314 \times 20^2}{16} = & 7850''^4 \\
\text{19 in. ring.} & I = \dfrac{\pi}{64}(D_1^4 - D_2^4) = \dfrac{\text{Area}}{16}(D_1^2 + D_2^2) & \\
& \quad = \dfrac{\text{Area}}{8} \times D^2 \text{ (mean) as limit} & \\
& \quad i.e., \quad \dfrac{84 \times 19^2}{8} = & \underline{3790} \\
& I = & 11{,}640''^4
\end{array}
$$

$$
\begin{aligned}
f_c &= \frac{150{,}000}{398} \pm \frac{150{,}000 \times 10}{11{,}640} \\
&= 380 \pm 129 = 510 \text{ lbs./sq. in. as maximum} \\
f_s &= 15 \times 510 = 7600 \text{ lbs./sq. in.}
\end{aligned}
$$

[1] The student will find it advantageous to familiarize himself with the easy slide-rule method of finding the area of circles. For the ordinary Mannheim rule, set the right hand index of the B-scale under the mark indicating $\frac{\pi}{4} = 0.785$ on the A-scale. Place the runner cross-line on the diameter on the C-scale and read the area on the A-scale, thus solving Area $= \frac{\pi D^2}{4}$.

The direct solution of a circular column is not practicable if there is tension over part of the section.

Example 28. Same as Example 26 except that the point of application of the load is 5″ from the axis, Fig. 41*a*.

Solution. If the load on a homogeneous column acts on an axis of symmetry outside the middle third there is tension on the section. In this case the presence of the steel modifies the limiting boundaries of the area within which the load must act if compression only is to exist, but the load is so far without the middle third that preliminary investigation by the method of the problem just outlined is hardly necessary. A glance at the previous figures quickly confirms this judgment.

To solve this problem consider a short section of the column, *abcd*, shown in elevation in Fig. 41*b*, which is in equilibrium under the action of the forces shown acting upon it; the given 150,000 lbs. on the end *ab*, and the internal fiber stresses on the end *cd*. The diagram

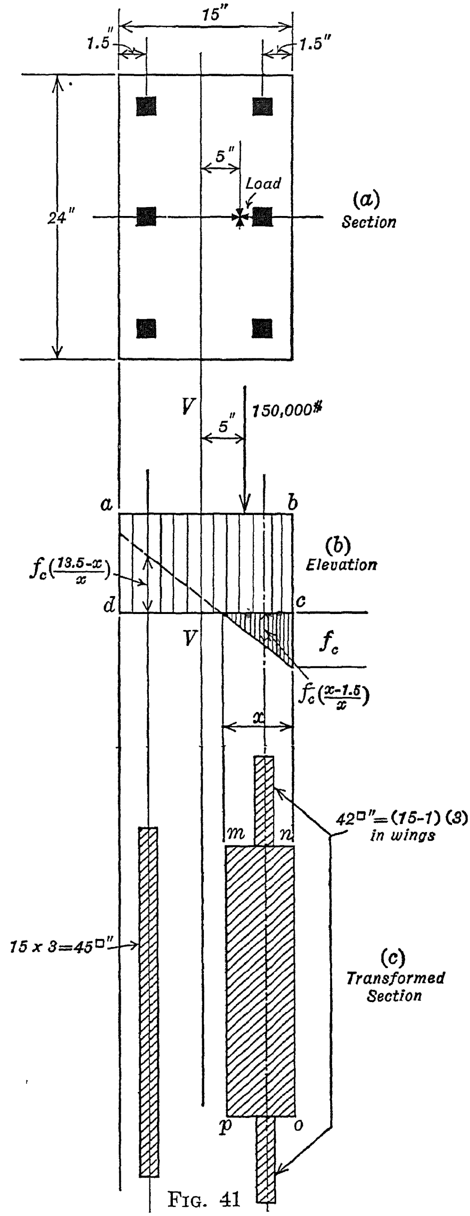


Fig. 41

of stress variation over this end resembles those already met with in beams,

the neutral axis being at an unknown distance, x, from the compression face, and the extreme compressive fiber stress, f_c, also being unknown. These two unknowns may be found by solving the equations written by application of the two conditions of equilibrium of a coplanar system of parallel forces. The algebraic work is simplified by taking the center of moments on the line of action of the known force, as thus an equation results containing x only: $\Sigma M = 0$ (compare Fig. 41*b*–*c*).

$$\begin{bmatrix} \text{Compression} \\ \text{on } mnop = \\ \frac{1}{2} f_c(24\,x) \end{bmatrix} \begin{bmatrix} \text{Arm} \\ \frac{x}{3} - 2.5 \end{bmatrix} - \begin{bmatrix} \text{Compression} \\ \text{on wings} = \\ f_c\left(\frac{x - 1.5}{x}\right)(42) \end{bmatrix} \begin{bmatrix} \text{Arm} \\ 1 \end{bmatrix}$$

$$- \begin{bmatrix} \text{Tension} = \\ f_c\left(\frac{13.5 - x}{x}\right)(45) \end{bmatrix} \begin{bmatrix} \text{Arm} \\ 11 \end{bmatrix} = 0$$

$$\text{giving } 4\,x^3 - 30\,x^2 - 42\,x + 63 - 6683 + 495\,x = 0$$
$$x^3 - 7.5\,x^2 + 113\,x = 1655$$
$$x = 11.0 \text{ in. closely.}$$

Applying the condition $\Sigma v = 0$ gives:

$$\tfrac{1}{2} f_c(24\,x) + f_c\left(\frac{x - 1.5}{x}\right)(42) - f_c\left(\frac{13.5 - x}{x}\right)(45) - 150{,}000 = 0$$

$$\text{or} \quad 132\,f_c + 36.3\,f_c - 10.3\,f_c = 150{,}000$$
$$f_c = 150{,}000 \div 158 = 950 \text{ lbs./sq. in.}$$

If this is ordinary concrete with a 28-day strength of 2000 lbs./sq. in., this stress is too high.

It is very convenient to be familiar with the method of solving a case of direct compression and bending by the transformed section as the plots given in most texts are of limited range and sections are often met with, particularly in arch design, where a careful solution is desired.

Frequently the moment applied to a column is not in the plane of a principal axis. When the magnitude of such a moment is sufficient to cause any great amount of tension, practically the only methods available are the graphic ones given in various treatises.[1]

[1] "Stresses in Composite Structural Members," Rich and Bigelow in *Journal of the Boston Society of Civil Engineers*, February, 1926. "Graphical Analysis," M. S. Wolfe; "Concrete Engineers Handbook," Hool and Johnson, p. 406.

When there is compression over the whole section the familiar method of dividing the moment into components in the planes of the principal axes is used, as illustrated by the following example.

Example 29. Same column as that treated in Ex. 26, Fig. 39. A load of 100,000 lbs. acts 1 in. from the Y-axis and 3 in. from X in the quarter toward corner b. What is the fiber stress in each corner? $n = 15$.

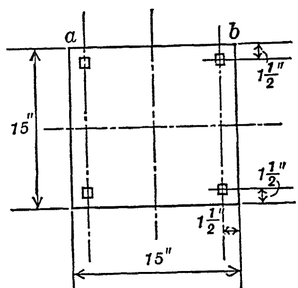
Solution. This section is called on to carry a direct load of 100,000 lbs., a moment of 100,000 in. lbs. about axis YY and one of 300,000 in.-lbs. about axis XX. The effect of each moment is determined independently as in Ex. 26. The transformed section for the 100,000 in.-lbs. moment is shown in Fig. 39b and that for the 300,000 in.-lbs. in Fig. 39c. Then

$$\begin{aligned} f_c &= \frac{P}{A} \pm \frac{M_1 y_1}{I_1} \pm \frac{M_2 y_2}{I_2} \\ &= \frac{100{,}000}{444} \pm \frac{100{,}000 \times 1 \times 7.5}{9770} \pm \frac{100{,}000 \times 3 \times 12}{23{,}440} \\ &= 226 \pm 77 \pm 154. \end{aligned}$$

The maximum stress is 457 lbs./sq. in. at b and the minimum 5 lbs./sq. in. tension at d.

Problem 16. At a certain section of this 15×15 column, the compression in the concrete at all points on edge ab is 600 lbs./sq. in. and the neutral axis is at the center of the column. $n = 15$. (a) What is the intensity of the load on the column? (b) Where is its line of action? (c) What is the bending moment at this section?

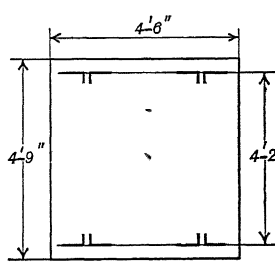
Ans. (a) 32,800 lbs.
(b) 10.2 in. from axis.
(c) 28,000 ft.-lbs.



Reinforcement: 4–1" square bars

Problem 17. The thrust in this reinforced concrete arch rib is 500,000 lbs. and it acts 31 in. above the axis. Determine the maximum stress in the concrete and in each layer of reinforcement. $n = 15$.

Ans. Max. compression in concrete
= 560 lbs./sq. in.
Max. compression in steel
= 7200 lbs./sq. in.
Max. tension in steel
= 5600 lbs./sq. in.



Reinforcement: 8 ∟s 6" × 4" × $\frac{5}{8}$"

CHAPTER VIII

FORMULAS, DIAGRAMS AND TABLES

59. In routine office practice it is essential that all work be carried on with the greatest speed consistent with excellence and accuracy. The general methods that have been outlined are fundamental but their application is somewhat cumbersome. In order to save time all designers provide themselves with many data in convenient form, and especially with tables and diagrams for design, which are based upon relations or formulas developed by means of the method of the transformed section. The principal use for the many formulas found in this and other textbooks on reinforced concrete is to compute tables and charts which give values for the various relations for different conditions. Since the formulas are easily written upon any diagram it is foolish to memorize any of these literal expressions whose relations cannot be easily visualized, especially as they are liable to subtle metamorphoses during periods of misuse and become thus a source of error. Their unthinking use tends to obscure the nature of the fundamental process being employed. In the absence of diagrams or tables recourse should be had to the method of the transformed section.

A large number of tables and diagrams have been published for use in reinforced concrete design. Many charts are good but some of the more comprehensive type are of very limited use on account of lack of precision and difficulty of reading. In general tables are quicker to use than diagrams and fewer mistakes are made in taking from them the desired data. Curves however offer many advantages as they show the changing values of the variables plotted and generally permit of direct reading without interpolation. Limitations of space make it impossible to include a comprehensive range of designing data in this volume. Only a few typical curves and tables have been printed.

The formulas and notation employed here are those made standard by the Joint Committee and in part will be found repeated in Appendices B and C.

60. Rectangular Beams with Tension Reinforcement. Notation:

f_s = tensile unit stress in steel;
f_c = compressive unit stress in extreme fiber of the concrete;
E_s = modulus of elasticity of steel;
E_c = modulus of elasticity of concrete;
n = E_s/E_c;
M = moment of resistance or bending moment in general;
b = breadth of beam;
d = depth of beam to center of steel;
A_s = cross-sectional area of tension steel reinforcement;
k = ratio of depth of neutral axis to depth, d;
j = ratio of lever arm of resisting couple to depth, d;
z = depth from compression face to resultant of the compressive stresses;
jd = $d - z$ = arm of resisting couple. (Hitherto called a.)
p = steel ratio = A_s/bd. Often expressed as a percentage.

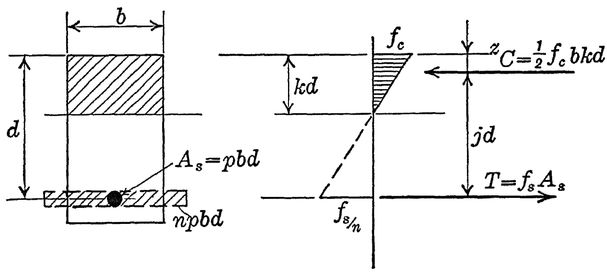


FIG. 42

By applying to the solution of the beam shown in Fig. 42 the method of the transformed section, a useful series of formulas may be derived. In terms of the steel the moment of resistance is

$$M = Tjd = (f_s pj)bd^2 \tag{10}$$

and in terms of the concrete

$$M = Cjd = (\tfrac{1}{2} f_c kj)bd^2. \tag{11}$$

The neutral axis may be located by finding the center of gravity of the transformed section:

$$bkd \times \frac{kd}{2} = npbd(d - kd)$$

whence

$$k = \sqrt{2\,pn + (pn)^2} - pn. \tag{12}$$

This formula is of service when investigating a beam where the steel area is known. When the fiber stresses are known, as in design, a simple proportion gives this result:

$$\frac{kd}{d} = \frac{f_c}{f_c + f_s/n}$$

$$k = \frac{1}{1 + \dfrac{f_s}{nf_c}}, \tag{13}$$

a formula in quite general use. For the purpose of this text it is best to return always to the original proportion instead of solving the formula and no table is given to aid in its use.

In design, where it is desired that both limiting stresses be realized simultaneously, the steel area must be such that the neutral axis lies at the level indicated by the expression just derived. It becomes necessary, therefore, to express the steel ratio, p, which measures the steel area, in terms of the fiber stresses, which may be done by equating T and C (Fig. 42):

$$f_s pbd = \tfrac{1}{2} f_c bkd$$

$$p = \frac{f_c}{2\,f_s} k.$$

Eliminating k by inserting its value from Eq. (13) gives:

$$p = \frac{1}{2} \frac{1}{\dfrac{f_s}{f_c}\left(\dfrac{f_s}{nf_c} + 1\right)} \tag{14}$$

which is the value of the steel ratio for "balanced reinforcement."

For determining the lever arm of the resisting couple it is to be noted that

$$j = 1 - k/3. \tag{15}$$

On Plate VI (page 391) are curves for k and j, Eqs. (12) and (15). Inspection of the curve for j shows that it varies little for wide variations of the steel ratio. Approximate values of j are therefore often used. The 1916 Joint Committee recommends a value

of $\frac{7}{8}$ for values of f_s ranging from 15,000 to 16,000 pounds per square inch, and for f_c from 600 to 650 pounds per square inch; the 1924 Joint Committee recommends a value of 0.86 for ranges of 16,000 to 18,000 pounds per square inch and 800 to 900 pounds per square inch. The use of these approximate values greatly facilitates much design work.

The most used formula in reinforced concrete work is this:

$$M = Tjd = f_s A_s jd, \tag{16}$$

which is so simple and useful a relation that it should be remembered, not arbitrarily as a collection of letters, but in the form of a pictured relation; *i.e.*, **the moment of resistance equals the tensile force of the resisting moment times the lever arm, — the tensile force equalling the unit stress multiplied by the area stressed.**

The formulas for moment of resistance, numbers (10) and (11), are best combined for use as

$$M = Rbd^2 \tag{17}$$

where $R = f_s pj$ or $\frac{1}{2} f_c kj$ according as the moment is expressed in terms of the steel or of the concrete. The quantity R, commonly called the coefficient of resistance, is seen to be a function of three variables, f_s or f_c, p and n (k being a function of p and n). In Plate VI curves are drawn showing the variation of R with the steel ratio p, n having the constant value 15, and a separate curve being drawn for each fiber stress desired. The steeper curves are for the values of f_s. Evidently the intersections of any two curves, as for $f_s = 20{,}000$ and $f_c = 800$, should be at the value of p determined for these stresses by Eq. (14).

The following examples illustrate the use of formulas and Plate VI and are the same as those previously given in Chapter VI.

Example 30. (Same as Ex. 5, Art. 44.) What are the fiber stresses for this beam? (Fig. 12.) $b = 10$ in., $d = 20$ in., $A_s = 2$ sq. in., $n = 15$, $M = 40{,}000$ ft.-lbs.

Solution. The first step in investigating a beam is to compute the value of the steel ratio.

$$p = 2 \div (10 \times 20) = 0.0100 \text{ or } 1.00\%.$$

Also $$R = \frac{40{,}000 \times 12}{10 \times 20^2} = 120.$$

The value of R is the second important criterion as to the status of a beam. The designer quickly forms the habit of basing his judgments on these two factors. In this case the intersection on Plate VI of the two ordinates just computed gives stresses of about 14,000 lbs./sq. in. for f_s and 670 lbs./sq. in. for f_c.

Example 31. (Same as Ex. 6, Art. 44.) What is the maximum moment that can be carried by the beam of Example 30 if the limiting fiber stresses are $f_s = 16{,}000$ lbs./sq. in. and $f_c = 650$ lbs./sq. in.?

Solution. As before, first determine the steel ratio, $p = 0.0100$. Enter Plate VI with this value and follow vertically upward to the 650 line, where $R = 117$, with the 16,000 line lying higher, giving a still larger value of R. Therefore, the allowable moment is

$$M = Rbd^2 = 117 \times 10 \times 20^2 \div 12 = 39{,}000 \text{ ft.-lbs.}$$

Evidently the beam is limited by the concrete, as a moment that stresses the steel to 16,000 lbs./sq. in. causes a stress of about 760 lbs./sq. in. in the concrete, estimating from the plate the reading at the intersection of $p = 1\%$ and $f_s = 16{,}000$ lbs./sq. in. The intersection of the 16,000 and 650 lines gives the value of p needed if these stresses are to be realized simultaneously, that is, 0.0077. If more steel is used the beam is over-reinforced and the concrete limits.

Example 32. (Same as Ex. 7, Art. 44.) Design a beam to carry a moment of 40,000 ft.-lbs. with stresses of $f_s = 16{,}000$ lbs./sq. in. and $f_c = 650$ lbs./sq. in. $n = 15$.

Solution. Plate VI gives the information that for these stresses $p = 0.0077$ and $R = 108$. Accordingly, $bd^2 = 40{,}000 \times 12 \div 108 = 4440$; and for $b = 10$ this gives $d = 21.1$. For the steel, $A_s = pbd = 0.0077 \times 10 \times 21.1 = 1.63$ sq. in.

Example 32a. (Same as Ex. 7*a*, Art. 44.) What is the steel area required for the beam of Example 32 if d is made 22 in. and $b = 10$ in.?

Solution. The actual value of R is $40{,}000 \times 12 \div (10 \times 22^2) = 99$. On Plate VI following horizontally from this figure the 650 line is reached first, but using the steel ratio there indicated would give a steel stress of about 19,000 lbs./sq. in. Further to the right the intersection with the 16,000 line calls for $p = 0.0071$ and $A_s = 0.0071 \times 10 \times 22 = 1.56$ sq. in.

Example 32b. (Same as Ex. 7*b*, Art. 44.) What is the steel area required for the beam of Example 32 if d is made 20 in. and $b = 10$ in.?

Solution. $R = 40{,}000 \times 12 \div (10 \times 20^2) = 120.$

The intersection of this horizontal with the 16,000 line on Plate VI indicates too high a concrete stress; its intersection with the 650 line is at $p = 0.0110$ giving $A_s = 0.0110 \times 10 \times 20 = 2.20$ sq. in.

In designing reinforced concrete beams keep in mind that **for any given value of the steel ratio (p) the neutral axis (k) is fixed and also the ratio of fiber stresses (f_s/f_c); that in order to realize any given fiber stresses in a beam it is necessary to employ the proper value of the steel ratio.**

61. Tee Beams. Notation as before except (see Fig. 43):

b = width of flange;
b' = width of stem;
t = thickness of flange.

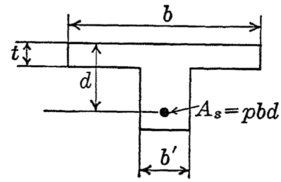


FIG. 43

When the neutral axis lies in the flange the formulas for rectangular beams are to be used. When the neutral axis lies in the stem the following are the standard approximate formulas which neglect the compression in the stem. Their derivation follows the general procedure of the preceding article. No exact formulas taking account of the compression in the stem will be given as no diagrams are available for their solution. The exact formulas are unnecessary as well as cumbersome. An exact analysis of a tee beam may be made by the procedure explained in Ex. 9, Art. 46. A similar problem solved by tables is given in Ex. 34*aa*, Art. 64.

Approximate Formulas, neglecting compression in the stem:

$$kd = \frac{2\,ndA_s + bt^2}{2\,nA_s + 2\,bt} \tag{18}$$

$$z = \frac{3\,kd - 2\,t}{2\,kd - t} \cdot \frac{t}{3} \tag{19}$$

$$jd = d - z \tag{20}$$

$$f_c = \frac{Mkd}{bt\left(kd - \frac{t}{2}\right)jd} \tag{21}$$

$$= \frac{f_s}{n} \cdot \frac{k}{1-k}. \tag{22}$$

$$k = \frac{1}{\left(\frac{f_s}{nf_c} + 1\right)}. \tag{23}$$

The expression for kd (18) can be written thus:

$$k = \frac{2\,pn + t^2/d^2}{2\,pn + 2\,t/d}. \tag{24}$$

Plate VII (page 392) gives curves for k with $n = 15$. Note that the right-hand termination of each curve marks where $k = t/d$, that is, where the neutral axis lies at the edge of the flange. For convenience the ratios of f_s/f_c corresponding to the values of k are set down at the right.

By substituting in Eq. (20) the values of z and k from formulas (19) and (24) there results:

$$j = \frac{6 - 6(t/d) + 2(t/d)^2 + \dfrac{(t/d)^3}{2\,pn}}{6 - 3(t/d)} \tag{24}$$

Curves for j from this equation are plotted on Plate VII.

Equation (21) is more easily understood when expressed thus, the first parenthesis giving the unit stress at the center of the flange:

$$f_c = \left(\frac{C = M/jd}{bt}\right)\left(\frac{kd}{kd - t/2}\right). \tag{25}$$

This formula can also be written

$$\frac{M}{bd^2} = f_c\left(1 - \frac{t}{2\,kd}\right)(t/d)j \tag{26}$$

and from it Plate VIII (page 393) was prepared, taking $n = 15$ and $f_s = 16{,}000$ pounds per square inch.

The following examples illustrate the use of these formulas and diagrams.

Example 33. (Same as Ex. 8, Art. 46, Fig. 17.) Locate the neutral axis of this tee beam. $b = 60$ in., $b' = 10$ in., $t = 6$ in., $d = 20$ in., $A_s = 2$ sq. in., $n = 15$.

Solution. $t/d = 6 \div 20 = 0.30$

$p = 2 \div (60 \times 20) = 0.0017.$

Entering Plate VII with the above value of t/d and looking for the $p = 0.002$ line, locates a point to the right of and above the right-hand ends of the k curves, showing that t/d is greater than k and that this is essentially a rectangular beam. From Plate VI, k is estimated approximately as 0.2, making $kd = 4''$.

Example 34. (Same as Ex. 9, Art. 46, Fig. 17, with $t = 3\frac{1}{2}$ in.) Same data as for Example 33 except that $t = 3.5''$. The beam carries a total moment of 40,000 ft.-lbs. What are the maximum fiber stresses?

Solution. $p = 2 \div (60 \times 20) = 0.0017$

$t/d = 3.5 \div 20 = 0.175.$

From Plate VII $j = 0.94$, approximately, and $f_s/f_c = 60$.

Then
$$f_s = \frac{40{,}000 \times 12}{2 \times 0.94 \times 20} = 12{,}800 \text{ lbs./sq. in.}$$
$$f_c = 12{,}800 \div 60 = 220 \text{ lbs./sq. in.}$$

Example 35. (Same as Ex. 10, Art. 46, Fig. 17, with $t = 3\frac{1}{2}$ in.) Beam of Example 33 except that $t = 3\frac{1}{2}$ in. What is the maximum moment of resistance for limiting stresses of $f_s = 16{,}000$ lbs./sq. in. and $f_c = 650$ lbs./sq. in.?

Solution. As in all cases of investigation, compute p and t/d, 0.0017 and 0.175 respectively, as in the previous example. The ratio f_s/f_c for 16,000–650 is 24.6. For this beam, according to Plate VII, the value of p and t/d fixes the ratio of fiber stresses at 60, showing it to be limited by the steel, with a maximum f_c of $\frac{16{,}000}{60} = 270$ lbs./sq. in.

Reading $\quad j = 0.94$ from Plate VII

gives $\quad M = 16{,}000 \times 2 \times 0.94 \times 20 \div 12 = 50{,}000$ ft.-lbs.

or, from Plate VIII with $R = 25$, f_c having been determined as 270 lbs./sq. in.
$$M = 25 \times 60 \times 20^2 \div 12 = 50{,}000 \text{ ft.-lbs.}$$

Example 36. (Same as Ex. 11, Art. 46.) Beam of same dimensions as that of Example 33, except that $t = 3\frac{1}{2}$ in., carrying 100,000 ft.-lbs. What is the steel area required? Stresses, $f_s = 16{,}000$ lbs./sq. in., $f_c = 650$ lbs./sq. in.

Solution. As in Example 11 it is possible to assume the lever arm and compute the steel area as 4.11 sq. in. However it is easy to make a slightly closer approximation than before by figuring $t/d = 0.175$ and inspecting the probable value of j as given by Plate VII, which may be estimated at about 0.92. Then
$$A_s = \frac{100{,}000 \times 12}{16{,}000 \times 0.92 \times 20} = 4.08 \text{ sq. in.}$$

A revision by computing p, reading the more exact value of $j = 0.925$, making $A_s = 4.05$ sq. in., adds nothing materially to the precision of the solution. Since the ratio of fiber stresses is about 35, f_c is plainly less than 650 lbs./sq. in.

Problem 18. Using the transformed section and standard notation, work out the complete derivations of all formulas listed in Art. 61.

62. Beams Reinforced for Both Tension and Compression. The formulas proposed by the Joint Committee for this problem are approximate in that they make no allowance for the holes left in

the concrete by the compression steel upon transforming the section, making the wings equal nA_s' instead of $(n - 1)A_s'$ (Fig. 21, etc.). The diagrams usually employed with them are cumbersome and not particularly advantageous. Accordingly there are here given exact formulas and a simple plot (Plates IX and X, pages 394 and 395)[1] based upon them, for use in design and, to a limited extent, in investigation. The beginner will find it most advantageous to use the method of the transformed section where these plots do not apply.

The following additions are made to the notation:

A_s' = area of compressive steel;

p' = steel ratio for compressive steel $= \dfrac{A_s'}{bd}$;

f_s' = compressive unit stress in steel;

d' = depth of center of compression steel from compression face of beam.

For locating the neutral axis:

$$k = \sqrt{2pn + 2p'(d'/d)(n-1) + (pn + p'(n-1))^2} - (pn + p'(n-1)). \tag{27}$$

The moment of resistance in terms of the stress in the concrete:

$$\begin{aligned} M &= \tfrac{1}{6}(f_c bd^2)(3k - k^2 + (6p'/k)(n-1)(k - d'/d)(1 - d'/d)) \\ &= Rbd^2. \end{aligned} \tag{28}$$

Curves showing the variations of R with changing steel ratios are given on Plates IX and X, the constants being $n = 15$ and actual fiber stresses of $f_s = 16{,}000$ pounds per square inch, $f_c = 650$ and 750 pounds per square inch.

The use of these diagrams in design is limited, of course, to the stresses indicated and is too simple to require illustration. They can be used in investigation to a limited degree if only the adequacy of the beam is in question and exact determination of the actual stresses is not desired.

Example 37. (Same as Ex. 12, Art. 47.) What are the maximum fiber stresses in this beam? $b = 10$ in., $d = 20$ in., $A_s = A_s' = 2$ sq. in., $n = 15$, $M = 40{,}000$ ft.-lbs.

Solution. Compute first the steel ratios, $p = p' = \dfrac{2}{10 \times 20} = 0.01.$

[1] After a diagram made by Sven G. Roeblad.

Also $d'/d = 1/10$ and $R = M/bd^2 = \dfrac{40{,}000 \times 12}{10 \times 20^2} = 120$. Reference to Plate X shows that for $R = 120$ and $d'/d = 1/10$ the required values of p and p' are both less than 0.01. Accordingly the beam is over-reinforced so far as the stresses 16,000–650 are concerned and the actual stresses must be less than these. An exact solution cannot be made by this diagram.

Example 38. (Same as Ex. 13, Art. 47.) Same beam as in Example 37. If the limiting fiber stresses are $f_s = 16{,}000$ lbs./sq. in and $f_c = 650$ lbs./sq. in. what is the maximum moment of resistance of this beam?

Solution. Using the data obtained in the previous problem, enter Plate X with $p = 0.01$ and $d'/d = 1/10$. The reading gives $R = 142$ and the required value of p' for balanced reinforcement as 0.006. The beam has more compression steel than is needed. Disregarding this fact an approximate solution gives

$$M = Rbd^2 = 142 \times 10 \times 20^2 \div 12 = 47{,}300 \text{ ft.-lbs.}$$

Example 39. (Same as Ex. 15, Art. 48.) What areas of tension and compression steel are required for this beam? $b = 10$ in., $d = 20$ in., $d' = 3$ in., $f_s = 16{,}000$ lbs./sq. in., $f_c = 650$ lbs./sq. in., $n = 15$, $M = 60{,}000$ ft.-lbs.

Solution. In order to use Plate X there are required the values of $d'/d = 3/20 = 0.15$ and $R = \dfrac{60{,}000 \times 12}{10 \times 20^2} = 180$. The plate gives $p = 0.013$ and $p' = 0.016$, making $A_s = 2.6$ sq. in. and $A_s' = 3.2$ sq. in.

Problem 19. Using the transformed section with standard notation work out the complete derivations of Equations (27) and (28).

63. Columns. Additions to notation:

P = total safe load on ordinary short column;

A = total effective area of column cross-section; *i.e.*, that within the spiral of spiral columns or within the protective covering of tied columns exposed to fire hazard; the total cross-sectional area of tied columns where the protective covering is considered as part of the load-carrying section;

p = ratio of area of longitudinal reinforcement to effective column area $= A_s/A$;

A_c = area of concrete within the spiral or within the protective covering $= A - A_s = A(1 - p)$;

f_c' = 28-day ultimate compressive strength of concrete.

Using the method of Art. 56 and Ex. 23 the following general formula for safe load on the usual short column is obtained:

$$P/A = f_c(1 + (n - 1)p). \tag{29}$$

The 1916 Joint Committee required the concrete of exposed tied columns to be regarded as protective covering to a depth of $1\frac{1}{2}$ inches, thus making the effective area of a 20-inch square column 17 inches × 17 inches. The 1924 Joint Committee reduced the working stress for this type but defined A in Equation (29) as the total cross-sectional area with no provision for protective covering. Considering the lack of toughness and bending resistance of the tied column it would have been better to reduce the unit stress without changing the requirement as to fireproofing. Both committee reports require the steel to be placed 2 inches clear from the surface of the concrete.

Formulas for columns with spirals based on Considère's work take the general form

$$P/A = f_c(1 + (n - 1)p) + 2.4\, nf_cp_1$$

where p_1 is the ratio of the volume of the spiral wire to the volume of the enclosed concrete. The Chicago code replaces the 2.4 by 2.5. The New York code replaces $2.4\, nf_c$ by $2\, f_s$, with no relation specified between f_c and f_s.

The following working unit stresses were recommended by the 1916 Joint Committee:

For tied columns $\quad f_c = 0.225\, f_c'. \qquad (30)$

For spiral columns $\quad f_c = 0.35\, f_c'. \qquad (31)$

In both types the steel ratio was limited to between 1 per cent and 4 per cent. The 1924 report specified:

For tied columns $\quad f_c = 0.20\, f_c' \qquad (32)$

with p between $\frac{1}{2}$ per cent and 2 per cent.

For spiral columns $\quad f_c = 300 + (0.10 + 4\, p)f_c' \qquad (33)$

with p between 1 per cent and 6 per cent.

Both reports gave rules as to size and spacing of lateral ties and spirals which will be considered later.

Plate XI (page 396) is a designing chart based on Equation (29) and the stresses just noted.

Example 40. (Same as Example 24, Art. 56.) Design a tied column to carry a load of 100,000 lbs. $f_c = 450$ lbs./sq. in. $n = 15$.

Solution. The less steel used the cheaper the column, so assume the minimum percentage of steel, $p = 1\%$ (1916 Joint Committee). Inspection of Plate XI shows P/A to be 513 lbs./sq. in., whence $A = 100{,}000 \div 513 = 195$ sq. in. A section 14 in. square contains 196 sq. in. The steel area is $0.01 \times 195 = 1.95$ sq. in.

If the concrete area chosen differs much from that required by the diagram the actual value of P/A should be computed and used to determine the new value of p from the chart.

Problem 20. Demonstrate that Equations (42) and (44) in the 1924 Joint Committee report (Appendix B), are essentially the same as Equation (29) in Art. 63.

64. Tables. For the design of members, tables are extremely convenient and for the most part are easily constructed. In the Appendix (pages 402–406) are given several pages reprinted by permission from "Reinforced Concrete Design Tables," by Thomas and Nichols, which illustrate the possibilities very clearly. In view of the previous chapters and articles these tables are for the most part self-explanatory, both as to their construction and use. Table 11 for tee beams covers only cases where "the neutral axis lies below the slab and gives only values for the compression areas above the bottom of the slab, neglecting the compression in the beam stem between the bottom of the slab and the neutral axis." When it is desired to take account of this compression, as is sometimes necessary with large beams, it may be done by combining the use of Tables 9 and 11 as explained in Example 34*aa*. The usual procedure with these tables is illustrated by the following examples.

Example 32aa. (Same as Examples 32 and 32*a*, Art. 60.) Design a rectangular beam to carry a moment of 40,000 ft.-lbs. with stresses of $f_s = 16{,}000$ lbs./sq. in. and $f_c = 650$ lbs./sq. in. $n = 15$.

Solution. Table 9. A beam 12 in. wide (b) and 22 in. deep (d) can carry 52,000 ft.-lbs. and requires 2.02 sq. in. of steel. To carry 40,000 ft.-lbs., the width should be $\frac{40}{52} \times 12 = 9.2$ in. and the steel area $\frac{40}{52} \times 2.02 = 1.56$ sq. in. Use a 10-in. beam with this given amount of steel. This is not exact but the approximation is sufficiently close.

Example 34a. Given a tee beam of the dimensions of that in Example 34 ($b \lessapprox 60$ in., $b' = 10$ in., $t = 3\frac{1}{2}$ in., $d = 20$ in., $A_s = 2$ sq. in., $n = 15$), carrying 100,000 ft.-lbs. What is the steel area required? $f_s = 16{,}000$ lbs./sq. in., $f_c = 650$ lbs./sq. in., $n = 15$.

Solution. Table 11. A tee beam with a 12-in. flange and $d = 20$ in., $t = 3\frac{1}{2}$ in. carries a moment of 32,200 ft.-lbs., with a steel area of 1.31 sq. in. By direct proportion of the moments the required width of flange is 37 in., which is less than the Joint Committee limit of $16\,t + b' = 66$ in. (Appendix B, Art. 115.) The steel area, similarly, is 4.07 sq. in.

Example 34aa. It is desired to design the tee beam of the preceding problem exactly, taking account of the compression in the stem.

Solution. Table 9 gives the information that a rectangular beam 10 in. wide and 20 in. deep (d) will carry a moment of $\frac{10}{12} \times 43{,}000 = 35{,}800$ ft.-lbs., and requires steel to the amount of $\frac{10}{12} \times 1.84 = 1.53$ sq. in. This leaves a moment of $100{,}000 - 35{,}800 = 64{,}200$ ft.-lbs. to be provided for by adding a $3\frac{1}{2}$ in. flange and more tension steel. Table 11 shows that 12 in. width of flange will add 32,200 ft.-lbs. of resisting moment and requires 1.31 sq. in. of steel. So the total width that must be added is $\frac{64.2}{32.2} \times 12 = 24$ in., making a total width of 34 in., which is less than the 66 in. limit. The total steel area is $1.53 + \frac{64.2}{32.2} \times 1.31 = 4.14$ sq. in.

This result indicates that the usual approximate method errs slightly on the safe side so far as the concrete is concerned and on the unsafe side as regards steel.

Example 39a. (Same as Example 39.) What are the areas of tension and compression steel required for this rectangular beam? $b = 10$ in., $d = 20$ in., $d' = 3$ in., $f_s = 16{,}000$ lbs./sq. in., $f_c = 650$ lbs./sq. in., $n = 15$; moment $= 60{,}000$ ft.-lbs.

Solution. Table 9 shows, as found above in Ex. 34*aa*, that this beam will carry 35,800 ft.-lbs., when reinforced only with 1.53 sq. in. of tension steel. Table 12 adds the information that 1 sq. in. of compression steel (with the proper additional amount of tension steel) will supply a moment of 7780 ft.-lbs. To supply $60{,}000 - 35{,}800 = 24{,}200$ ft.-lbs., there is required 3.12 sq. in. The additional tension steel equals

$$\frac{T}{f_s} = \frac{24{,}200 \times 12}{17} \times \frac{1}{16{,}000} = 1.07 \text{ sq. in.,}$$

making a total of 2.60 sq. in.

Example 40a. (Same as Example 40.) Design a square column reinforced with vertical steel, secured by ties, to carry a load of 100,000 lbs.; $f_c = 450$ lbs./sq. in., $n = 15$.

Solution. It is desired to use a minimum steel ratio of 1%. Table 13 gives the solution directly, a 14×14 in. column with 1.96 sq. in. of steel. This size must be increased by most specifications to provide fireproofing if fire hazard exists as in a building.

65. Direct Stress and Bending. Probably the most convenient diagrams that have been published hitherto for the solution of sections carrying direct stress and bending are those given by Professors Turneaure and Maurer in their book "Principles of Reinforced Concrete Construction," which are here reproduced by permission in Plates XII and XIII (pages 397 and 398). The following equations were used in plotting the curves there shown:

Plate XII. No tension on the section. Symmetrical reinforcement.

$$12\,k(1 + 2\,np)\left(\frac{e}{h}\right) = 1 + 24\,np\left(\frac{a^2}{h^2}\right) + 6(1 + 2\,np)\left(\frac{e}{h}\right) \quad (34)$$

$$\frac{M}{bh^2f_c} = \frac{1}{12\,k}\left(1 + 24\,np\left(\frac{a^2}{h^2}\right)\right) \quad (35)$$

Plate XIII. Both tension and compression on the section. Symmetrical reinforcement:

$$k^3 - 3\left(\tfrac{1}{2} - \frac{e}{h}\right)(k^2) + 12\,npk\left(\frac{e}{h}\right) = 6\,np\left(\frac{e}{h} + 2\frac{a^2}{h^2}\right) \quad (36)$$

$$\frac{M}{bh^2f_c} = \frac{1}{12}k(3 - 2\,k) + \frac{2\,pn}{k}\left(\frac{a^2}{h^2}\right) \quad (37)$$

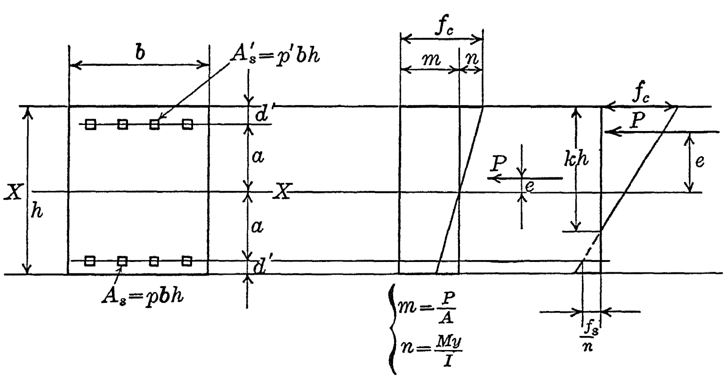


FIG. 44

The notation is clearly explained in Fig. 44. These four equations are derived by the method employed in the solution of Ex. 28, Art. 58, except that n was used instead of $(n - 1)$ in replacing the compression steel in the transformed section. They are thus approximate, as is shown in applying them to the problems pre-

viously considered. In using Plates XII and XIII it should be kept in mind that the reinforcement is symmetrical and that p is the ratio of one-half of the steel only. Other diagrams in very common use are expressed in terms of the total steel ratio.

A still more convenient type of diagram for the solution of sections carrying both bending and direct stress is shown in Plate XIV (page 399), which was devised by Sven G. Roeblad. Here a single chart covers both the case without tension and that with tension. It is peculiarly adaptable to the investigation of rectangular columns where the building code requires an allowance to be made for fireproofing because the steel is here assumed to be at the very edge of the effective section.

Example 41. (Same as Example 26.) What is the maximum concrete stress in this column? $b = 24$ in., $h = 15$ in., $d' = 1.5$ in., $A_s = A_s' = 3$ sq. in., $P = 150{,}000$ lbs., $e = 2$ in., $n = 15$.

Solution. First collect the following data:

$$p = p' = \frac{3}{15 \times 24} = 0.0083$$
$$d'/h = 1.5/15 = 0.10$$
$$e/h = 2/15 = 0.13.$$

Examination of these values shows that Plate XII applies and that there is no tension. The value of $\frac{M}{bh^2f_c}$ given by the plot is about 0.098. Then

$$f_c = \frac{150{,}000 \times 2}{24 \times 15^2 \times 0.098} = 570 \text{ lbs./sq. in.}$$

Example 42. Same as Example 28 and accordingly the same as the preceding except that $e = 5$ in.

Solution. The same data are collected except

$$e/h = 5/15 = 0.33.$$

Plate XIII applies and there is tension. The value of $\frac{M}{bh^2f_c}$ is found to be 0.148.

$$f_c = \frac{150{,}000 \times 5}{24 \times 15^2 \times 0.148} = 940 \text{ lbs./sq. in.}$$

Problem 21. (*a*) Work out the derivation of Equations (34) and (35), noting that n was used in place of $(n - 1)$ in transforming the section.

(*b*) The derivation of Equations (36) and (37).

Suggestion. To obtain Equation (35) (or 37), take moments about XX, Fig. 44. To obtain Equation (34) (or 36), combine (35) (or 37) with the expressions obtained by applying the other condition of equilibrium.

66. List of Principal Design Formulas.

Shear Intensity (Art. 51)	$v = \dfrac{V}{bjd}$ or $\dfrac{V}{b'jd}$	(1 or 1*a*)
Stress in Vertical Stirrup (Art. 53)	$S = v'bs$	(2)
Stress in Inclined Rod (Art. 53)	$S = \frac{3}{4}\, v'bs$	(7)
Length of Embedment (Art. 54)	$L = \dfrac{f_s}{4\,u} D = \dfrac{P}{u\Sigma o}$	(8)
Bond Stress in Tension Reinforcement (Art. 54)	$u = \dfrac{vb}{\Sigma o}$	(9)
Moment of Resistance of Any Reinforced Concrete Beam in terms of Steel Stress (Art. 60)	$M = f_s A_s jd$	(16)
Same, in terms of Coefficient of Resistance (Art. 60)	$M = Rbd^2$	(17)
Allowable Column Load (Art. 63)	$P = f_c(A + (n - 1)pA)$	(29)

Problems 22 to 28. Compute the values of the several ordinates indicated in these problems by use of the appropriate formulas and check results against the readings of the plots. Prepare also in each case a series of instructions and skeleton tabulation of computations such as could be given to a computer, ignorant of the theory involved, with orders to prepare the data for constructing the plots completely.

22. Plate VI. (*a*) for $p = 0.0100$, compute k, j, R for $f_s = 16{,}000$, and R for $f_c = 650$.

(*b*) Locate the intersection of the 16,000 and 650 lines by two independent methods.

23. Plate VII. (*a*) for $t/d = 0.2$ and $p = 0.004$, compute the values of k and j.

(*b*) Compute the value of the extreme right-hand ordinate of the $p = 0.004$ curve.

(*c*) Check the vertical location of the value of $f_s/f_c = 30$.

24. Plate VIII. Compute the ordinate of the $f_c = 500$ curve at the extreme right-hand end and that where $t/d = 0.2$.

25. Plate IX. (*a*) Compute the ordinates at the left-hand ends of all curves.

Suggestion. These values are for a single reinforced beam (that is, reinforced for tension only) with balanced reinforcement.

(*b*) Compute the ordinates for $R = 200$ and $d'/d = 0.10$.

Suggestion. Compute first the value of p'. Note that k is known.

26. Plate XI. Compute all the values needed for laying out this plate.

27. Plate XII. Compute the value of $\frac{M}{bh^2f_c}$ for $e/h = 0.10$ and $p = 1\%$.

28. Plate XIII. Compute the value of $\frac{M}{bh^2f_c}$ for $e/h = 1.0$ and $p = 1\%$.

CHAPTER IX

RETAINING WALLS

67. Retaining walls are built to restrain a mass of earth or similar material and are of two types: the gravity wall of plain concrete or other masonry, which depends for its stability principally upon its own weight, and the reinforced concrete wall which depends, in addition, upon the weight of a portion of the earth back of it. These two types are illustrated in Fig. 45. The gravity wall there shown resists solely by its own weight the thrust of the earth behind it which tends to slide the wall along its foundation and tip it over; the reinforced concrete wall can neither slide nor tip except as the earth resting on its heel to the left of plane *bd* slides or tips with it. Had the gravity wall been built with a sloping back the weight of the prism of earth above the sloping portion would assist the wall in retaining its position.

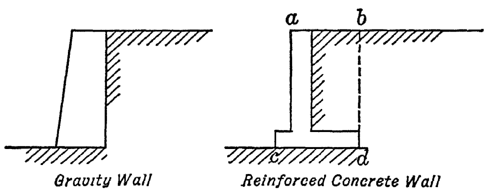


FIG. 45

The mathematical part of the design of walls consists in ascertaining the amount of earth thrust on the back, and proportioning the wall so that it shall be structurally sufficient in every part and stable against sliding and overturning, without exerting too large pressure upon the foundation. The exact determination of the pressure that a given mass of earth will exert is practically impossible, depending as it does upon so many variable and uncertain factors, such as the character of the material, its cohesion, its moisture content, the coefficient of friction of the particles of the mass one upon the other and of the earth upon the wall. The various theories that have been proposed assume a dry granular material without cohesion, and give reasonable values for the maximum possible pressure so long as the backing is not clay and there is no accumulation of water in the earth back of the wall, a

happening which will very largely increase the thrust. The theory most commonly used in the United States is that of Rankine, a brief statement of which is given in Appendix D.

This theory states that the line of action of the pressure exerted by a mass of earth upon a supporting wall is parallel to the earth surface; that the intensity of this pressure at any point in a vertical plane (perpendicular to a vertical plane containing the line of earth thrust) is given by the expression:

$$p = Cwh \tag{38}$$

where

$$C = \cos\theta\left(\frac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}\right) \tag{39}$$

In these equations the following notation is used:

w = weight of earth per cubic foot;
h = vertical depth in feet of point where pressure is taken;
θ = angle made by the earth surface with the horizontal;
ϕ = angle of repose of earth, usually assumed equal to angle of internal friction.

The general problem of retaining wall design is outlined by the following examples:

Example 43. Is the wall shown in Fig. 46 stable against overturning and sliding?

Data: Weight of earth 100 lbs./cu. ft., weight of wall 150 lbs./cu. ft., angle of repose of the material 1 : $1\frac{1}{2}$, coefficient of friction of concrete on earth 0.40, allowable pressure on foundation 6000 lbs./sq. ft.

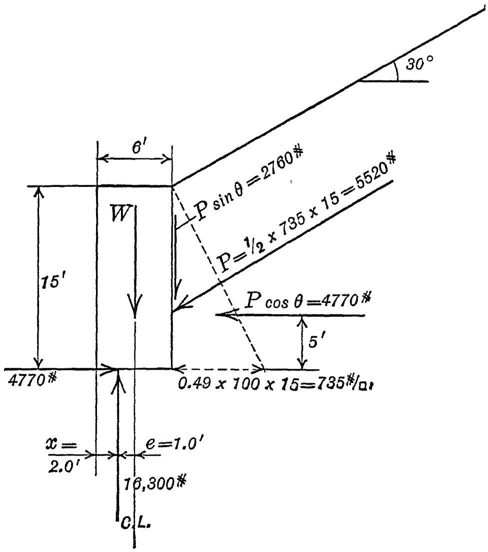
Solution. From Equation (38) it is plain that the intensity of pressure varies directly with the depth as is indicated in Fig. 46, and so the resultant pressure on the back of the wall acts at $\frac{2}{3}$ depth or 5 ft. from the bottom. Evaluating C gives (see Fig. 46, for computations):

$$C = 0.865\left(\frac{0.865 - 0.240}{0.865 + 0.240}\right) = 0.49.$$

The intensity of pressure on the vertical back at the bottom is 0.49 of the intensity of vertical pressure at that point, and the total thrust parallel to the surface on a one-foot length of wall is the mean intensity of pressure multiplied by the wall height. This should be sufficient explanation of the figures on the sketch.

The forces acting on a one-foot length of wall, in addition to its own

weight, are the earth thrust just described and the foundation pressure. Since this system of forces is in equilibrium, full information concerning the resultant of this unknown base pressure (but not concerning its distribution) may be obtained by applying the three equations of equilibrium. Accordingly, the horizontal component equals 4770 lbs. and the vertical



Data:

$\theta = 30°$

$\cos\theta = \frac{\sqrt{3}}{2} = 0.865$

$\cos^2\theta = 0.750$

$\phi = \tan^{-1}\frac{2}{3}$

$\cos^2\phi = \frac{9}{13} = 0.692$

$\sqrt{\cos^2\theta - \cos^2\phi} = \sqrt{0.058}$
$= 0.24$

$W = 6 \times 15 \times 150 = 13{,}500\#$

FIG. 46

13,500 + 2760 = 16,300 lbs. In order to locate the line of action of the resultant, moments may be taken about any convenient point in the plane of the force system, the toe being chosen in this case:

$$13{,}500 \times 3 + 2760 \times 6 - 4770 \times 5 - 16{,}300 \times x = 0$$
$$x = 2.0 \text{ ft.}$$

Assuming the pressure to vary uniformly on the base, the extreme intensities are given by the familiar

$$f = \frac{P}{A} \pm \frac{Mc}{I} = \frac{P}{A}\left(1 \pm \frac{6\,e}{b}\right) = \frac{16{,}300}{6}\left(1 \pm \frac{6 \times 1}{6}\right)$$

f = 5440 lbs./sq. ft. at the toe and = 0 at the heel, the resultant acting through the outer middle-third point. This maximum pressure is less than the allowable. The wall is safe against overturning. Often a factor of safety against overturning is used, defined as the ratio of the earth pressure that would cause the resultant on the base to pass through the toe to the actual earth pressure, that is the value of P that would make

$x = 0$ in the above equation.[1] The factor of safety in this problem is 5.5. A factor as low as 2 is considered sufficient.

The resistance that may be developed to sliding equals $0.40 \times 16{,}300 = 6520$ lbs., which is greater than 4770 lbs., the greatest push to be expected, a factor of safety of 1.3. The wall may be regarded as safe against sliding.

In computing the stability of walls the vertical component of pressure upon a vertical back is often disregarded.

Example 44. Same data as for Example 43 except $\phi = 30°$ and $\theta = 0$, the surface of the ground back of the wall being a level storage ground with a maximum loading of 500 lbs./sq. ft. What is the pressure on the back of the wall?

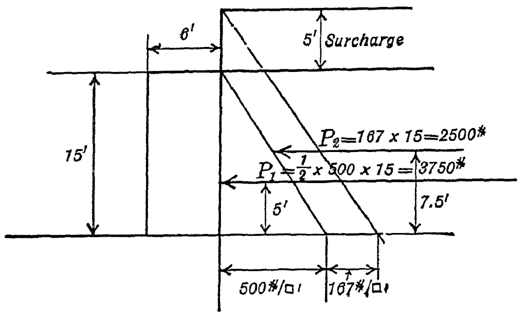


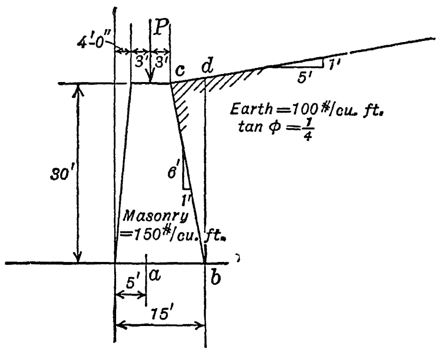
FIG. 47

Solution. Fig. 47. The value of C in Equation (38) for these data is $\frac{1}{3}$, a value often used when conditions are not accurately known. It is seen that this is equivalent to taking the pressure on the back of the wall as that caused by a liquid back-fill having a weight of 33.3 lbs./cu. ft. Many engineers use such an equivalent liquid filling in computing pressures on walls, 25 and 30 lbs./cu. ft. being common unit weights chosen. Such figures should not be used without careful investigation of the conditions of the problem and of the success of walls of known proportions in similar situations.

The effect of the loaded surface may be taken as represented by the vertical pressure of a depth of earth, called a surcharge, sufficient to give the same unit vertical pressure. Then the horizontal pressure on the top of the wall is $\frac{1}{3} \times 100 \times 5 = 167$ lbs./sq. ft., and that at the bottom $\frac{1}{3} \times 100 \times 20 = 667$ lbs./sq. ft. The trapezoid of pressure may be considered as made up of a triangle and a parallelogram representing the two forces, P_1 and P_2, shown, whose sum equals the total pressure sought. It is not necessary to locate this resultant in order to find the base pressure.

[1] Sometimes the factor of safety is defined as the ratio of resisting moment to overturning moment, 2.4 in this case.

Problem 29. The line of action of the resultant pressure acting on the base of this wall passes through *a*. What is the magnitude of the load (P) in lbs. per ft. of length of wall?



Suggestion. Find the earth pressure on the vertical plane *db* and consider the weight of the prism of earth *dbc* as a vertical force acting on the wall.

Ans. $P = 57{,}800$ lbs.

68. Cantilever Retaining Wall. The rectangular beam is one of the simplest problems met with in reinforced concrete design, particularly so when the proportions are made such that no diagonal tension reinforcement is required. The most common form of rectangular beam is the slab, a member of great width as compared with the depth. Accordingly it is fitting to choose for the first example of actual design in this text a cantilever retaining wall of the sort shown in Fig. 48, consisting as it does of the three simple elements, each a cantilever slab, the vertical stem (a), the heel (b), and the toe (c), the last two together constituting the base. The forces acting on the stem (a in Fig. 49) are the earth thrust and the internal resisting shear and moment (V and M); on the heel (b in Fig. 49), its own weight, the downward weight of the mass of earth above, the upward pressure of the foundation bed and the resisting shear and moment; on the toe (c in Fig. 49), its own weight and that of the earth above it, the upward pressure of the foundation bed and

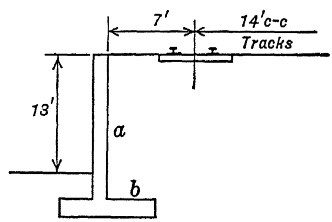


FIG. 48

the internal stresses at the support. The downward weight of the earth above the toe is never considered as this fill may not be in place when the wall is first loaded.

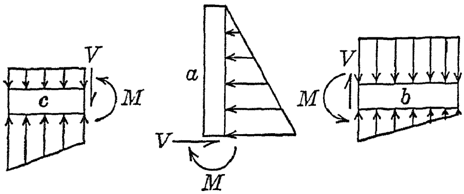


FIG. 49

This type of wall is economical for moderate heights, up to about 18 feet or 20 feet. Higher walls are generally made with brackets, called counterforts when inside and buttresses when outside of the vertical slab which is fastened to them. (See Fig. 50.) A counterfort wall is a much more complicated problem than the cantilever type.

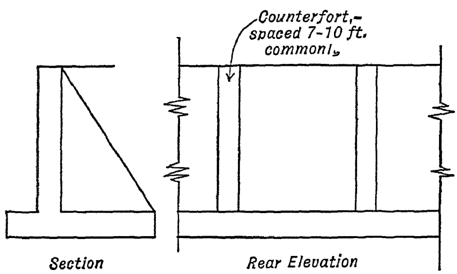


FIG. 50

When it is desired to build a wall close to a property line beyond which no encroachment is possible, the vertical slab is placed at the extreme end of the base giving an L-shaped wall, without a projecting toe.

69. Data for a Cantilever Wall. Detailed consideration will now be given to the design of a cantilever retaining wall to fill the situation outlined below. The necessary figures and sketches are shown together on a series of computation sheets. In the text is given a description of the various operations covering all the significant details. It is expected that the student will model

his own work on the lines suggested by these sheets, making plain the various steps by clear subject headings and sketches and refraining from lengthy written descriptions. Slovenly workmanship, such as hasty sketches and crude, illegible letters and numerals, easily leads to error and results in sheets that are difficult to check. The engineering computer must always keep in mind that his work is to pass under the critical eye of a superior for check and execute it so that it will be easily understood. The student should study these computation sheets with three points in mind, best reserving the third however for separate consideration: (*a*) the application of the theory of earth pressure to the determination of the external forces acting on the wall and its several parts; (*b*) the application of the theory of reinforced concrete to the proportioning of the several sections; (*c*) the precision of the computations, justifying the approximations made and seeking others to shorten the work.

In connection with this chapter the student should read Section I of Chapter XI, 1924 Joint Committee Report in Appendix B. It is also advisable to compare all the specified fiber stresses in this example, together with other details, with the several recommendations of the Joint Committee elsewhere in the same chapter.

Data: Design a cantilever wall for track elevation, the details of the situation being shown in Fig. 48. Loading for the tracks: Cooper E-60 locomotive. Weight of earth fill, 100 lbs./cu. ft. Allowable pressure on earth, 3000 lbs./sq. ft. Angle of repose, 1 vertical to $1\frac{1}{2}$ horizontal. ($\phi = 33° 42'$.) Angle of friction, concrete on earth, 22°. (Tan 22° = 0.40.) Allowable unit stresses: tension in steel, 16,000 lbs./sq. in.; compression in concrete, 650 lbs./sq. in.; shear, no diagonal tension reinforcement being used, 40 lbs./sq. in.; bond, 100 lbs./sq. in. Ordinary concrete with a 28-day strength of 2000 lbs./sq. in. is assumed, with $n = 15$. The steel used is structural grade, deformed bars.

70. First Steps in Design. (Computation Sheet W1.) The first thing for the designer to do is to assemble his information, as completely as may be necessary, on a computation sheet and make a sketch of the wall about as he judges it will appear when designed; all of which appears on Sheet 1 herewith. In making the sketch the question of the depth of foundation comes up at once. In order to prevent movement by the freezing and the thawing of the ground, the base must be set at or below the frost line, which is here assumed to be 4 feet below the surface. In

the northern United States from 4 feet to 5 feet is the usual depth found necessary to get below the frost.

Another question to be settled is the position of the vertical stem on the base, which depends upon the limitations placed upon the line of action of the resultant foundation pressure. In this case the wall is not on rock but on compressible material, and some settlement may be expected. The conservative practice in this situation is to require the resultant to strike near the center of the base so that the intensity of pressure will be nearly uniform, resulting in even settlement without tipping of the wall.[1] Comparative studies have shown that for the most economical wall the front of the stem should be placed approximately over the point where it is desired that the resultant strike the base. These considerations explain the dimensions shown on this sketch. A top width of wall of 12 inches is a common minimum to allow easy pouring of concrete.

After the sketch is made, the preliminary work is completed by determining the relation between the vertical and horizontal pressures at any point in the earth backing; that is, evaluating C in Eqs. (38) and (39). Here the horizontal pressure is 0.29 times the vertical intensity at the same point.

71. Base Width. (Computation Sheets W1–W2.) Designers endeavor to tell as much of the story of their work by sketches as possible, and use a minimum of written description. The two diagrams under this heading give full information of what is being done.[2] In the first the effect of the track load was reduced

[1] The figure for allowable unit pressure upon a foundation is set low enough so that the settlement will not be excessive. In this case the line of pressure was kept near the center of the base so that there would be uniform settlement and no tipping. This is an extremely severe and expensive requirement as a little settlement at the toe with consequent moving forward of the top of the wall is not a serious matter in a wall of this sort. If it is objected to on account of appearance the stem can be given a slight batter to the back to compensate for the expected movement.

[2] In studying any section of these computations first read the sketch and check the calculations made on it. Values that appear without explanation are either repeated from an earlier sketch, or are calculated in the accompanying computations or follow so directly from the data shown that details were not considered necessary. The first two sketches on Sheet W2 illustrate this. The horizontal force, 4200 lbs., on the first sketch is from the second diagram on the preceding sheet; the moment, 45,600 ft.-lbs., on the second sketch was calculated by means of figures alongside the diagram; the values of the horizontal forces follow directly from their curves of pressure variation.

to an equivalent mass of superimposed earth by distributing the weight on one driving axle, 60,000 pounds for E–60, over the area measured by the driver spacing along the track and the track spacing. No impact was considered. The pressure on the vertical plane, *ab*, at the heel, was divided into two parts: that occurring with empty tracks (4200 pounds acting at $\frac{2}{3}$ depth) and the increment due to the locomotive loading (4400 pounds acting at mid-depth). (Compare Ex. 44.) The tendency of this pressure to overturn the wall was obtained by taking moments about any point in the plane of the base, giving a total overturning moment of 61,400 foot-pounds acting on a 1-foot length of wall. As an approximation in determining the base width the total weight, W, of the wall with the earth above it was taken as equal to that of a prism of earth, *mnbc*, and the toe was neglected. It was desired that the resultant pressure cut the base at c, and for this condition the moment of W about c must be equal and opposite to the overturning moment. Equating these moments and solving gave a trial base width.

Next came consideration of the possibility that a wider base might be required when the outside track is empty and the inside ones loaded. (See second sketch.) In this position of the live load there is no surcharge assisting in holding down the heel. The recommendation of the American Railway Engineering Association is that the full surcharge be taken as above if its edge comes over the heel, and none if the edge comes a distance from the heel equal to the depth, and proportionately for intermediate positions of the edge of the surcharge. The second sketch shows the edge of the track load, which is 7 feet from the center line of track, to be 9 feet from the intersection of the ground, with a 45-degree line through the heel. In conformity with the recommendation the surcharge was made $\frac{9}{17}$ of that previously found necessary, and it was made to extend up to the heel. On this basis a trifle wider base than required for the first loading was found desirable. A lesser value was chosen, however, 14 feet, which is an extremely wide base for a wall of this height. The refusal to be bound by the larger figure is explained by noting that a 45-degree line from the wall end of the second track ties does not hit the vertical stem of the wall at all and it would not seem that much overturning effect could properly be attributed to the load on the inside track. Tests by Professor Enger (*Engineering News-*

Record, Jan. 22, 1916) would indicate that the influence of this load would not be felt outside a line sloping at 60 degrees with the horizontal.

72. Stem Thickness. (Computation Sheet W2.) The complete design of any element of this wall is impossible until the others are also designed. So a trial computation was made to find out how thick the vertical stem must be made at the bottom, assuming a base 2 feet thick. The stem thickness must be sufficient to make it safe in bending and in shear, shear being a measure of the diagonal tension. Since no stirrups or bent rods are to be used, a low shear stress has been set. Equation (1), Art. 51, was used to give the shear limit for the depth to the steel (d), and Eq. (17), Art. 60, to give the limit set by moment. The total thickness of the stem was made 3 inches greater than the larger of these two values in order to furnish a generous cover of concrete over the steel to protect it from corrosion. (See Art. 27.)

73. Base Pressure and Sliding. (Computation Sheets W2 and W3.) The final determination of the intensities of base pressure was now possible since variations in the base thickness produce entirely negligible differences. The computations follow closely the model set in Ex. 43 and require little explanation.

The earth in front of the toe cannot be counted on to resist sliding, not only because it may be absent when needed, as previously noted concerning the earth above the toe, but also because after it is in place the fill may shrink and draw away from contact with the concrete.

The resistance offered to sliding by the cut-off wall may be estimated by assuming that all or part of the passive resistance of the earth in front of it is available. Here the passive pressure is $(1 + 0.55) \div (1 - 0.55) = 3.4$ times the vertical pressure. Theoretically the 1-foot projection develops $3.4 \times 1515 = 5100$ pounds resistance. To ensure this in any reasonable degree the earth must be thoroughly compacted in front of the key wall. Evidently the efficiency of this arrangement depends on the amount of movement that takes place in developing the necessary resistance. No large movement can take place except as the 8 feet of earth in front of the projection is pushed along ahead of it, shearing or sliding along a horizontal plane. The resistance offered to shear (earth sliding on earth) is very considerable, approximately

COMPUTATIONS FOR CANTILEVER RETAINING WALL

Sheet W1

Data:

Cantilever Retaining Wall.

Surcharge: Cooper E60 Loading.

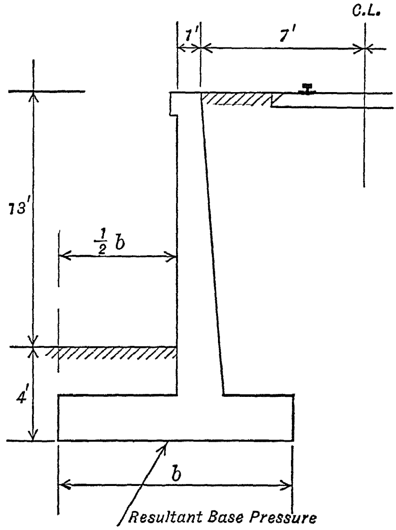
Earth: $w = 100\#/cu.\ ft.$

$$\phi = 33^\circ\ 42' = \tan^{-1}\frac{1}{1.5}$$

Concrete; $w = 150\#/cu.\ ft.$

Foundation Pressure limited to $3000\#/\square'$.

Angle of Friction — Concrete on Earth $= 22^\circ$



Fibre Stresses: —

$f_c = 650\#/\square''$ $n = 15$

$f_s = 16{,}000\#/\square''$

$u = 100\#/\square''$

$v = 40\#/\square''$

Earth Pressure: —

$\theta = 0$ $\cos\theta = 1$

$\phi = 33^\circ\text{-}42'$ $\sin\phi = 0.55$

$$C = \cos\theta\,\frac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}$$

$$= \frac{1 - \sin\phi}{1 + \sin\phi} = \frac{0.45}{1.55} = 0.29$$

$$p = Cwh$$
$$= 0.29\ wh$$

Base Width:

Surcharge: — All tracks loaded.

$$\frac{60{,}000}{5 \times 14} = 860\#/\square'$$

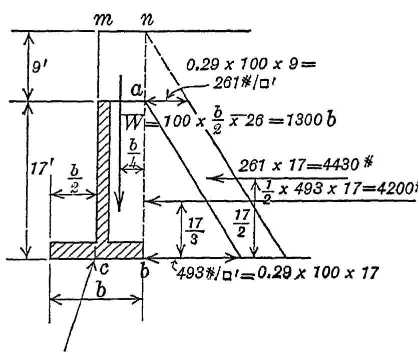
$\simeq 9'$ *of earth approximately.*

ΣM *about* $c = 0$

$$\left\{\begin{array}{ll} 4200 \times \frac{17}{3} = 23{,}800'\# \\ 4430 \times \frac{17}{2} = 37{,}600 \\ \hline 8630\# \quad 61{,}400 = \end{array}\right.$$

$$W \times \frac{b}{4} = \frac{1300\,b^2}{4}$$

$b = 13.75\ ft.$ *Try* $14'$



$\frac{1}{1.5} \times 8 \times 1550 = 8200$ pounds $\left(\text{tangent of angle of friction} = \frac{1}{1.5}\right)$.

Note the record made of the slope of the pressure line (bottom of Computation Sheet W3), 9.3 pounds per square foot variation of pressure for each foot along the base. This ratio was used in obtaining the intermediate intensities in preference to setting up a proportion from the similar triangles involved. This manner of dealing with sloping lines facilitates checking.

No factor of safety against overturning was computed as its significance is somewhat uncertain. Assuming that the surcharge over the heel does not increase with the increasing earth push, this factor would equal the actual thrust, 8600 pounds for the first loading (Sheet W2), divided into the thrust that would bring the resultant line of base pressure through the toe, that is, give an overturning moment of 207,500 foot-pounds, an increase of 146,100 foot-pounds. Assuming this increase in moment results from a surcharge outside the wall, the additional thrust acting at mid-depth is $146{,}100 \div 8.5 = 17{,}200$ pounds, making a total thrust of 25,800 pounds. On this basis the factor of safety is $25{,}800 \div 8600 = 3.0$, which is ample.

Another factor (sometimes called the factor of limitation), used to estimate the stability of walls, is the ratio of that thrust which would cause the line of action of the base pressure to pass through the outer middle-third point, to the actual thrust. The values allowed this factor vary from 1 for rock beds to 2 for compressible material.

As a matter of precision, it is to be noted that in setting down the lever arms of the several weights the $\pm$ sign indicates that the distance was estimated, not computed.

74. Design of Base. (Computation Sheets W4 and W5.) In this design the simplest form of wall is being used, with a base of uniform thickness throughout, and without fillets at the juncture of stem and base. The toe appears first on the sheets simply because it was judged that probably it would prove the critical portion, which turned out to be the case. Usually the heel is the decisive factor.

The first matter to attract attention is that 4 inches is used for cover for the steel instead of 3 inches. This is because the con-

COMPUTATIONS FOR CANTILEVER RETAINING WALL

Sheet W2

Base Width: (Continued)
Inside Track Loaded: Outside Track Empty. Trial 14'-0"

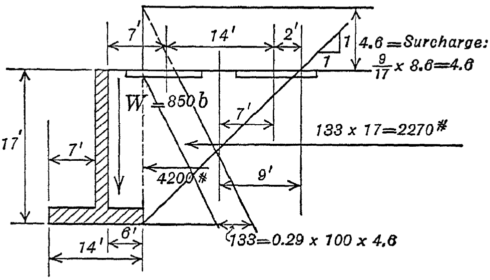
$\Sigma M_c = 0$

$4200 \times \frac{17}{3} = 23,800'\#$

$2270 \times \frac{17}{2} = 19,300$

$6470 \qquad 43,100'\# = 850\,b \times \frac{b}{4}$

$b = 14.2$ ft. Try 14'



Stem Thickness:

Assume 2 ft. base

$V \,\&\, M/ \quad 3260 \times 5 = 16,300'\#$

$3900 \times 7.5 = 29,300$

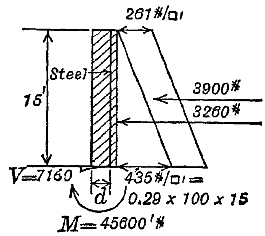
$V = \quad 7160\# \quad M = 45,600'\#$

Thickness:

Shear: $d = \dfrac{V}{bjv} = \dfrac{7160}{12 \times \frac{7}{8} \times 40} = 17$ in.

Moment: $d = \sqrt{\dfrac{M}{bR}} = \sqrt{\dfrac{45,600 \times 12}{12 \times 108}} = 20.6$ in. or 21 in.

Add cover 3
Total 24 in.



$R = 108$ for 16,000 − 650
$n = 15$
Plate VI.

Trial: 24" d = 21" to Steel

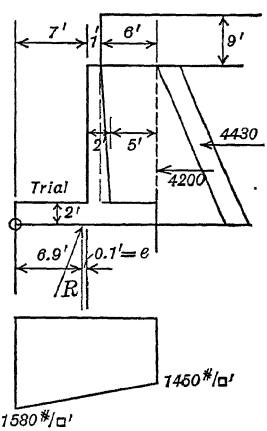
Base Pressure: Outside Track Loaded.

ΣM toe = 0:	Weight ×	Arm =	M
Base	2 × 14 × 150 = 4200#	7.0'	29,400
Stem	1.5 × 15 × 150 = 3375	7.8'	26,400
Earth	6 × 9 = 54□'	±	
	5.5 × 15 = 82.5		
	100 × 136.5 = 13650	11.1' ±	151,700
	21225#		207,500'#

Less overturning moment from sheet W1 61,400

21,200# | 146,100'#

Distance from toe to R 6.90 ft.



crete of the base is poured without bottom forms. On account of the dirt stirred up by the pouring, the concrete at the very bottom of the base is probably of poorer quality than at the top.

The steel area was determined by Eq. (16), Art. 60, using an average value of j. It is convenient to know the area required per inch of length of wall, as here obtained, since no tables for bar spacing are printed in this textbook. It is plain that the moment in foot-pounds per foot of length of wall is equal numerically to the moment in inch-pounds per inch of length, making this area a simpler one to compute than that required per foot. Knowing the number of square inches of steel that must be furnished every inch of wall, the number of inches of length that any one bar can provide for, which is the required spacing for that size of bar, equals the bar area divided by the required area. The student should copy the table of bar sizes and areas in Art. 12 and place it in easy range of vision when at work. It is then a simple operation to decide upon a bar and its spacing.

The degree of approximation involved in the use of the average value of $\frac{7}{8}$ for j is of interest. An exact solution of the required area for the steel in the toe (Sheet W4) proceeds thus: first compute R in Eq. (17),

$$R = \frac{M}{bd^2} = \frac{30{,}900}{1 \times 21^2} = 70.$$

Then from Plate VI, $n = 15$, with 16,000–650 the stresses, the steel ratio, p, is found to equal 0.0049; whence $A_s = 0.0049 \times 1 \times 21 = 0.103$ square inch per inch of length. This is so close to the approximate value previously found that no change can be made. For the area required for the heel (Sheet W4) an exact value cannot be found by the same method since the value of R falls outside the limits of Plate VI. Computation shows A_s to be about 0.050 square inch per inch, which permits changing the spacing from $11\frac{1}{4}$ inches to 12 inches on center. (Abbreviated o.c. on the computation sheets.) In this particular case any reduction of the amount of steel would not seem advisable since a steel ratio of 0.0025 is generally considered about the minimum for main reinforcement.

The bars in the top of the base are stressed to their maximum at a point vertically beneath the face of the stem. They must

COMPUTATIONS FOR CANTILEVER RETAINING WALL

Sheet W3

Unit Pressure:

$$f = \frac{P}{A}\left(1 \pm \frac{6\,e}{b}\right) = \frac{21{,}200}{14}\left(1 \pm \frac{6 \times 0.1}{14}\right) = 1515\ (1 \pm 0.043)$$
$$= 1580 \text{ and } 1450\#/\square' < 3000\#/\square' \text{ limit}$$

Base Pressure: Outside Track Empty: others loaded. See 1st sketch, Sheet W2.

$\Sigma M_0 = 0$	Weight	Arm	M'#
Base	4200	7.0	29,400
Stem	3375	7.8±	26,400
Earth	8250	11.2±	92,300
	15,825		148,100
Less overturning moment (Sheet W2)			43,100
		15,800#	105.000'#
Distance from toe to R = i.e., 0.3' in front of c.l.			6.7'

Unit pressures are plainly less than above

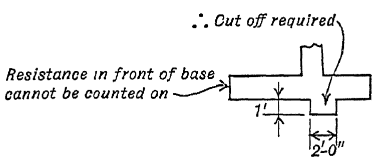
14 ft. base O.K.

Resistance to Sliding.

Tan 22° = 0.40 (From Sheets W2–W3)

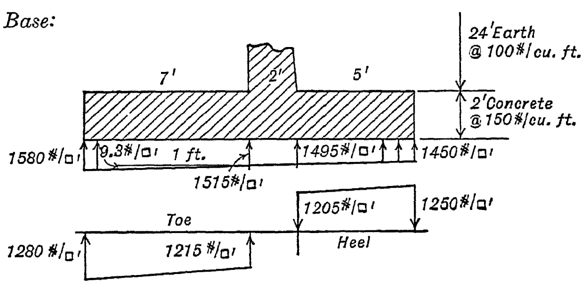
Outside Track Loaded $0.4 \times 21{,}200 = 8480\# < 8630\#$

" " *Empty* $0.4 \times 15{,}800 = 6310\# < 6470$



Use Cut-off Wall

Loads on Base:



extend into the heel a sufficient distance to develop their stress by bond. At the same section the shear is a maximum and with it the bond stress. The formulas and methods of Art 54 explain the computations as regards these details.

Justification of the apparently high bond stress in the first choice made for heel reinforcement was obtained by following the specifications of the 1924 Joint Committee which allows higher bond stress than the usual limit when a bar is anchored at either end in accordance with their requirements. It is plain that if a bar is securely anchored in a mass of solid concrete at either end, the effect of local slip between the points of anchorage, due to high bond stress, is much lessened if not entirely neutralized. Formula No. 35 of the Joint Committee, put in words, says that the total anchorage at one end, measuring from the place where the tensile stress is computed, must be sufficient to develop that stress by bond, the distance from the point of stress to where the anchorage proper begins (distance y) being rated at a modified limit (Qu) of bond stress. In this case y is zero and the expression assumes the form of Eq. (8), Art. 54. Since the length of anchorage (x) may always be made whatever is desired, bending the bar back along itself with a 180-degree bend if needful, the first term to the right of the equality sign of formula (35) need never be considered unless desired. So far as this end of the bar is concerned a bond stress in excess of the rated limit (and the Joint Committee sets no limit to this excess) is allowable if anchorage is provided sufficient to develop the working stress in the steel with the usual limit of bond in the anchorage.

The other end must be anchored in accordance with the requirements of Art. 140*b* of the Joint Committee report. That anchorage must be enough to develop one-third of the working stress in a length back from the heel sufficient to stress the concrete of the base to a maximum of 40 pounds per square inch in direct fiber stress, considering it to be unreinforced. This length is about 31 inches and since the rod will extend to within 1 or 2 inches of the end of the base its embedment is bound to be at least 29 inches, which is more than the 35/3 inches required. Both ends of the rods are thus properly secured by using straight lengths of 8′-0″. Since the embedment required at the stem is 35 inches and the bar cannot be expected to be closer than 1 inch to the end of the heel, the bar length can be made 4′-11″ plus 35″ = 7′-10″. It is

COMPUTATIONS FOR CANTILEVER RETAINING WALL

Sheet W4

Base — Toe:

See Sheet W3 for forces acting.

Shear	× Arm	= Moment
$1215 \times 7 = 8500$	$\frac{7}{2}$	$29,800'\#$
$65 \times \frac{7}{2} = 230$	$2 \times \frac{7}{3}$	$1,100$
$V = 8730\#$		$M = 30,900'\#$

Thickness:

$$d = \frac{V}{bjv} = \frac{8730}{12 \times \frac{7}{8} \times 40} = 21'' \qquad d = \sqrt{\frac{M}{bR}} = \sqrt{\frac{30,900 \times 12}{12 \times 108}}$$

Add cover 4

Total $25''$

$= 17'' < 21''$

Thickness 25″ d = 21″ to steel

Steel: $A_s = \frac{M}{f_s jd} = \frac{30,900}{16,000 \times \frac{7}{8} \times 21} = 0.105\square''/''$

Try $\frac{7}{8}''\phi$ @ $5\frac{3}{4}''$ o.c.

Bond: $u = \frac{vb}{\Sigma_0} = \frac{40 \times 5.75}{\pi \times \frac{7}{8}} = 84\#/\square'' < 100$ O.K.

Embedment: $L = \frac{f_s}{4u} D = \frac{16,000}{4 \times 100} \times \frac{7}{8} = 35''$

Use $\frac{7}{8}''\phi$ 10′-0″ in bottom $5\frac{3}{4}''$ o.c.

Base: — Heel:

Shear	× Arm	= M
$1205 \times 5 = 6025$	2.5	$15,100$
$45 \times \frac{5}{2} = 113$	$\frac{2}{3} \times 5$	400
$V = 6140\#$		$M = 15,500'\#$

For $d = 21$ $v = \frac{6140}{12 \times \frac{7}{8} \times 21} = 28\#/\square'' < 40$ O.K.

Steel: $A_s = \frac{15,500}{16,000 \times \frac{7}{8} \times 21} = 0.053\square''/''$

Try $\frac{7}{8}''\phi$ @ $11\frac{1}{4}''$ o.c. in top

Bond: $u = \frac{28 \times 11.25}{\pi \times \frac{7}{8}} = 115\#/\square'' > 100\#/\square''$ *Limit*

$\therefore$ *Investigate requirements of J. C. 1924*

By formula 35. J. C. 1924. Art. 137

$$F = Qu\Sigma oy + u\Sigma ox$$

$$16,000 \times \frac{\pi}{4} \times \overline{0.875}^2 = 0 + 100 \times \pi \times 0.875\,(x)$$

$$x = \frac{16,000}{4 \times 100} \times \frac{7}{8} = 35''$$

---

better practice to cut rods in multiples of 3 inches for short lengths and of 6 inches for long.

The Joint Committee (Art. 184, Sec. I, see Appendix B) very wisely gives consideration to the possibility that settlement or some other cause may shift the foundation pressure to the forward part of the base. If this happens the heel should be strong enough to carry the largely increased downward load, higher working stresses being permissible on account of the small chance of this happening. On Sheet W5 it is shown that this requirement very materially increases the amount of steel, changing the spacing of $\frac{7}{8}$-inch round bars from $11\frac{1}{4}$ to $7\frac{3}{4}$ inches o.c. After the steel was chosen for the stem it was noted that 1-inch squares at 13 inches o.c. gives the required area without excessive bond stress and with the advantage that the spacing fits easily in with that of the vertical steel, and a change was made to the large size.

75. Reinforcement of the Stem. (Computation Sheets W5–W6.) The shear and moment were found at two intermediate sections of the stem, the values previously found for the 15-foot depth being considered sufficiently close to require no revision. The steel areas in the last column of the table (Sheet W5) of course were got by Eq. (16) as before. It would not be economical to run all the heavy 1-inch square bars to the top of the wall, so the arrangement shown on Sheet W6 was adopted. A five-bar group would make the maximum spacing in the upper part more than twice the wall thickness, so it was not chosen. Some designers would set a limit of $2\frac{1}{2}$ times the wall thickness and use a 6-bar grouping in this case. A neat free-hand sketch on quadrilled paper suffices for the plot of steel areas required at various depths. Here it was found that the intermediate length of bar begins to come into play 7 feet down, and the short length, about 10 feet down. These bars are brought 1 foot, more or less, closer to the surface than required and bent toward the compression face, since without this anchorage they would not begin to come into action at the sections desired.[1] All of these bars are stressed to their limit at the bottom, and accordingly must extend 40 diameters above that point. Similarly, the long and the intermediate bars must both extend 40 diameters above the 10-foot section, where again they are stressed to their working maximum due to the cutting off of the short bars.

[1] See Art. 95*g*, page 203.

COMPUTATIONS FOR CANTILEVER RETAINING WALL

Sheet W5

Base: Heel (Continued)

Anchorage. Art. 140b. J. C. 1924 As plain concrete beam

$$MR = \tfrac{1}{6} fbh^2 = \tfrac{1}{6} \times 40 \times 12 \times \overline{25}^2 \times \tfrac{1}{12} = 4170'\#$$

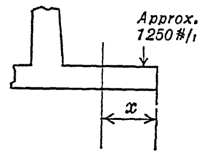
$$= \frac{1250\ x^2}{2}$$

$$x = 2.6 \text{ ft. about } 31 \text{ inches.}$$

Required Length to develop

$$\left.\begin{array}{l} \dfrac{16{,}000}{3} \#/\square'' \quad = \quad 12\ '' \end{array}\right\}$$

$\therefore$ *No hook needed*



By § 184. J. C. 1924. $f_s = 1.5 \times 16{,}000 = 24{,}000\#/\square''$

$f_c = 1.5 \times 650 = 975\#/\square''$

Neglecting base pressure

$$M = 5 \times 2700 \times 2.5 = 33{,}800'\#/'$$

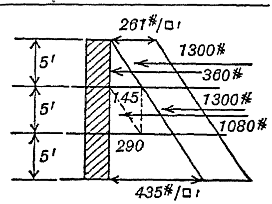
Approximately $A_s = \dfrac{33{,}800}{24{,}000 \times \frac{7}{8} \times 21} = 0.077\square''/''$

First choice: $\frac{7}{8}''\phi$ @ $7\frac{3}{4}$ *o.c. in top;* $< 11\frac{1}{4}''$ *above*

Final choice: $1''\ \square$ $8'\text{-}3''$ @ $13''$ *o.c.*

Stem — (continued from sheet W2).

Base taken as 2'-0" thick



Section	V = Shear	$d = \dfrac{V}{12 \times \frac{7}{8} \times 40}$	M '#/' or "#/"		$d = \sqrt{\dfrac{M}{108}}$	d actual	$A_s\square''/''$
5'	1660	4"	$360 \times \frac{5}{3} = 600$ $1300 \times \frac{5}{2} = 3250$	3850	6.0"	13"	0.021
10'	4040	10"	$1440 \times \frac{10}{3} = 4800$ $2600 \times 5 = 13000$	17800	12.8"	17	0.075
15'	7160	17"		45600	20.6"	21	0.155

Actual Shear $v = \frac{17}{21} \times 40 = 33\#/\square''$. *Try* $1''\ \square$ @ $6\frac{1}{2}''$.

" *Bond* $u = \dfrac{33 \times 6.5}{4 \times 1} = 54\#/\square'' < 100.$ O.K.

Anchorage in the base is secured by running the bars into the cut-off wall and hooking the ends. This hook should be a 180-degree bend of diameter equal to 8 bar diameters, and preferably with a straight extension beyond the hook of about 4 bar diameters.

In order to prevent vertical cracks due to shrinkage and to temperature stresses, horizontal reinforcement is required, the usual amount being an area equal to from 0.2 per cent to 0.3 per cent of the section area. This is not entirely sufficient to accomplish the end desired and long walls should be built in sections of 60 feet or less.

76. Conclusion. (Computation Sheet W6.) A sketch showing the complete design is shown on Sheet W6, and is made with enough detail to give all the information needed by a draftsman preparing the working drawings.

Certain additional steel is here shown which has not previously been mentioned: $\frac{3}{4}$-inch rounds vertically in the stem to support the horizontal temperature steel and $\frac{1}{2}$-inch rounds longitudinally in the base to tie the whole together and ensure it acting as a unit.

In addition to the drains shown, a layer of porous material should be laid the length of the heel to above the level of the drain opening. This is a common requirement of specifications and one too frequently disregarded.

A construction joint is shown between the stem and the base, with a tongue, say, $3'' \times 6''$. The width of this key must be sufficient to carry the total shear at a limit of about 120 pounds per square inch for its unit value.

Problem 30. Make a complete design for a wall for the situation shown in Fig. 48, allowing the resultant base pressure to strike near the outer middle-third point. Use the data of Art. 69.

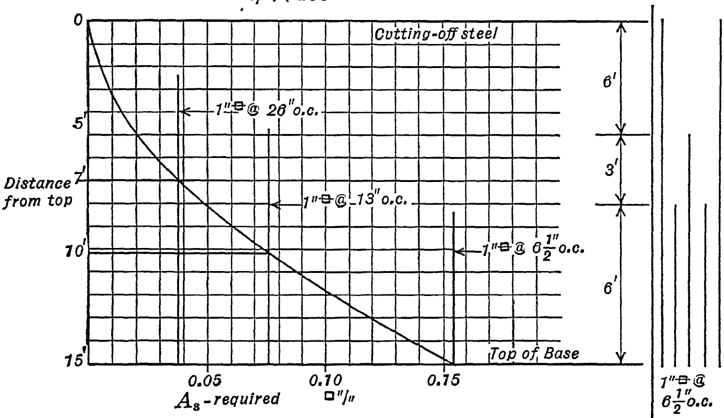
Ans. Base, $10'$-$6'' \times 28''$ with cut off wall. Stem $23''$ thick at bottom. Steel, in toe, $\frac{7}{8}'' \phi$ at $10\frac{1}{2}$ in. o.c.; in heel, $1'' \phi$ at $10''$ o.c. Other details as before.

COMPUTATIONS FOR CANTILEVER RETAINING WALL

Sheet W6

Stem: (Continued)

Embedment required: $L = \dfrac{16{,}000}{4 \times 100} \times 1 = 40''$



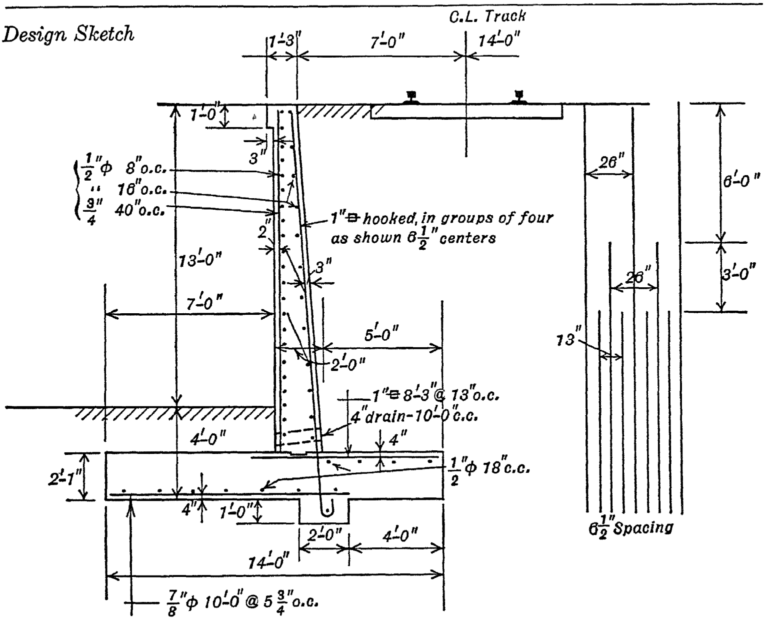
Temperature Steel $\frac{1}{2}''\phi$ @ 8″ o.c. front, $\frac{1}{2}''\phi$ @ 16″ o.c. back

$0.002 \times 12 \times 18 = 0.44\square''/'$

mean thickness

Use $\frac{2}{3} \times 0.44 = 0.29\square''/'$ in face

Design Sketch



CHAPTER X

HIGHWAY BRIDGES

77. Highway bridges of reinforced concrete are built as arches, as cantilevers and as continuous and non-continuous beams. This chapter deals with only the most common of these forms, the simple span, of which there are three general types: the slab bridge, used with economy for spans up to about 20 feet; the through and the deck beam bridge; the through and the deck girder bridge. In a beam bridge the load of the road slab is carried by beams which rest on the abutments; in a girder bridge, floor beams, with or without stringers, carry the floor slab, and in turn are supported by the main girders. A girder is usually defined as a beam that receives its principal load from other beams.[1] The ordinary limit of span for beam bridges ranges from 40 to 60 feet. For longer spans girder bridges are the rule on account of their greater rigidity. Perhaps the longest span on record is the 142 feet of the Salt River Bridge in California, described in the Engineering News Record of February 26, 1920. This is a half-through structure with both stringers and floor beams, 16 ft. roadway. Ordinarily the arch replaces the beam or girder for spans greater than about 60 feet if the foundation and other conditions permit.

In this chapter are given the design computations for a slab and a deck beam bridge. A through beam bridge is similar in cross-section to the slab bridge shown on page 149, differing from it in that the light balustrade is replaced by heavy beams carrying the roadway slab, which spans across from side beam to side beam. This type cannot be used economically for wider roadways than about 20 feet.

78. Concentrated Loads on Slabs. Highway bridge slabs must be designed to carry the heavy concentrations brought upon them by the wheels of modern motor trucks. The question arises

[1] A floor-beam is a transverse member at right angles to the axis of the bridge; stringers are longitudinal beams spanning between floor beams. The terms "beam" and "girder" are often used interchangeably.

at once as to the width of slab which supports any given concentrated load. This is illustrated in Fig. 51, where a wheel is shown resting at the center of a wide slab, supported along the edges *ab* and *cd*. The strip of slab, of width T, beneath this wheel cannot deflect under the load without at the same time causing the deflection of the adjacent strips, and in this way the effect of

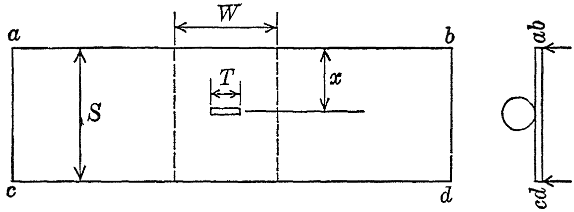


FIG. 51

the load is distributed over an indefinite width. Several experimenters have studied the problem, and on the basis of their data various rules have been proposed for use in design. Two well known specifications are reprinted below with some abbreviations, and their use is illustrated in the succeeding design computations.

American Railway Engineering Association: "*General Specifications for Steel Highway Bridges*," printed in the 1924 Proceedings, page 247. (These specifications are tentative and are now undergoing revision.)

(*31.*) In calculating the stresses due to wheel loads on concrete slabs, the wheel load shall be considered as distributed laterally over a strip of slab. The width of this strip measured parallel to the supports shall be determined by the following formulas in which

W = width of strip in feet, but never more than 6 feet.
T = width of tire in feet, taken as one inch for each 1000 lbs. of wheel load.
x = distance in feet from the center of the nearer support to the middle point of the line of contact of the tire with the slab. For shear the distance "x" shall be taken as $2\frac{1}{2}$ times the effective depth of the slab.

(*a*) Axles perpendicular to the supports:

$$W = \frac{4x}{3}.$$

(*b*) Axles parallel to the supports:

$$W = \frac{4x}{3} + T.$$

(*c*) If the position of the wheel loads is such as to make the individual strips overlap more than $\frac{1}{5}$ of the width of the narrower strip, the slab shall be designed for the sum of the intensities of the loads.

(*32.*) In calculating bending moments in stringers and floor beams wheel loads shall be assumed as concentrated at a point.

(*33.*) Each interior stringer and each floor beam without stringers shall be assumed to support that portion of the wheel loads determined by the following formulas, in which

L = load carried by one stringer or floor beam.
P = the wheel load.
S = the spacing of stringers or floor beams in feet.

For Stringers.

When the floor is designed for one truck

$$L = \frac{PS}{6}.$$

When the floor system is designed for two trucks

$$L = \frac{2\,PS}{9}.$$

When the stringer spacing is such that " L " would exceed " P " the stringer loads shall be determined by the reactions of the truck wheels, assuming the flooring between stringers to act as simple beams.

(*34.*) The live load supported by outside stringers shall be the reactions of the truck wheels assuming the flooring to act as a simple beam, but this live load shall in no case be less than would be required for interior stringers under the requirements specified.

For Floor Beams Without Stringers.

$$L = \frac{PS}{6}.$$

When the floor beam spacing is such that " L " would exceed " P," the floor beam loads shall be determined by the reactions of the truck wheel assuming the flooring between floor beams to act as simple beams.

(*35.*) In calculating the end shears and end reactions of stringers and floor beams, no lateral or longitudinal distribution of concentrated loads shall be assumed.

Specifications for distribution of concentrated loads proposed by Professor Ketchum in his "*Structural Engineers Handbook,*" 3d. Ed. enlarged, page 150. Using the same notation as before:

(*a*) The distribution of concentrated wheel loads for bending moments in reinforced concrete slabs with longitudinal supports shall be calculated by the formula

$$W = \tfrac{2}{3}\,(S + T)$$

with a maximum limit of 6 ft.

(*b*) The distribution of concentrated wheel loads for bending moments in reinforced concrete slabs with transverse girders shall be calculated by the formula

$$W = 2\,S/3 + T$$

with a maximum limit of 6 ft.

(*c*) The distribution of concentrated wheel loads for bending moments in slabs of girder bridges, in which the span of the bridge is not less than the width of bridge center to center of girders, shall be calculated for spans of 9 ft. or over by the formula

$$W = 2\,S/3$$

with a maximum limit of 12 ft.

(*d*) The effective width for shear in beams carrying concentrated loads shall be taken the same as for bending moment (*i.e.*, as calculated by the first two formulas above given) with a minimum effective width of 3 ft. and a maximum effective width of 6 ft. The total shear for an effective width of 3 ft. shall be considered as punching (pure) shear. The total shear for an effective width of 4.5 ft. and over shall be considered as beam shear (a measure of diagonal tension); for effective widths between 3 ft. and 4.5 ft. the total shear shall be divided proportionally between beam shear and punching shear; beam shear shall be used in calculating bond stress and as a measure of diagonal tension.

(*e*) In the design of longitudinal joists or stringers with concrete floors the fraction of the concentrated load carried by one stringer for spacings 6 ft. or less will be taken equal to the stringer spacing in feet divided by 6 ft. Outside stringers are to be designed for the same load as intermediate stringers.

(*f*) In the design of transverse stringers or floor beams with concrete floors, the fraction of the concentrated load carried by one floor beam for floor beams spaced 6 ft. or less, will be taken equal to the floor beam spacing divided by 6 ft. For floor beams spaced 6 ft. or over the entire reactions are assumed to be carried by one floor beam. Axle loads are assumed as distributed on a line 12 ft. long.

The reader is referred to the complete specifications for general information concerning the loads for which modern highway bridges should be designed.

79. Design of a Slab Bridge. (Computation Sheets SB1–SB2.) The example chosen was the design of a slab bridge to carry a 26-foot macadam roadway across a 16-foot clear opening. The loads are those usual for heavy traffic and are shown with other necessary data on the first computation sheet. The working stresses are those designated by the 1916 Joint Committee for the quality of concrete here used. The designer completes the assembly of data on this first sheet with a cross-sectional view that shows the general features of the structure. The balustrade and

curb will be poured after the slab has set and so the deeper section at the edge will be of no assistance in carrying load. The slab, then, acts as a rectangular beam about 29 feet wide with supports about 17 feet apart center to center.

The A.R.E.A. formula was chosen (see Computation Sheet SB1) to determine the width of slab supporting one wheel, but the same result, 6 feet, would follow from the use of Ketchum's specification. The moment and shear were computed for a strip of slab 12 inches wide, maximum moment occurring with the heavy wheel at the center of the span and maximum shear with it at the end, bringing the lighter wheel 5 feet on the bridge. Equations (17) and (1) were used to determine the depth required by the given stresses in bending and in shear respectively. The value of R was taken from Plate VI. As it happens both shear and moment require practically the same depth (d) and 2 inches were added to this value to give the total thickness. This provides about $1\frac{3}{4}$ inches of concrete below the steel to protect it from corrosion.

The actual value of d taken for the slab is so near to that theoretically necessary for balanced design that the steel area was calculated without discernible error by Equation (16) (see Computation Sheet SB2) using the approximate value for j of $\frac{7}{8}$. (Compare Art. 74.) The operation of choosing the steel and its spacing consists in dividing a number of bar areas by the area required per inch, thus determining the spacing for each size. To do this easily, set the area required per inch of slab on the B-scale of the ordinary Mannheim slide rule, below the left index of the A-scale; above the several bar areas on the B-scale read on the A-scale the corresponding spacing. There are only eleven standard bars and it is probably simpler to indicate each of these areas by a penknife scratch on the B-scale than to bother with the use of tables of bar sizes and spacings. The bond stress was found by Equation (9).

Even when no diagonal tension steel is needed, as in this case, it is customary always to bend up a portion of the main reinforcement, as shown on Computation Sheet SB2, to make the beam more dependable. This bent steel is arranged so that a few sloping bars cross the vertical plane through the slab above the face of the abutment, thus lessening the danger of any vertical crack at this section, such as might result from shrinkage or temperature restraint.

COMPUTATIONS FOR SLAB BRIDGE

Sheet SB1

Data:

Slab Bridge: *26′ Highway*

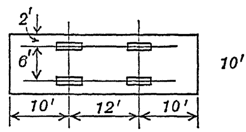
16′-0″ Span — Clear —

Wearing Surface: *Ordinary macadam surface carried across bridge*

Loading: *100#/□′*

20 Ton Trucks covering area →

14 T. on rear axles



Impact: *30%*

Materials: *Concrete: 2000#/□″ Ultimate Strength @ 28 ds.*

Steel: *Structural Grade — Deformed Bars*

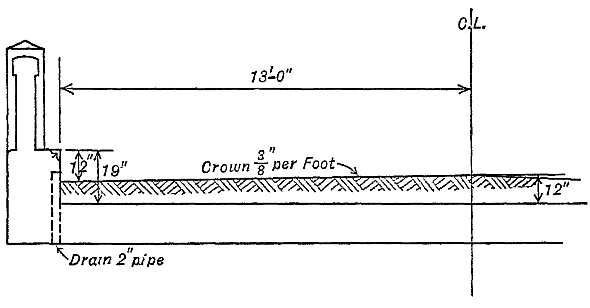
Stresses:

$f_s = 16{,}000\#/□''$. $f_c = 650\#/□''$. $n = 15$

$v = 40\#/□''$ *without diagonal tension reinforcement.*

$= 120$ *with diagonal tension reinforcement.*

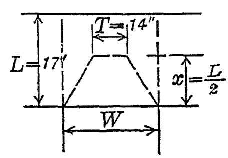
$u = 100\#/□''$



Slab:

Width of Slab supporting wheel load —

Assume 12″ abutment making span 17′-0″ c-c bearings



$W = \frac{4}{3}x + T$ (*A.R.E.A.*)

$= \frac{4}{3} \times \frac{17}{2} + 1.17 = 12.5' > 6'$.

Take $W = 6'$

Moment:

Live:	$\frac{14{,}000}{6} \times 17 \times \frac{1}{4} =$	*9,920′#*
Impact:	*30%*	*2,980*
Dead:		
12″ Rd. Surface	*130#/□′*	
Try 18″ Slab	*225*	
	$355\#/□' \times 17^2 \times \frac{1}{8} =$	*12,800*
		25,700′#

The cross reinforcement serves several useful functions. It binds the whole slab together and prevents longitudinal cracking. This ensures the integrity of the section, which is necessary if the effect of a concentrated load is to be distributed over a width of slab greater than the width of the area under bearing. The principal agent of this distribution is the shearing resistance of the concrete which tends to cause equal deflections of the strip of slab beneath the load and the strips adjacent on either side. These cross rods are often called distributing bars because they assist this action by carrying any cross tension incident to it. These bars are also useful in construction in holding the main reinforcement in place, the two sets of rods being wired together.

As a guide to the amount of this steel, the usual rule for temperature reinforcement was used (Art. 75), although of course the resistance to transverse contraction and expansion offered by the abutments is very much less than would be provided by rigid restraint along the sides of the span. A common practice is to use $\frac{1}{2}$-inch square bars spaced 12 inches on centers which furnishes about the same area as that here used.

With the short spans of slab bridges it is not necessary to take elaborate precautions to prevent temperature stresses due to the restraining action of the supports. A double layer of heavy tar paper at one end prevents adhesion of the bridge slab to the abutment. It is not uncommon with short spans to provide for movement at both ends in this same way.

The following is ordinary reinforcement for a hand rail such as that here used: vertically, $\frac{1}{2}$-inch square bars 12 to 18 inches on centers, with hooked ends well anchored in the slab; longitudinally, $2\frac{1}{2}$-inch square bars in the top and nominal temperature reinforcement below.

In order that no water shall stand on the bridge deck weep-holes or drains should be provided as shown. For pleasing appearance a slight camber, $\frac{1}{20}$ inch per foot, a total of about $\frac{3}{8}$ inch, should be given the structure. A slight sag below the true straight level of the supports, due, for example, to deflection of the form-work on pouring the bridge, is very unsightly.

80. Design of a Deck Beam Bridge. (Computation Sheets B1–B8.) For this example a deck beam bridge was chosen, with a clear span of 40 feet. The general data as to roadway, loads and stresses are the same as in the previous article (Sheet

COMPUTATIONS FOR SLAB BRIDGE

Sheet SB2

Slab: Continued

Shear	*Live* $\frac{1}{6}$ $(14{,}000 + \frac{5}{17} \times 6000) =$	$2630\#$
	Impact: 30%	790
	Dead: $355 \times 17 \times \frac{1}{2}$	3020
		6440

Depth (d) Moment: $d = \sqrt{\dfrac{M}{bR}} = \sqrt{\dfrac{25{,}700 \times 12}{12 \times 108}} = 15.4''$ $17\frac{1}{2}''$ *slab*

Shear: $d = \dfrac{V}{bjv} = \dfrac{6440}{12 \times \frac{7}{8} \times 40} = 15.3''$ "

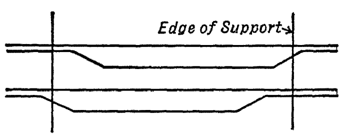
$17\frac{1}{2}''$ *thick — $15\frac{1}{2}''$ to steel.*

Steel:

$$A_s = \frac{M}{f_s j d} = \frac{25{,}700}{16{,}000 \times \frac{7}{8} \times 15.5} = 0.118\square''/'' \text{ of width.}$$

Try $\frac{7}{8}''\phi$ @ $5''$ *o.c.* $= 0.120\square''/''$.

$$\text{Bond} — u = \frac{vb}{\Sigma o} = \frac{40 \times 5}{\pi \times \frac{7}{8}} = 73 < 100\#/\square'' \text{ if all bars straight.}$$



Bend up $\frac{1}{2}$ of steel for diagonal tension.

Alternate straight and bent bars in bottom of slab

Practically $\frac{3}{4}$ of steel available for bond at end

$\therefore u = \frac{4}{3} \times 73 < 100$ O.K.

Main Reinforcement $\frac{7}{8}''\phi$ @ $5''$ *o.c.*

Cross Bars —

$0.002 \times 12 \times 17.5 = 0.42\square''/' \approx \frac{3}{4}''\phi$ @ $12''$ *o.c.*

Cross Reinforcement $\frac{3}{4}''\ \phi$ @ $12''$ o.c.

B1), except that a 2-inch bituminous wearing surface is used. (The method of laying such a surface is described on page 159 of Ketchum's Structural Engineers Handbook, 3d Edition, Enlarged.) The standard 40-foot bridge of the Division of Highways, Massachusetts Public Service Commission (1922), closely resembles the structure here designed, the same stresses and live loading being used for both.

81. Slab. (Computation Sheets B1 and B2.) The slab is treated as a rectangular beam, continuous over the several longitudinal supporting beams, and with a width equal to the length of the bridge. For simplicity a typical strip one foot wide is used in the computation. The first design, based on the A.R.E.A. specification, resulted in a 10-inch thickness which is considerably heavier than that ordinarily used in similar cases. By Ketchum's specifications a $7\frac{1}{2}$-inch slab is possible. It is good practice to set the limit of minimum thickness for bridge slabs at 6 inches.

Since the floor slab is continuous over the several longitudinal beams that support it the bending moment was reduced 20 per cent, the reason for this being discussed in the next chapter. It was assumed that the maximum positive moment at mid span equals the maximum negative moment over the supports, which is probably reasonably near the truth.

Perhaps the most peculiar feature in these figures at first glance is that the maximum shear is not computed, but, instead, the shear that accompanies maximum moment. This is because shearing stress is used as a measure of diagonal tension and bond, both of which will be extremely small when the shear is a maximum, there being practically no moment when the load is placed $16\frac{1}{4}$ inches (the value of x, A.R.E.A., par. 31; quoted in Art. 78) from the center of the support. If the maximum shear is computed the proper unit stress to use in judging its effect is that commonly set for true or punching shear, 6 per cent of the 28-day ultimate strength.

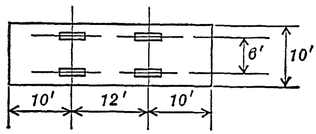
The provision near the top of Sheet B2 for a higher shearing stress with anchored bars is that of Art. 191*a*, 1924 Joint Committee. (See Appendix B.)

The bond stress in the chosen design (the $7\frac{1}{2}$-inch slab) is more than 100 pounds per square inch and so special anchorage must be given the bars. A similar situation is discussed in Art. 74, page 138. This bond stress is for the tension reinforcement in the top

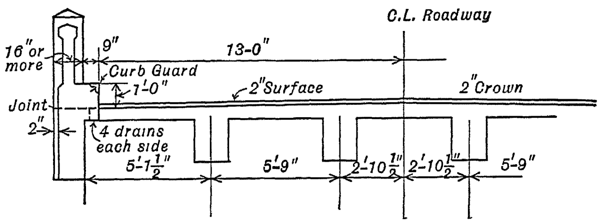
COMPUTATIONS FOR BEAM BRIDGE

Sheet B1

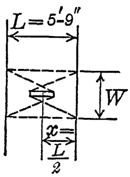
Data:
Beam Bridge: 26 ft. Highway. 40 ft.-0 in. Clear Span.
Bituminous Wearing Surface: 2 in. thick.
Loading: 100 lbs. per sq. ft.
20 Ton Trucks covering area →
14 Ton rear axle
Impact: 30%



Material: Concrete: 2000#/□″ Ultimate Strength @ 28 ds.
Steel: Structural Grade: Deformed Bars
Stresses: $f_s = 16{,}000\#/\square''$. $f_c = 650\#/\square''$. $n = 15$
$v = 40\#/\square''$ *without diagonal tension reinforcement.*
$= 120\#/\square''$ *with diagonal tension reinforcement.*
$u = 100\#/\square''$



Slab:
Moment. Assume 15″ beams making clear span of slab = 4′-6″

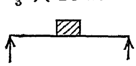


W = *width of slab supporting 1 wheel*
$= \frac{4}{3}x$ *(A.R.E.A.)*
$= \frac{4}{3} \times \frac{69}{2} = 46'' = 3.83'$

M:

Live: $\frac{14{,}000}{3.83} \times 4.5 \times \frac{1}{4} \times \frac{8}{10} =$		$3290'\#$
Impact: 30%		990
Dead: 2″ Surface	$25\#/\square'$	
Try 8″ slab	100	
	125	
$125 \times \overline{4.5}^2 \times \frac{1}{8} \times \frac{8}{10} =$		250
$M =$		$4530'\#$

Shear: Here x *(see Art. 78, par. 31)* $= 2.5 \times 6.5 = 16.25''$ *for 8″ slab,* $d = 6.5''$
$W = \frac{4}{3} \times 16.25 = 21.7'' = 1.8'$



V:

←*Live:* $\frac{14{,}000}{1.8} \times \frac{1}{2}$	$3890\#$
Impact: 30%	1170
Dead: $125 \times 4.5 \times \frac{1}{2} =$	280
$V =$	$5340\#$

of the slab over the supports. The anchorage required is the extension of a portion of the steel beyond the point of inflection. (See Arts 139 and 140*a*, Joint Committee, Appendix B.) This requirement is far exceeded by the arrangement of steel described in the next paragraph.

Equal areas of tension steel are required in the bottom of the slab between beams and in the top over the beams. It is possible to furnish the required area of top steel satisfactorily by a combination of straight and bent rods, various arrangements being common. The use of an equal number of straight rods in the top and in the bottom is believed to give a better solution of the problem. When two trucks are passing on the bridge there will be need of tension steel in the top across the whole width of the middle panel. A single truck at mid panel anywhere on the bridge will cause tension in the top of the slab in the adjacent panels for the whole width also. There should be steel in the slabs placed to carry these stresses.

82. Inside Beams. (Computation Sheets B3–B5.)

(*a*) *Loads and Stresses.* While differing in detail both specifications give in this case the same proportions of the weight on an axle for the load on a beam, namely, one wheel load. The sketches on the third sheet of computations show the various loadings tried in search of the maximum live load shear and require no explanation. The sketch for maximum moment shows a front and a rear wheel of a single truck placed on the beam in such a position as to cause maximum moment in the span, in accordance with the well known theorem that the center of the span should lie midway between the resultant of the loads and the proper adjacent wheel, in this case, plainly, the heavier one. In addition, the uniform live load of 100 pounds per square foot, 575 pounds per foot of beam, was placed to cover all of the beam not covered by the truck, and the possible effect of this uniform load upon the position of the truck for maximum moment was neglected.

Before the dead load stresses are known the size of the beam stem must be fixed, which may be done approximately by computations on scratch paper, giving a trial size to be used as the basis of further investigation. The stem of a simple tee beam, such as these, must be large enough to provide ample resistance to the diagonal tension stresses, measured by shearing stress, and also large enough to allow the longitudinal reinforcement to be

COMPUTATIONS FOR BEAM BRIDGE

Sheet B2

Depth: (*d*) *Moment:*

$$d = \sqrt{\frac{M}{bR}} = \sqrt{\frac{4530}{108}} = 6.5''. \quad 8'' \text{ slab}$$

Shear:

$$d = \frac{V}{bjv} = \frac{5340}{12 \times \frac{7}{8} \times 40} = 12.7''. \quad 14'' \text{ slab}$$

By anchoring bars v may be raised to 60#/□″

$$d = \frac{40}{60} \times 12.7 = 8.5'' \quad 10'' \text{ slab}$$

As 10″ slab is very heavy try results by Ketchum's Specifications.

Alternate Design for Slab:

W = width of slab supporting wheel (Sketch-Sheet B1)

$= \frac{2}{3}(S + T) = \frac{2}{3}(69 + 14) = 55.3$ in. $= 4.6'$ (Ketchum)

Moment:	L	$\frac{14{,}000}{4.6} \times 4.5 \times \frac{1}{4} \times \frac{8}{10}$	= 2740′#
	I	30%	820
	D	(8″ slab)	250
			3810′#/′
Shear	L	$\frac{14{,}000}{4.6} \times \frac{1}{2}$	= 1520#
	I	30%	460
	D		280
			2260#/′

Depth: Shear $d = \frac{V}{bjv} = \frac{2260}{12 \times \frac{7}{8} \times 40} = 5.4''$

Since W > 4.5′ beam shear only to be taken.

Moment $d = \sqrt{\frac{M}{bR}} = \sqrt{\frac{3810}{108}} = 6''.0$ *Use $7\frac{1}{2}''$ slab.*

Steel. $A_s = pbd = 0.0077 \times 1 \times 6 = 0.0462\square''/'' \approx \frac{5}{8}''\phi$ @ $6\frac{1}{2}''$ o.c.

Bond $u = \frac{vb}{\Sigma o} = \frac{\left(\frac{5.4}{6} \times 40\right) 6.5}{\pi \times \frac{5}{8}} = 119\#/\square''$ $\therefore$ *Anchor bars*

Use $7\frac{1}{2}''$ slab, 6″ to steel, $\frac{5}{8}''\phi$ @ $6\frac{1}{2}''$ o.c. top and bottom anchored.

Longitudinal Steel — $7\frac{1}{2}''$ slab; 4′-6″ clear span.

$A_s = 0.002\,(7.5)(4.5 \times 12) = 0.81\square''$ $5\text{-}\frac{1}{2}''\phi = 0.98\square''$

Use 3-$\frac{1}{2}''\phi$ in bottom 2-$\frac{1}{2}''\phi$ in top per panel.

placed with proper clearances. It is also desirable that the proportions be such that the design be economical. The preliminary scratch computations whose results are used on Sheet B3 were based on the need of providing a beam with sufficient shearing (diagonal tension) strength and the recorded calculation of the required cross-section area shows that the assumed size and weight is satisfactory for that purpose. The proportions shown in the sketch are within the limits set by the common rule of making the depth between two and three times the breadth, but the depth is somewhat shallower than that set by another equally common rule which makes the depth in inches about equal to the span in feet.

(*b*) *Proportions.* In this simple situation a formula for economic proportions has some justification. The following

$$d = \sqrt{\frac{rM}{f_s b'}} + t/2$$

was devised by Professors Turneaure and Maurer in their book "Principles of Reinforced Concrete Construction" (third edition, page 186), and is here used for illustration (Sheet B4). The term r is the ratio of the cost per unit volume of concrete and steel, that for steel being increased by 25 to 40 per cent to cover the cost of stirrups and other items not considered in the derivation.[1] Its use shows that for economy a considerable increase should be made in the depth of the 14-inch beam, the total becoming 39.9 + 3.5 = 43.5 inches. This is relatively rather slender, so the 16-inch breadth with d = 38 inches, total 41.5 inches, was chosen arbitrarily.

[1] The derivation is as follows, using the data of Fig. 52: It is assumed that no appreciable difference is made in the cost by the small difference in form work involved. Let c = unit cost of the concrete and rc = unit cost of the steel. Then the total cost of the stem per unit length, exclusive of the portion below the center of the steel, is

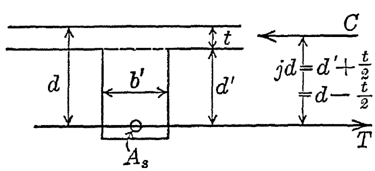


FIG. 52

$$C = c\left(b'd' + \frac{rM}{f_s(d' + \frac{1}{2}t)}\right).$$

Differentiating with regard to the variable d', and equating the result to zero, gives for minimum cost

$$d' + t/2 \quad \text{or} \quad d - t/2 = \sqrt{rM/f_s b'}.$$

COMPUTATIONS FOR BEAM BRIDGE

Sheet B3

Inside Beams:

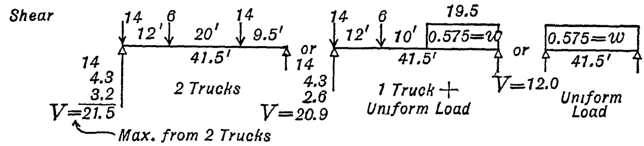
Assume 18″ abutments: $\therefore$ *Span* $= 40' + 1.5' = 41.5'$

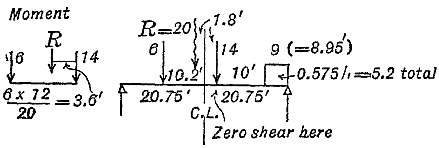
Proportion of one wheel load carried by one beam

$$L = \frac{2\,PS}{9} = \frac{2 \times 5.75}{9} P > P \text{ (A.R.E.A.)}$$

$\therefore$ *Assume one beam carries total load on wheel. (Assuming 21″ clear between trucks.)*

Live Load Stresses:

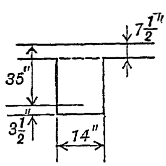




Moment: $\dfrac{20 \times \overline{18.95}^2}{41.5} = 173{,}000\#'$ or $\dfrac{wL^2}{8} =$

$5.2 \times \dfrac{4.5}{41.5} \times 22.55 = \dfrac{12{,}800}{185{,}800'\#}$ — *Max.* $\dfrac{0.575 \times 41.5^2}{8} = 124{,}000$

Stresses: Dead:



Trial Size

Wearing Surface	$25\#/\square'$	
7½″ Slab	94	
	119×5.75	$= 684\#/'$
Stem (Trial)		452
Total		$1140\#/'$

$\dfrac{150}{144} \times 14 \times 31 = 452\#/'$

Shear: $V = \frac{1}{2} \times 1140 \times 41.5 = 23{,}700\#$

Mom: $M = 1140 \times \overline{41.5}^2 \times \frac{1}{8} = 245{,}000'\#$

Stress Summary

	V	M
Dead:	23,700#	245,000′#
Live:	21,500	185,800
Impact 30%	6,500	55,800
	51,700	486,600

Stem:

Shear $\left\{ b'd = \dfrac{V}{jv} = \dfrac{51{,}700}{\frac{7}{8} \times 120} = 492\square'' \right.$ $b' = 14''$, $d = 35''$

total depth $= 38.5''$

(*c*) *Steel.* Choice was made of eight large bars to give the required steel area (Sheet B4) as trial calculations show that the use of more than eight makes necessary three layers of rods, which is not desirable. It is customary, though not universal practice, to use an even number of bars and make the beam symmetrical in every respect about the vertical axis. It is also good practice to use not more than two sizes of bars in any beam and make those two adjacent sizes, that is, within $\frac{1}{8}$ inch of each other. The requirements as to bar clearances are illustrated by the sketch on this sheet. (Compare Art. 63, 67 of the Joint Committee 1924, in footnote.)[1] It is obvious that there must be a sufficient mass of concrete about each rod to transmit the shearing stresses set up. (See the derivation of Equation (9), Art. 54.) It would be an interminable task to calculate this embedment afresh for each beam designed, and unnecessary, as the standard spacing rules give safe results for ordinary beams with not more than two layers of steel. When there are more than three layers the variation of stress in them may be of importance; the spacing of the rods, both horizontally and vertically, should be generous. For this beam the spacings conform closely to the rules laid down by Taylor, Thompson and Smulski in "Concrete, Plain and Reinforced," Vol. I, page 273. Horizontally the clear spacing is limited to $1\frac{1}{2}$ times the diameter of a round rod, or two times the side of a square rod, or $1\frac{1}{4}$ times the maximum size of aggregate, with a minimum of 1 inch. Vertically the clear spacing is limited to 1 inch. A more conservative rule limits the vertical spacing to 1 inch or to

[1] (**63.**) **Placing.** Metal reinforcement shall be accurately positioned, and secured against displacement by using annealed iron wire of not less than No. 18 gauge, or suitable clips at intersections, and shall be supported by concrete or metal chairs or spacers, or metal hangers. The minimum clear distance between parallel bars shall be one and one-half ($1\frac{1}{2}$) times the diameter of round bars, or one and one-half ($1\frac{1}{2}$) times the diagonal of square bars; if the ends of bars are anchored as specified in Section 140, the clear spacing may be made equal to the diameter of round bars, or to the diagonal of square bars, but in no case shall the spacing between bars be less than 1 in., nor less than one and one-quarter ($1\frac{1}{4}$) times the maximum size of the coarse aggregate. Bars parallel to the face of any member shall be embedded a clear distance of not less than one (1) diameter from the face.

(**67.**) **Moisture Protection.** Metal reinforcement in wall footings and column footings shall have a minimum covering of 3 in. of concrete. At surfaces of concrete exposed to the weather metal reinforcement shall be protected by not less than 2 in. of concrete.

COMPUTATIONS FOR BEAM BRIDGE

Sheet B4

Inside Beams: (*Continued*)

Economical depth

$$d = \sqrt{\frac{rM}{f_s b'}} + \frac{t}{2} \quad \text{where } r = \frac{492 \times 0.045}{12.00/27} = 50 \text{ approx.}$$

$$= \sqrt{\frac{50 \times 486{,}600 \times 12}{16{,}000 \times b'}} + 3.75$$

$$= 39.9'' \quad \text{for } b' = 14''$$
$$= 37.5'' \quad = 16''$$
$$= 35.5'' \quad = 18''$$

Try
$b' = 16''$
$d = 38''$
Total depth $= 41.5''$

Revised Shear and Moment

$$\Delta w = 7 \times 16 \times \frac{150}{144} = 117\#/'$$

$$\Delta V = \tfrac{1}{2} \times 117 \times 41.5 = 2400\# \qquad \text{Total } V = \underline{54{,}100\#}$$

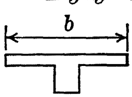
$$\Delta M = \tfrac{1}{8} \times 117 \times \overline{41.5}^2 = 25{,}300 \qquad \text{"} \quad M = \underline{512{,}000'\#}$$

$$\text{Unit Shear: } v = \frac{54{,}100}{16 \times \frac{7}{8} \times 38} = 102\#/\square'' < 120 \qquad \text{O.K.}$$

$$\text{Steel: } A_s = \frac{M}{f_s jd} = \frac{512{,}000 \times 12}{16{,}000 \times 0.90 \times 38} = \frac{11.24}{11.08} \qquad \text{Try } \begin{array}{l} 4\text{-}1\tfrac{1}{4}''\square = 6.24 \\ 4\text{-}1\tfrac{1}{8}''\square = \underline{5.07} \\ \qquad\qquad 11.31\square'' \end{array}$$

Check of f_c $\qquad 0.915$

$$b \begin{cases} L/4 = 41.5/4 = 10.4' \\ 16 \times 7\tfrac{1}{2} + 16 = 136'' \\ \text{Spacing} = 5'\text{-}9'' = 69'' \leftarrow \end{cases}$$

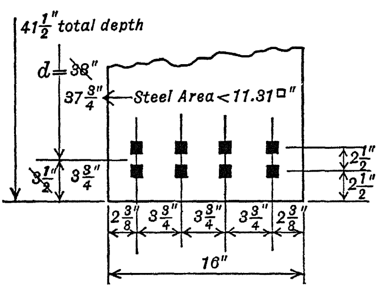


$$\text{Pl. VII} \begin{cases} p = \dfrac{11.31}{69 \times 38} = 0.0043 \\ t/d = 7.5/38 = 0.2 \end{cases}$$
$$j = 0.915$$

No change in steel.

$f_s/f_c = 31 \pm$

$$f_c < \frac{16{,}000}{31} < 520\#/\square''$$



Steel Spacing

Spacing of Bars
$3 \times 1\tfrac{1}{4} = 3\tfrac{3}{4}''$

Top Layer — $4\text{-}1\tfrac{1}{8}''$ □ *bent*

Bottom Layer $4\text{-}1\tfrac{1}{4}$ □ *straight*

$$\text{Bond} — u = \frac{vb}{\Sigma o} = \frac{102 \times 16}{4 \times 4 \times 1\frac{1}{4}} = 82\#/\square'' < 100 \qquad \text{O.K.}$$

No anchorage required.

the diameter of the largest bar. This will allow mortar to work freely around the rods even if the maximum aggregate cannot pass. As shown on this sheet the rods are a trifle too close to the surface of the concrete, a minimum of 2 inches being desirable. The use of some one of the several varieties of bar supports and spacers now on the market is strongly urged to ensure the proper placing of this steel.

If the center of gravity of the steel area is assumed as halfway between the two layers, as is customary, the value of d is slightly less than that used in calculations. The difference is too small to be regarded.

These intermediate beams are tee beams and the limits to the width of slab that may be assumed to act integrally with the stem are indicated in the sketch on Sheet B4. These rules are those of the 1924 Joint Committee (Art. 115, Appendix B). The assumed value of j used in Equation (16) and the ratio of fiber stresses were checked by Plate VII. Since there is an excess of steel, f_s is less than 16,000 pounds per square inch but the exact stress is not of interest.

It was assumed (bottom of Sheet B4) that the lower layer of bars is left straight in the bottom the length of the beam and the bond stress was calculated accordingly.

The assumption was made that the maximum possible bending moment at any section of the beam is equal to the ordinate at that section to a parabola drawn (as shown on Sheet B5) with the maximum moment already computed as the center ordinate. This would be exact were the live load either a uniform or a single moving concentrated load and, as it is, gives results close to the truth. On this basis the limiting positions of the points of bend were found.

The design of these interior beams was completed by making a sketch to give all the essential information regarding them and to serve as basis for the design of the diagonal tension reinforcement. It was decided to arrange the bent steel so that one pair of bent rods cuts the vertical plane at the support, with the second pair close enough to the first so that no stirrups are required between the points of bend. Since the distance between these points, 24 inches, is less than $\frac{3}{4}$ d, this is accomplished if the sloping rods can carry the stress in that distance. For stirrup design it is sufficiently accurate to use the clear span, taking the shear

COMPUTATIONS FOR BEAM BRIDGE

Sheet B5

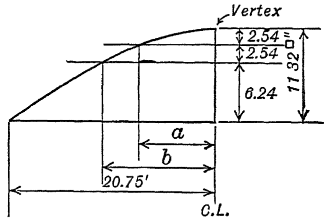
Inside Beams: (*Continued*)

Points of bend —

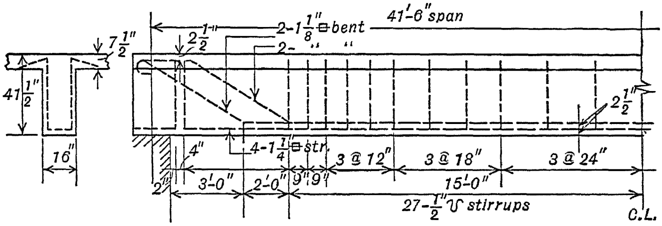
Assume parabolic moment curve →

$$a = 20.75\sqrt{\frac{2.54}{11.32}} = 9.9'$$

$$b = 20.75\sqrt{\frac{5.08}{11.32}} = 13.9'$$

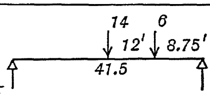


Design of Inside Beams.



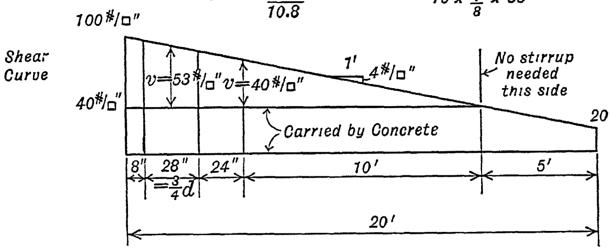
Diagonal Tension Reinforcement

Max. Shear @ C.L.



	7
	1.3
Live	8.3
30% Imp.	2.5
Dead	0.0
	10.8

$$v = \frac{10{,}800}{16 \times \frac{7}{8} \times 38} = 20\#/\square''$$



Stress in 1st pair of bent rods

$$f_s = \frac{\frac{3}{4}v'bs}{A_s} = \frac{\frac{3}{4} \times 53 \times 16 \times 28}{2 \times \overline{1.125}^2} = 7100\#/\square'' < 16{,}000\#/\square'' \quad \text{O.K.}$$

∴ *2nd pair also O.K.*

Stirrups. Try $\frac{1}{2}''\ \phi$ ⊔ @ *16,000#/□″* ≈ *2 × 0.196 × 16,000* ≈ *6270# each*

Spacing:

End.

$$s = \frac{S}{v'b} = \frac{6270}{60 \times 16} = 6\tfrac{1}{2}'' \quad \textit{Use two stirrups} —$$

Inside of bent bars. *Max. Spacing* $= \frac{d}{2} = 19''$

$$s = \frac{6270}{40 \times 16} = 10''$$

at the edge of the support as 100 pounds per square inch, with a straight line variation of shearing stress down to the maximum intensity at the center of the beam. Any possible shear curve due to any loading will fall within the shear curve thus drawn, and accordingly the reinforcement proportioned by its use will be adequate.

The intensity of stress in the pair of bent rods nearer the support was computed by aid of Equation (7) (assuming that the concrete carries diagonal tension to an amount measured by a unit shear of 40 pounds per square inch) and is very small. It is plain the stress in the second set of rods is still less and that the length from the lower bend nearer the centre to a distance of $\frac{3}{4} \times 38 = 28$ inches toward the support from the other bottom bend is cared for properly by the bent rods without stirrups.

Stirrups are needed in the 8 inches at the end of the beam not reinforced by the large bars. The $\frac{1}{2}$-inch round stirrups used must be spaced $6\frac{1}{2}$ inches apart, or less, at the end and so two are required. The necessary stirrups in the 10 feet inside the bent rods were placed by making enough spaces, about equal to the minimum computed, to reach to the section where the shear permits change to the next practical spacing, and so on. If the sketch is to scale and the rate of change of shear per foot is shown, it is simple to do this mentally. Stirrups should be used throughout the beam for the purpose of tying the stem and flange together even though not required by shear. Many designers would place the stirrups in this beam without counting at all upon the bent rods.

Problem 31. Compute the maximum total moment at the quarter point of an interior beam and compare with the value given by the assumption of parabolic variation of moment made on Computation Sheet B5.

Ans. 390,800′# $\frac{3}{4} \times 512{,}000 = 384{,}000$′#.

Problem 32. Compute the maximum intensity of shear at the quarter point of an interior beam and compare with the value given by the assumed shear curve on Computation Sheet B5.

Ans. 58 lbs./sq. in. 60 lbs./sq. in.

83. Outside Beams. (Computation Sheets B6–B7.) The outside beams were designed for the same live moment and shear as the intermediate ones, although normal traffic will not cause these stresses. The possibilities of accident make this a wise plan

COMPUTATIONS FOR BEAM BRIDGE

Sheet B6

Outside Beams:

Live Shear and Moment same as for inside beams

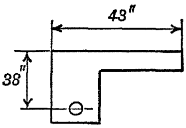
Dead V & M:

Weight of parapet above slab (and curb)		500#/′
Weight of slab and wearing surface $\frac{1}{2} \times 4.5 \times 119$		268
Weight of beam:	Try 16×42	700
	$d = 38''$	1468#/′

$$V = \tfrac{1}{2} \times 1468 \times 41.5 = 30{,}500\#$$
$$M = \tfrac{1}{8} \times 1468 \times 41.5^2 = 316{,}000'\#$$

	Shear	Moment
Live	21,500	185,800
Impact	6,500	55,800
Dead	30,500	316,000
	58,500	557,600

Trial Section: $d = 38''$



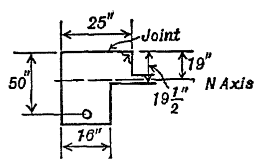
$$v = \frac{58{,}500}{16 \times \frac{7}{8} \times 38} = 110\#/\square''$$

$$A_s = \frac{557{,}600 \times 12}{16{,}000 \times 0.91 \times 38} = 12.09 \qquad \begin{aligned} 6\text{-}1\tfrac{1}{4}\,\square &= 9.38 \\ 2\text{-}1\tfrac{1}{8}\,\square &= 2.53 \\ &\overline{11.91\square''} \end{aligned}$$

$$\left.\begin{aligned} p &= \frac{11.91}{43 \times 38} = 0.0073 \\ t/d &= 7.5/38 = 0.2 \end{aligned}\right\} \begin{aligned} j &= 0.91 \\ \frac{f_s}{f_c} &= 22 \end{aligned} \qquad f_c = \frac{16{,}000}{22} = 730$$

$$> 0.7 \times 650$$
$$\therefore \text{VOID}$$

Try as unsymmetrical rectangular beam.



$$R = \frac{M}{bd^2} = \frac{557{,}600 \times 12}{25 \times \overline{50}^2} = 107 \qquad \begin{cases} k = 0.375 \\ p = 0.0076 \end{cases}$$

From Pl. VI stresses $= 16{,}000 - 650$

$$A_s = pbd = 0.0076 \times 25 \times 50 = 9.63\square''$$

$$\begin{cases} 6\text{-}1\tfrac{1}{8}\square = 7.59 \\ 2\text{-}1''\square = 2.00 \end{cases} \qquad \overline{9.59\square''}$$

Shear: $v = \dfrac{V}{b'jd} = \dfrac{58{,}500}{16 \times \frac{7}{8} \times 50} = 84\#/\square''$

Bond — on 4-$1\frac{1}{8}$ □ straight $u = \dfrac{vb}{\Sigma o} = \dfrac{84 \times 16}{4 \times 4 \times 1\frac{1}{8}} = 75 < 100\#/\square''$

---

and it is required by both of the load specifications. The dead moment and shear were figured, assuming a section below the slab about equal to that given the interior beams.

The first investigation of this section (Sheet B6) was on the assumption that a construction joint comes at the top of the slab and that the slab acts as a flange, giving a non-symmetrical *L* section. Taylor, Thompson and Smulski recommend ("Concrete, Plain and Reinforced," Vol. I, page 142) that for such a beam the allowable stresses be reduced 30 per cent.

This section being entirely inadequate, study was next made of the rectangular beam (Sheet B6) made by including the 12-inch curb, which must be poured at the same time as the slab to permit of this. The full width of 25 inches was taken on first trial and the lack of symmetry ignored, noting that the neutral axis comes in the slab. Since a curb guard, made of a steel angle section, will extend the length of the bridge, this full width is not properly available. The lack of symmetry is not a desirable feature. Since a sturdy beam is wanted the 8 bars (6–$1\frac{1}{8}$-inch, and 2–1-inch squares) found necessary were changed to the largest size available, $1\frac{1}{4}$ inches square, and approximate computations made to determine the width of compression face called into play without exceeding the given stresses. A direct computation by means of the transformed sections would be rather laborious. So a width was assumed and by aid of Equation (17) and Plate VI the steel area required was found. Evidently a width between 21 inches and 23 inches requires the 12.5 square inches furnished. Since this arbitrary increase in steel results in neglecting only between 2 and 3 inches of the total top width it seemed advisable to use the 8-inch heavy rods. The rest of the design is similar to that of the interior beams.

84. Expansion Rocker. (Computation Sheet B8.) To prevent high temperature stresses and undesirable cracking in bridge and abutments, provision must be made for free expansion and contraction of the structure with temperature changes. The manner of making this provision varies with the length of span and no definite rule of practice can be given. Some standards would require a rocker bearing at one end for a span as short as this and others would use a sliding support of metal plates for spans up to 50 feet. In this design expansion is provided for by supporting one end of each beam on a cast iron rocker, placed

COMPUTATIONS FOR BEAM BRIDGE

Sheet B7

Try b — required for $8\text{-}1\tfrac{1}{4}\ \square = 12.5\square''$ *max. area possible in 2 layers.*

$\text{for } b = 23''\qquad R = 114.\quad p = 0.0092.\quad kd = 21''\pm$

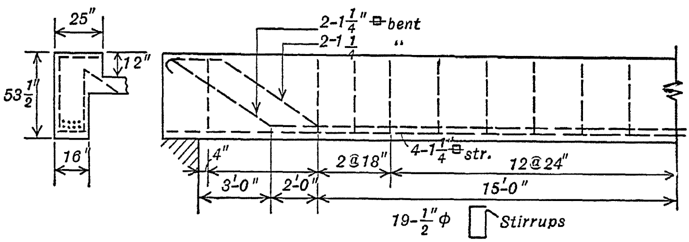
$A_s = 10.6\square''$

$\text{for } b = 21''\qquad R = 125.\quad p = 0.0128.\quad kd = 23''\pm$

$A_s = 13.4\square''$

Use $8\text{-}1\tfrac{1}{4}\ \square$ *to compensate for lack of symmetry — uncertain action of curb — neutral axis below slab — etc.*

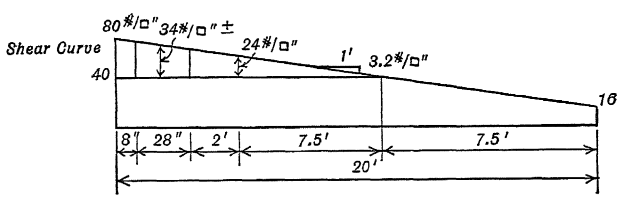
Outside Beams — Design



Diagonal Tension Reinforcement.

@ *Centre*

$$v = \frac{10{,}800}{16 \times \frac{7}{8} \times 50} = 16\#/\square''$$



From previous computations — (Sheet B5) bent bars are O.K.

Stirrups: $\tfrac{1}{2}''\phi$ U @ $6270\#/\square''$

Spacing:

End: $\quad s = \dfrac{6270}{40 \times 16} = 10''.$ *Use 1* U *stirrup*

Inside Bent Bars — Max. $s = \dfrac{d}{2} = 25'' \qquad s = \dfrac{40}{24} \times 10 = 17''$

between steel bearing plates, proportioned to bring the bearing stresses on the concrete within the given limit of 500 pounds per square inch. The bearing of the cast iron rocker on steel is limited to $300D$ pounds per inch of length, where D is the diameter of the rounded surface of contact. The length of the rocker is made 2 inches less than the width of the beam, giving a net width of 12 inches and a required diameter, and also height of 16 inches. A thickness of $2\frac{1}{2}$ inches, about the minimum advisable, results in very low column stresses. Two small lugs are arranged to key into holes in each plate and prevent the overturning of the rocker.

When the beams are ready for pouring each rocker is in place in its asphalt-filled pocket, with the upper steel plate resting on it and on the felt joint. This plate is kept in place during pouring by a mortar joint about the edge. A piece of tar paper on top keeps concrete from clogging the key holes.

COMPUTATIONS FOR BEAM BRIDGE

Sheet B8

End Bearings:

Fixed End — $f_b = 500\#/\square''$ *allowable*

$$A = \frac{57{,}300}{500} = 115\square'' \quad \text{With } 18'' \text{ abutment}$$

$$16 \times 18 = 288\square'' \text{ available. O.K.}$$

Expansion End. C. I. Rocker

Allowable bearing of C. I. Rocker on steel plate —

$f_b = 300\ D$ *per inch length (Ketchum's Spec. for Hy. Br.)*

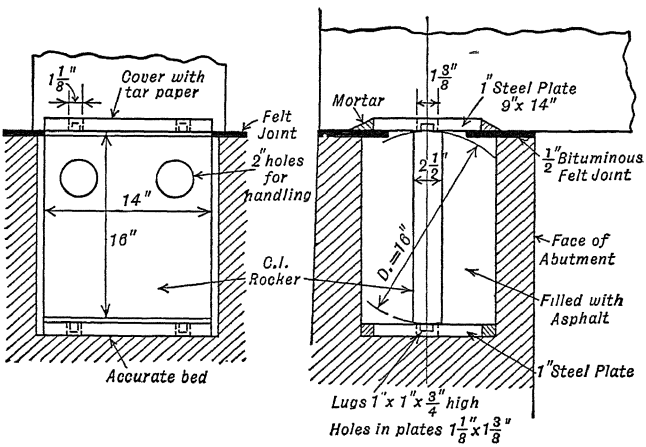
$$LD = \frac{57{,}300}{300} = 191 \qquad \begin{matrix} D = 14'' \text{ or } 16'' \\ L = 14'' \text{ or } 12'' \end{matrix} \qquad \left.\begin{matrix} D = 16'' \\ L = 14'' \end{matrix}\right\} \text{Used}$$

Expansion $= 80 \times 0.0000065 \times 41.5 \times 12 = 0.26''$

Plates — $14 \times 9 = 126 > 115$ *required*

Rocker thickness — $f_b = 9000 - 40\ L/r$ *(Ketchum)*

$$f_b = \frac{57{,}300}{14 \times 2.5} = 1700\#/\square'' \quad \text{O.K.}$$

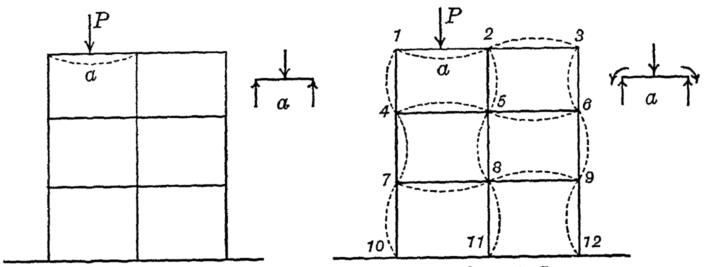


The rocker here shown is closely the same as that on P412, Reinforced Concrete and Masonry Structures, Hool and Kinne.

CHAPTER XI

CONTINUOUS BEAMS AND RIGID FRAMES

85. A reinforced concrete structure is, in effect, a monolith, a unit in itself, and not merely an assemblage of individual beams and columns. Any load causes stress, not only in the members immediately supporting it, but also in every other member of the frame, the magnitude of this effect rapidly decreasing with the distances of the unit from the load. In ordinary steel construction the joints are not generally rigid enough to develop large bending resistance. The only parts of the steel frame shown in



Steel Frame *Reinforced Concrete Frame*

Shape under load shown by dotted lines.

FIG. 53

Fig. 53, stressed by the load P, are the beam beneath the load and the two supporting columns. In the similar reinforced concrete frame, Fig. 53, the rigidity of the joints causes every member to deform under the load. Accordingly the steel beams are designed as simple end-supported members, and the columns as members carrying direct stress only.[1] In a reinforced concrete frame the beams are restrained at the ends by their rigid connections, and these end moments must be considered in design. The columns likewise are subject to both direct stress and bending.

[1] The steel frames of tall buildings exposed to the horizontal force of the wind are made rigid enough to carry the resultant stresses by rigid construction at the joints. The columns and beams are proportioned to resist the resulting combination of direct stress and bending. Every steel frame must be made stable by proper bracing.

The perfect continuity of reinforced concrete structures is weakened by construction joints, formed when fresh concrete is poured in contact with concrete that has already set. Commonly reinforcing steel crosses such joints and the break in continuity is not regarded. To make such a joint capable of transmitting shear as well as bending the surfaces of contact must be keyed together, either by a formed mortise or by roughening the surface and the use of projecting stones. If the joint is smooth with no steel crossing it, neither shear nor bending can be considered as transmitted.

It is evident that the moments and shears in any beam of a rigid frame cannot be determined by the principles of statics alone since there are two unknown end moments, two unknown vertical reactions and possibly two unknown horizontal reactions, a total of 6 unknowns, 3 more in number than the conditions of equilibrium of a non-concurrent co-planar force system. The exact determination of the maximum moments, shear and direct stress in any member of a monolithic structure is a very complicated matter, and in consequence, for ordinary structures, simple rules have been devised for obtaining these stresses that give results close enough to the truth for safe and economical design. For unusual structures special methods must be used. The two most important of these methods, those of Least Work and Slope Deflection, are described in this chapter.

86. Continuous Beams, Theorem of Three Moments. A simple case of a continuous beam of reinforced concrete was furnished by the floor slab of the beam bridge designed in Chapter X. Here the slab is to be poured at the same time as the beam stems and so forms a rectangular beam, rigidly attached to its supports, continuous over several spans. The Theorem of Three Moments offers a convenient means of studying the stress in such a case. This theorem is an equation expressing the relation that exists between the bending moments in a continuous beam at any three consecutive supports, and for a beam of uniform moment of inertia, supported on knife edges, all either at the same level or at the proper elevation to fit the unloaded beam, it takes this form:

$$M_1L_1 + 2\,M_2(L_1 + L_2) + M_3L_2 = -\tfrac{1}{4}\,w_1L_1^3 - \tfrac{1}{4}\,w_2L_2^3 - \Sigma P_1L_1^2(K_1 - K_1^3) - \Sigma P_2L_2^2(2\,K_2 - 3\,K_2^2 + K_2^3), \quad (40)$$

where (see Fig. 54)

M_1 = the bending moment at support 1 in foot-pounds;
M_2 = the bending moment at support 2 in foot-pounds;
M_3 = the bending moment at support 3 in foot-pounds;
L_1 = the length of the left-hand span, 1–2, in feet;
L_2 = the length of the right-hand span, 2–3, in feet;
w_1 = the uniform load on the left-hand span in pounds per linear foot;

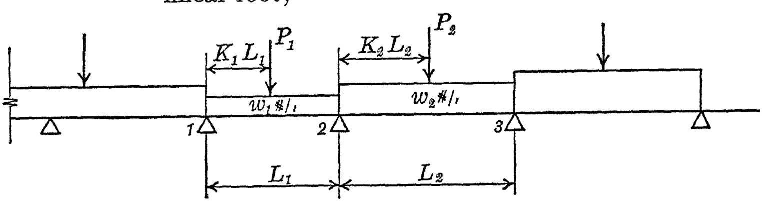


FIG. 54

w_2 = the uniform load on the right-hand span in pounds per linear foot;
P_1 = any concentrated load on span 1–2, a distance K_1L_1 from 1;
P_2 = any concentrated load on span 2–3, a distance K_2L_2 from 2.

The signs used with this equation are those relating to beams, where negative moment is that which causes tension in the top fiber.

The equation can be extended to care for a uniform load covering only a part of a span by substituting in the term giving the effect of a concentrated load in the given span, wdx, for P, x for KL, and replacing the summation sign by that for integration.

The following derivation of the Theorem of Three Moments for uniform loading is given to make possible a ready review. For more general statements of the theorem and its derivation the reader should consult a good text on the strength of materials.

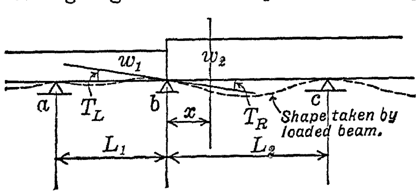


FIG. 55

In Fig. 55 are shown any two consecutive spans of a continuous beam of uniform cross-section and level supports, with a loading that is uniform over any one span. By use of the equation for the elastic curve, $M = EI\dfrac{d^2y}{dx^2}$, the slope of the tangent

at any support, as b, may be expressed, first, in terms of the load, etc., to the right, and then in terms of the similar factors to the left. These two expressions are for the same quantity and so are equal to each other. Their combination gives the Three Moment Equation.

The moment at any section in the right span equals

$$M = EI\frac{d^2y}{dx^2} = M_b + S_r x - \tfrac{1}{2}w_2 x^2,$$

where S_r is the shear to the right of support b. Integrating

$$EI\frac{dy}{dx} = M_b x + \tfrac{1}{2}S_r x^2 - \tfrac{1}{6}w_2 x^3 + (\text{Constant} = T_R EI). \qquad (1)$$

Here the constant of integration is expressed as the product of three constants and T_R is chosen to represent the tangent at b expressed in terms of the right span factors. Integrating a second time

$$EIy = \tfrac{1}{2}M_b x^2 + \tfrac{1}{6}S_r x^3 - \tfrac{1}{24}w_2 x^4 + T_R EIx + (\text{Constant} = 0). \qquad (2)$$

Placing in equation (2) L_2 for x, $y = 0$, and substituting the value of S_r obtained from the expression for the moment at c, ($M_c = M_b + S_r L_2 - \frac{1}{2}w_2 L_2^2$) gives the following expression for the slope of the tangent to the curve of the beam at b in terms of the right-hand span elements:

$$T_R = \frac{1}{EI}\left(-\tfrac{1}{3}M_b L_2 - \tfrac{1}{6}M_c L_2 - \tfrac{1}{24}w_2 L_2^3\right).$$

A similar expression for the slope at b in terms of the left-hand factors may be written by analogy from the expression for that at c (T_c) obtained by writing equation (1) with $x = L_2$, substituting the value of S_r as before,

$$T_L = \frac{1}{EI}\left(\tfrac{1}{6}M_a L_1 + \tfrac{1}{3}M_b L_1 + \tfrac{1}{24}w_1 L_1^3\right).$$

Equating $T_R = T_L$ gives

$$M_a L_1 + 2M_b(L_1 + L_2) + M_c L_2 = -\tfrac{1}{4}w_1 L_1^3 - \tfrac{1}{4}w_2 L_2^3.$$

The use of the theorem is illustrated by the following examples:

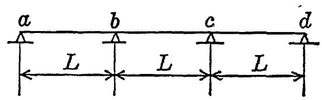


FIG. 56

Example 45. What is the maximum moment at b in the beam shown in Fig. 56? Dead load in pounds per foot $= w_1$; live load $= w_2 = 3\,w_1$.

Solution. Note that $M_a = M_d = 0$, and for dead load, $M_b = M_c$, the beam being symmetrical. For dead load the three-moment equation takes this form

$$M_a L + 2M_b(L + L) + M_c L = -\tfrac{1}{4}w_1 L^3 - \tfrac{1}{4}w_1 L^3.$$

Solving:

$$5M_b = -\tfrac{1}{2}w_1 L^2$$
$$M_b = -\tfrac{1}{10}w_1 L^2, \text{ the dead load moment.}$$

In order to determine the spans which should be loaded to produce negative moment at b place a single load on each of the three spans in turn (Fig. 57a–b–c) and consider the resulting deflection of the beam. For example: a load on ab, with supports c and d removed, brings the beam to the position abd'. The action of the c support is to pull the beam down into the position $abcd''$, causing tension in the top of the beam at b and thus negative moment at that point. The subsequent pushing up of the beam into place by support d does not change this. Loads anywhere on

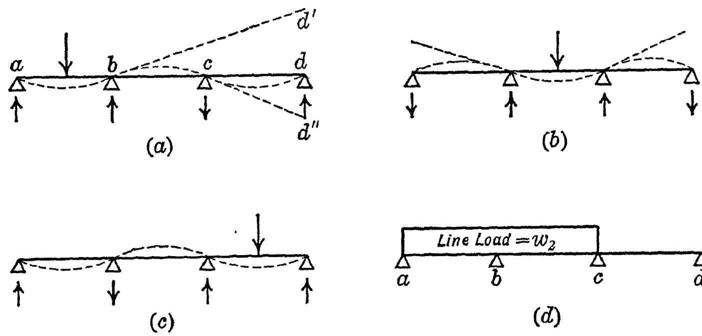


FIG. 57

spans ab and bc cause negative moment at b, and so both these spans should be covered with the live load for a maximum value of that function. For this loading (Fig. 57d) there are two unknowns, M_b and M_c ($M_a = M_d = 0$), and two simultaneous equations involving these two unknowns may be written by applying the Theorem of Three Moments twice, first to the three supports abc, and then to the three bcd:

$$M_aL + 2\,M_b(L + L) + M_cL = -\tfrac{1}{4}\,w_2L^3 - \tfrac{1}{4}\,w_2L^3$$

and

$$M_bL + 2\,M_c(L + L) + M_dL = -\tfrac{1}{4}\,w_2L^3.$$

Solving

$$M_b = -\tfrac{7}{60}\,w_2L^2 \text{ for live load moment.}$$

The maximum moment equals the sum of the dead and live load moments and would be expressed in terms of the total load per foot $w = w_1 + w_2 = w_1 + 3\,w_1 = 4\,w_1$ giving

$$M_b = -\tfrac{1}{10}\cdot\frac{w}{4}\cdot L^2 - \tfrac{7}{60}\cdot\frac{3\,w}{4}\cdot L^2 = -\tfrac{9}{80}\,wL^2.$$

In expressions for moment the multiplier of wL^2 is known as the **Moment Factor** or **Moment Coefficient.**

Example 46. What is the shear at a section 2 ft. from the right end of this beam (Fig. 58*a*)?

Discussion. The difficulty introduced by the fixed end at c can be met as follows. Imagine the support at that point changed to a knife edge and the beam to be continued to the right, a distance L, to another support d. (Fig. 58*b*.) The beam with its extension deflects more than the original beam because the restraint offered by the span cd at c is not sufficient to make the tangent to the elastic curve horizontal at that point. However, the shorter the distance L, the stiffer the span cd and the flatter the tangents at c and d. When $L = 0$ these tangents coincide and are horizontal, giving a condition of fixity at c. Accordingly the theorem of three moments can be applied to this problem by replacing the fixed end with an unloaded span of zero length, supported on knife edges.

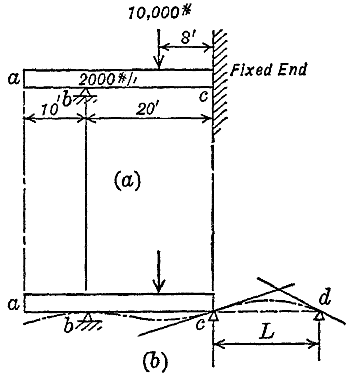


FIG. 58

Solution. The beam with its added span is continuous over three supports, at which there is only one unknown moment, that at $c(M_c)$. $M_d = 0$; $M_b = -2 \times 10 \times 5 = -100$ kip-ft. (A kip is 1000 lbs.) Writing the equation of Three Moments:

$$-100 \times 20 + 2\,M_c \times 20 + 0 = -\tfrac{1}{4} \times 2 \times \overline{20}^3 - 10 \times \overline{20}^2\,(0.6 - \overline{0.6}^3)$$
$$M_c = -88.4 \text{ kip-ft.}$$

The negative sign of this moment indicates that it causes tension in the top fiber and therefore it acts in a clockwise direction upon the beam at c as shown in Fig. 59. Either of the unknown vertical reactions may be found by applying the condition of equilibrium, $\Sigma M = 0$. In using this condition it is customary to name a clockwise moment positive. It is important to distinguish clearly between these two conventions of signs: that for bending moment in beams and that for moment in force systems acting on (assumed) rigid bodies.

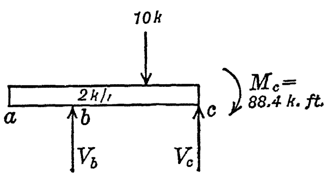


FIG. 59

$$\Sigma M_c = 0$$
$$V_b \times 20 - 2 \times 30 \times 15 - 10 \times 8 + 88.4 = 0$$
$$V_b = 44.6 \text{ kips.}$$

The positive sign of the result indicates that the assumed upward direction is correct. The desired shear is:

$$44.6 - 2 \times 28 - 10 = -21{,}400 \text{ lbs.}$$

87. Moment Factors for Continuous Beams and Girders. In Example 45 is illustrated the process of finding the moment factors to use in designing a given continuous beam carrying uniform loads. Applying the same methods to beams of a varying number of equal spans the following table was constructed. The columns headed " Dead " loads give the maximum values of the positive and

COEFFICIENTS[2] FOR MAXIMUM MOMENTS IN CONTINUOUS BEAMS (x)

$$M = xwL^2$$

Number of spans	Intermediate spans and supports				End span and second support			
	At center (+)		At support (−)		At center (+)		At support (−)	
	Dead	Live	Dead	Live	Dead	Live	Dead	Live
Two	...	. .	. ....	.	0 070	0 095	0 125	0 125
Three	0 025	0 075			*0 080*	*0 100*	0 100	*0 117*
Four	0 036	0 081	0 071	*0 107*	0 071	0 098	*0 107*	0 120 (0 115)[1]
Five	*0 046*	*0 086*	0.079	0 111 (0 106)[1]	0 072	0 099	0 105	0 120 (0 116)[1]
Six	0.043	0 084	*0 086*	0 116 (0 106)[1]	0 072	0 099	0 106	0 120 (0 116)[1]
Seven	0 044	0 084	0 085	0 114 (0 106)[1]	0 072	0 099	0 106	0 120 (0 116)[1]

[1] Where two adjacent spans only are loaded.

negative moment to be found in the beam due to the dead load, which of course covers the whole length. The columns headed " Live " loads give the maximum moments, positive and negative, that can be caused anywhere in the beam by the live load, by placing it only on those spans where its effect is to increase the moment under consideration. Reasonable maximum values of all those given in the table are indicated by italics, the larger values being rejected as involving unreasonable assumptions as to the position of the live loads. In the following table these

[2] This table and the following one are reproduced with some variation from " Principles of Reinforced Concrete Construction," by Turneaure and Maurer.

maximum values found above for live and dead loads are combined into a single term for various ratios of live and dead, 1 to 2 being the usual range of that ratio.

COEFFICIENTS FOR MAXIMUM MOMENTS IN CONTINUOUS BEAMS OF THREE OR MORE EQUAL SPANS DUE TO COMBINED DEAD AND LIVE LOADS (x)

$$M = xwL^2$$

Ratio of live : dead	Intermediate spans		End spans	
	At center	At support	At center	At support
1 : 1	0.066	0.097	0.090	0.112
2 : 1	0.073	0.099	0.093	0.114
5 : 1	0.079	0.104	0.097	0.115

Based on this study the following conclusions are usually drawn (compare Art. 107, 1924 Joint Committee, Appendix B): that for continuous beams of three or more equal spans, carrying uniform loads, the maximum positive and negative moments to be expected in interior spans may be taken as $\frac{wL^2}{12}$, the width and rigidity of the supports serving to reduce the theoretical values; for end spans the maximum positive and negative moment to be expected is $\frac{wL^2}{10}$, the negative moment being that at the first interior support. For two span beams the maximum positive moment is taken as $\frac{wL^2}{10}$ and the negative moment as $\frac{wL^2}{8}$.

Certain of the above factors are somewhat less when the outside support is a reinforced concrete column and offers considerable resistance to bending. The 1924 Joint Committee report (Art. 110, Appendix B) makes recommendations covering the action of supporting columns, based on studies made by more elaborate methods than that just outlined.

For girders, that is, for beams carrying concentrated loads, it is sufficiently accurate ordinarily to compute the maximum positive moment as though it were a simple non-continuous

member, and then modify this result in the same ratio as for the same section in a uniformly loaded beam of the same number of spans.

Example 47. What are the maximum positive and negative moments in the center span of a girder continuous over three equal spans with a single load, P, at the center of each? Neglect the dead weight of the girder.

Solution. For a similar beam with uniform load the maximum positive moment at the center of the interior span is $\frac{wL^2}{12}$, and the maximum negative moment $\frac{wL^2}{10}$. The maximum positive moment for a simple girder with center load is $\frac{PL}{4}$; for a simple beam, uniformly loaded, $\frac{wL^2}{8}$. Since $\frac{wL^2}{12} = \frac{wL^2}{8} \times \frac{8}{12}$, the maximum positive moment in the center of the girder is $\frac{PL}{4} \times \frac{8}{12} = \frac{PL}{6}$; the maximum negative moment is $\frac{PL}{4} \times \frac{8}{10} = \frac{PL}{5}$.

Problem 33. A girder which is continuous over two equal spans carries equal loads at the center of each span. Compute the maximum moment at the center support (a) by means of the theorem of three moments, (b) by the approximate method described above. Neglect girder weight.

Ans. (a) $-0.19\ PL$. (b) $-0.20\ PL$.

88. Method of Least Work. The stresses in statically indeterminate structures, such as those formed of beams and columns meeting in rigid (hingeless) joints, may be determined by the method of Least Work. The Theorem of Least Work states that the internal stresses in any statically indeterminate structure are such that the total work done by them, as the structure deforms under load, is a minimum. It is possible, therefore, to make exact solutions of rigid frames by writing an expression for the internal work in the structure in terms of one or more unknowns (moments, shears and thrusts, the number being the excess of the total number of unknowns over those that may be obtained by application of the principles of statics), placing the several partial differentials of this expression equal to zero, and solving.

Expressions for the work done by internal fiber stresses are obtained as follows. Let

P = total axial stress in a piece;
A = cross-sectional area of piece;
L = length of piece;
E = modulus of elasticity of the material $= f/e$;
f = unit stress $= P/A$ for column or tie $= Mc/I$ for beam;
e = strain or deformation per unit length $= f/E$;
I = moment of inertia;
y = distance from neutral axis of a beam to the stressed fiber.

In a member subjected to axial stress the internal work equals the average force acting during deformation, multiplied by the total deformation,

$$W = \frac{P}{2} \times eL = \frac{P}{2} \times \frac{PL}{AE} = \frac{P^2L}{2\,AE}. \tag{41}$$

FIG. 60

A beam is simply an assemblage of a vast number of elementary columns and ties, such as that shown in Fig. 60, with dimensions $b \times dx \times dy$. The work done in any such prism equals

$$W = \frac{P^2L}{2\,AE} = \frac{1}{2}\left(\frac{M^2y^2}{I^2} \times \overline{bdy}^2\right)\left(\frac{dx}{bdyE}\right)$$

and the total work in the beam equals (noting $\int y^2bdy = I$)

$$W = \int \frac{M^2dx}{2\,EI}\,. \tag{42}$$

Example 48. What is the moment at the end of the beam a–b of the rigid frame a–b–c–d (Fig. 61a)? The section is uniform both as to size and material throughout.

Solution. The values of E, I and A are the same for all members. Cut the frame at any convenient point, as at a, Fig. 61b, and represent

by arrows the unknown thrust, moment and shear there acting. From the symmetry of the structure it is plain that the shear in ab at a equals one-half of the load; $S = 20{,}000$ lbs. It is also plain that the thrust in ab must be the same in amount and direction as that in dc. This thrust, T, then must equal zero as otherwise there would be two equal horizontal

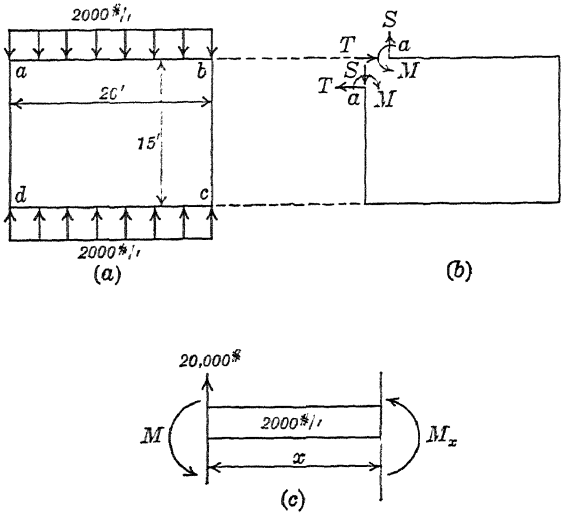


FIG. 61

forces acting on the vertical ad in the same direction, which is not consistent with equilibrium. The moment M_x, in ab at any distance x from a, equals (see Fig. 61c):

$$M_x = -M - \frac{wx^2}{2} + Sx = -M - \frac{2\,x^2}{2} + 20\,x.$$

The total work in the structure equals

$$W = 2\int_0^{20} \frac{(-M - x^2 + 20\,x)^2\,dx}{2\,EI} + 2\int_0^{15} \frac{M^2dx}{2\,EI} + 2\left(\frac{20^2 \times 15}{2\,AE}\right).$$

Differentiating in respect to M and placing the expression equal to zero,

$$\frac{dW}{dM} = 2\int_0^{20} \frac{2\,(-M - x^2 + 20\,x)(-1)dx}{2\,EI} + 2\int_0^{15} \frac{2\,Mdx}{2\,EI} = 0$$

$$(Mx + x^3/3 - 10\,x^2)\Big|_0^{20} + (Mx)\Big|_0^{15} = 0$$

$$20\,M + \frac{8000}{3} - 4000 + 15\,M = 0$$

$$M = +38{,}000 \text{ ft.-lbs.}$$

The positive sign indicates that the assumed direction for the moment at a happened to be correct, *i.e.*, negative moment, causing tension in the top fiber. The moment factor here equals $\frac{1}{21.1}$.

89. Method of Slope Deflection.[1] A method of analysis of rigid frames that has come into prominence within the last ten years is that of Slope Deflection which in general is much simpler of application than Least Work. Briefly described the method is as follows. A series of simultaneous equations is set up and solved, each equation expressing a relation between certain of the bending moments at the ends of the several members of the structure under consideration, and giving as a result the values of all these end moments. The relation most commonly used in setting up these equations is the equilibrium existing between all the end moments at any one of the rigid joints. The determination of the shears and direct stresses, completing the solution of the structure, is a simple matter once these moments are determined.

It should be noted that this method is an approximate one, as no account is taken by it of the change in length of any member due to axial stress. Its use gives results identical with those of the method of least work when the work due to axial stress is disregarded.

90. The General Slope Deflection Equation. The differential equation of the elastic curve ($d^2y/dx^2 = M/EI$) of a loaded beam provides a means of expressing the moment at the end of a restrained member in terms of the three variables, slope, deflection and load. The complete argument necessary for the deduction of a general expression for end moment for any member of uniform cross-section and constant modulus of elasticity, with any system of transverse loading, may be obtained by study of the special case of a restrained beam with a uniform load over its whole length.

Consider the beam shown in Fig. 62, of which mn is the original unloaded and unstrained position (or is parallel to it) and mn' the final position. The tangent to the elastic curve at m has rotated through the angle θ_m and that at n through the angle θ_n. The end n shows a deflection d from the original tangent through m; and conversely the end m the same deflection as regards n. The

[1] Presented by Professor George A. Maney, 1915.

directions assumed for the end shears are immaterial and are taken as shown.[1] For convenience the end moments are assumed as positive, that is, acting in a clockwise direction. Note that M_{mn} indicates the moment at the m end of the member mn and M_{nm} that at the n end.

The following convention of signs is adopted:

(*a*) external moment acting in a clockwise direction at the end of a member is taken positive; counter-clockwise, negative;

(*b*) clockwise rotation of the tangent at either end is called positive slope; counter-clockwise, negative;

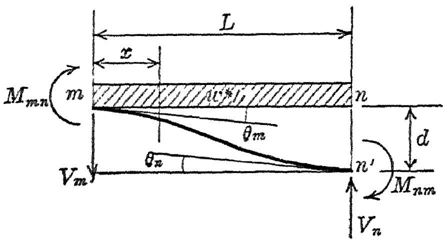


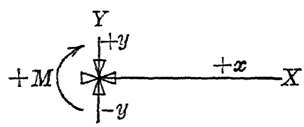
FIG. 62

(*c*) deflection is positive when the line connecting the two ends of the member revolves in a clockwise direction; negative when counter-clockwise.

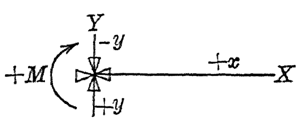
In Fig. 62 moments, slopes and deflection are all shown as positive.

The sign conventions for the equation for the elastic curve, $EI\,\dfrac{d^2y}{dx^2} = +M$, are as follows (see Fig. 63*a*): x positive to right; y positive upward; M positive when causing compression in the top fiber of a horizontal beam.

Since this convention differs from that assumed for the slope deflection method it becomes necessary to relate the two sets of signs and inspect the elastic curve equation to determine how it should be written to agree with the slope deflection signs.



(*a*) *for Beams*



(*b*) *Slope Deflection Method*

Sign Conventions

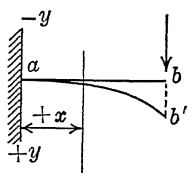
FIG. 63

For this derivation the origin is taken at the left (m) end of the beam with x positive to the right. Since clockwise rotation is positive, y is positive downward and negative upward. (Fig. 63*b*.) Clock-

[1] The student should verify this statement by assuming opposite directions for the shears and carrying the change through the derivation that follows.

wise moment at the left end is positive by both the slope deflection and beam sign conventions, such a moment causing compression in the top fiber. The only difference in the two sets of signs, then, is that relating to y (the origin being to the left).

The following considerations show that if the signs are taken as chosen for the slope deflection equation, the elastic curve equation must be written $EI \frac{d^2y}{dx^2} = -M$. Consider, for example, a cantilever beam ab (Fig. 64a) of which ab' is the elastic curve under load, and note that the moment acting on the beam at the support at a is counterclockwise and negative by both conventions. By the slope deflection convention the slope of this beam at any point away from the support is positive; that is, $\frac{dy}{dx}$ is positive. This slope increases with x; that is, $\frac{d^2y}{dx^2}$ is also positive. Since the moment acting is negative the equation must be written with the negative sign for M so that when the value of the moment is inserted the result will be positive as required. It is suggested that the student carry through this argument with a beam acted upon by two equal and opposite end moments as shown in Fig. 64b.



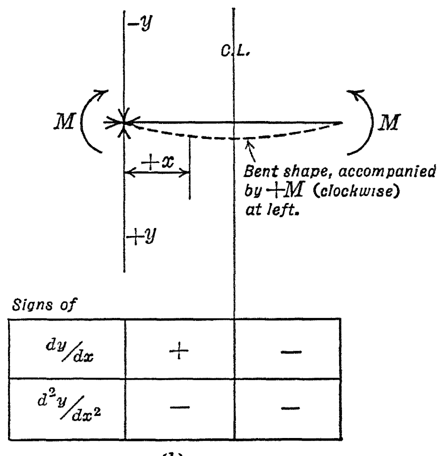


FIG. 64

The equation for the elastic curve of the beam of Fig. 62 is

$$EI \frac{d^2y}{dx^2} = -M = -\left(M_{mn} - V_m x - \frac{wx^2}{2}\right).$$

Integrating once gives

$$EI\frac{dy}{dx} = -M_{mn}x + \frac{V_m x^2}{2} + \frac{wx^3}{6} + (\text{Constant} = EI\theta_m) \quad (a)$$

since for $x = 0 \quad \dfrac{dy}{dx} = \theta_m$.

A second integration gives

$$EIy = -\frac{M_{mn}x^2}{2} + \frac{V_m x^3}{6} + \frac{wx^4}{24} + EI\theta_m x + (\text{Constant} = 0) \ (b)$$

since for $x = 0 \quad y = 0$.

For $x = L, \dfrac{dy}{dx} = \theta_n,\ y = d$, and these equations become

$$EI\theta_n = -M_{mn}L + \frac{V_m L^2}{2} + \frac{wL^3}{6} + EI\theta_m \qquad (a')$$

$$EId = \frac{-M_{mn}L^2}{2} + \frac{V_m L^3}{6} + \frac{wL^4}{24} + EI\theta_m L. \qquad (b')$$

Substituting in (b') the value of V_m obtained from (a') gives

$$EId = \frac{EI\theta_n L}{3} + \frac{M_{mn}L^2}{3} - \frac{wL^4}{18} - \frac{EI\theta_m L}{3} - \frac{M_{mn}L^2}{2} + \frac{wL^4}{24} + EI\theta_m L$$

whence

$$M_{mn} = \frac{2\,EI}{L}\left(2\,\theta_m + \theta_n - 3\frac{d}{L}\right) - \frac{wL^2}{12} \qquad (c)$$

It will be recognized that $-\dfrac{wl^2}{12}$ (the negative sign indicating counter-clockwise moment) is that moment that occurs at the left end of a fixed-ended beam of span L carrying a load of w pounds per foot over the whole length.

If the origin is taken at the right end of the beam (Fig. 62), another investigation of the relation between the two sign conventions becomes necessary, and, based upon their reconciliation, the derivation yields:

$$M_{nm} = \frac{2\,EI}{L}\left(2\,\theta_n + \theta_m - 3\frac{d}{L}\right) + \frac{wL^2}{12} \qquad (d)$$

The student should verify this statement, noting that the positive sign for the $\dfrac{wl^2}{12}$ term indicates clockwise moment such as exists at the right end of a fixed-ended beam.

An inspection of Equations (*c*) and (*d*) shows that each consists of two terms, one involving the changes in end slopes and the deflection, and the other the end moment occurring on a fixed ended beam with a uniform overall load. It is evident that this could have been anticipated by a preliminary consideration of the problem, for the moment at the end of any restrained beam can be considered as consisting of two parts: first, that existing at the end of the beam without load, assuming the final changes in end slopes and the final deflection to be realized; second, the moment made necessary when the load is applied in order to prevent further rotation of the end tangents; in other words, the moment required to fix the end of the beam. This second term is one that is easily found for any system of transverse loads by use of the Three Moment Equation as described in Example 46, Discussion, page 173. Tables of these values for common types of loading are given in several texts and handbooks.[1]

The general slope-deflection equation takes the following form, the end of the beam for which the moment is written being designated the " near " end:

$$M_{\text{near}} = 2\,EK(2\,\theta_{\text{near}} + \theta_{\text{far}} - 3\,R) \pm C_{\text{near}} \qquad (43)$$

where $K = I/L$ $\qquad E$ = modulus of elasticity

$R = d/L$ $\qquad I$ = moment of inertia of section

C_{near} = the moment caused at the near end of an identical fixed-ended beam carrying the same load as the given beam.

The sign of the C term is determined by the direction of the end moment of the fixed beam. For example, in writing this equation for the moment at the right end of the beam in Fig. 62, the C term is positive, for the left end, negative.

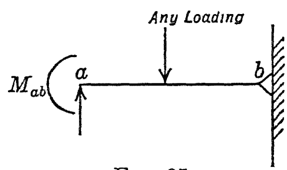


FIG. 65

For slope deflection problems it is desirable to have conveniently at hand a single sheet giving the general equation with explanation of signs and notation. Such a sheet should also show a special form of the general equation to use when one end of a member is hinged. This is derived

[1] For instance, in Bulletin 108, University of Illinois, " Analysis of Statically Indeterminate Structures by the Slope Deflection Method " by Wilson, Richart and Weiss.

as follows (Fig. 65):

$$M_{ab} = 2\,EK(2\,\theta_a + \theta_b - 3\,R) - C_{ab} \tag{a}$$
$$M_{ba} = 0 = 2\,EK(2\,\theta_b + \theta_a - 3\,R) + C_{ba}. \tag{b}$$

Solving for θ_b

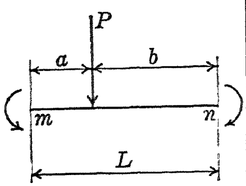
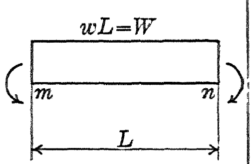
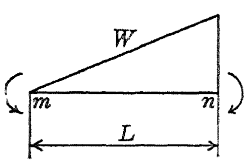
$$\theta_b = \tfrac{1}{2}\left(-\frac{C_{ba}}{2\,EK} - \theta_a + 3\,R\right)$$

Substituting in (a)

$$M_{ab} = 2\,EK\left(2\,\theta_a - \frac{C_{ba}}{4\,EK} - \frac{\theta_a}{2} + \frac{3\,R}{2} - 3\,R\right) - C_{ab}$$
$$= 2\,EK(1.5\,\theta_a - 1.5\,R) - (C_{ab} + \tfrac{1}{2}\,C_{ba}). \tag{c}$$

TABLE I

VALUES OF CONSTANTS C AND H

Loading	C_{mn}	C_{nm}	H_{mn}	H_{nm}
	$\frac{Pab^2}{L^2}$	$\frac{Pba^2}{L^2}$	$\frac{Pab}{2\,L^2}(L + b)$	$\frac{Pab}{2\,L^2}(L + a)$
	$\frac{WL}{12}$	$\frac{WL}{12}$	$\frac{WL}{8}$	$\frac{WL}{8}$
	$\frac{WL}{15}$	$\frac{WL}{10}$	$\frac{7}{60}\,WL$	$\frac{2}{15}\,WL$

The general form of this equation is:

$$M_{\text{near}} = 2\,EK(1.5\,\theta_{\text{near}} - 1.5\,R) \pm H_{\text{near}} \tag{44}$$

where H_{near} = the arithmetical sum, $(C_{\text{near}} + \frac{1}{2}\,C_{\text{far}})$, the sign being as for C_{near}. The values of the constants C and H for certain simple loadings are given in Table I.

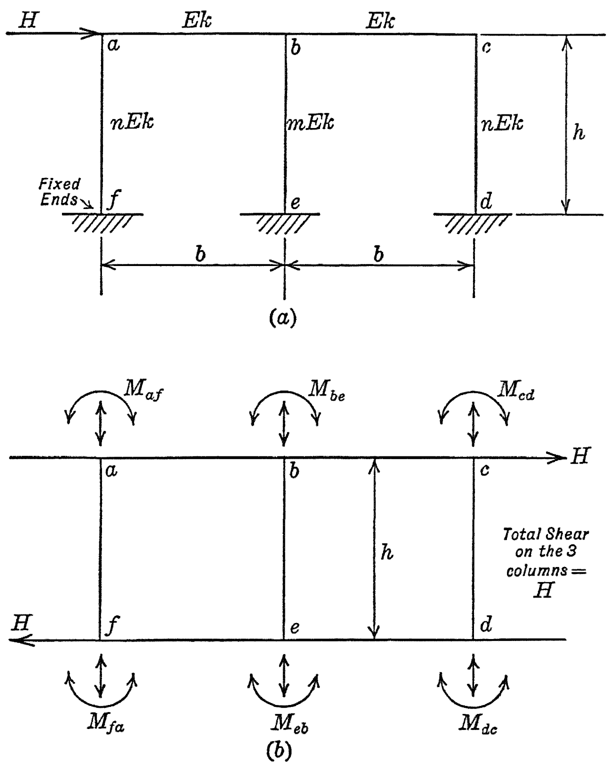


FIG. 66

Example 49. Compute the reactions for the three column bent shown in Fig. 66. The product of the modulus of elasticity by the ratio of moment of inertia to length ($I/L = K$) is taken as EK for the girders; for the outside columns, nEK; for the middle column, mEK.

Solution. Since the joints are rigid, all the tangents to the members meeting at any joint rotate equally with the deformation of the structure; that is, θ_a for member ab equals θ_a for member af. By use of the general

slope-deflection equation the end moments for all members may be expressed as follows:

$$
\begin{aligned}
M_{ab} &= 2\,EK(2\,\theta_a + \theta_b) \\
M_{af} &= 2\,EK(2\,n\theta_a - 3\,nR) \\
M_{ba} &= 2\,EK(2\,\theta_b + \theta_a) \\
M_{be} &= 2\,EK(2\,m\theta_b - 3\,mR) \\
M_{bc} &= 2\,EK(2\,\theta_b + \theta_c) \\
M_{cb} &= 2\,EK(2\,\theta_c + \theta_b) \\
M_{cd} &= 2\,EK(2\,n\theta_c - 3\,nR) \\
M_{fa} &= 2\,EK(n\theta_a - 3\,nR) \\
M_{eb} &= 2\,EK(m\theta_b - 3\,mR) \\
M_{dc} &= 2\,EK(n\theta_c - 3\,nR)
\end{aligned}
$$

It will be noted that in writing these expressions the value of $R = d/h$ was taken as equal for all three columns, the shortening of the girders under direct load being neglected. Similarly it was assumed that points a, b and c remained on the same level as regards each other. Also $\theta_f = \theta_e = \theta_d = 0$.

In this series of expressions for moment there occur four unknown quantities, θ_a, θ_b, θ_c and $R = d/h$, the evaluating of which will make it possible to determine the values of all the end moments and so lead to a complete solution of the structure. Four independent equations are required for the finding of these four unknowns. Three of these equations are found at once from the condition of equilibrium at each joint.

$$M_{ab} + M_{af} = 0 \tag{1}$$
$$M_{ba} + M_{bc} + M_{be} = 0 \tag{2}$$
$$M_{cb} + M_{cd} = 0. \tag{3}$$

The fourth is given by consideration of the three columns isolated from the rest of the structure by sections just below the connecting girder and just above the supports. (Fig. 66*b*.) Here the unknown direct stresses and end moments acting on the columns are represented by double-headed arrows and the total shear on the three columns (the distribution of which is unknown), by the horizontal arrows marked H. This system of forces is in equilibrium and taking moments about any point gives

$$M_{af} + M_{fa} + M_{be} + M_{eb} + M_{cd} + M_{dc} + Hh = 0. \tag{4}$$

In these equations the unknown end moments are assumed positive. The moment of the shear is positive, being clockwise for any section. The direct stresses balance and do not appear in the moment equation. The first of these equations, when rewritten with the two moments expressed in terms of slope and deflection, takes this form

$$2\,EK(2\,\theta_a + \theta_b + 2\,n\theta_a - 3\,nR) = 0. \tag{1'}$$

The solution of four simultaneous equations written out in this form would be inconvenient and so the four equations are tabulated as follows.

A general solution is cumbersome and accordingly the table continues with the equations as they appear for $n = m = 1$, for which values a solution is obtained.

No.	Operation	Coefficients of θ_a, θ_b, etc.				Right side of equation	
		θ_a	θ_b	θ_c	$(-3\,R)$	$\frac{1}{2\,EK}$	Check
1	General Form	$2+2\,n$	1	...	n	0	...
2		1	$4+2\,m$	1	m	0	...
3		...	1	$2+2\,n$	n	0	...
4		$3\,n$	$3\,m$	$3\,n$	$4\,n+2\,m$	$-Hh$	..
1	For $n = m = 1$	4	1	..	1	0	6
2		1	6	1	1	0	9
3			1	4	1	0	6
4		3	3	3	6	$-Hh$	15
1′	#1 ÷ 4	1	0.25	..	0.25	0	1.5
4′	#4 ÷ 3	1	1	1	2	$-0.333\,Hh$	5
5	#2 − #1′		5.75	1	0.75	0	7.5
6	#2 − #4′	.	5	..	−1.00	$0.333\,Hh$	4
5′	#5 ÷ 5.75	.	1	0.174	0.1306	0	1.30
6′	#6 ÷ 5	.	1	..	−0.200	$0.0667\,Hh$	0.80
7	#3 − #5′		.	3.826	0.8694	0	4.70
8	#3 − #6′			4	1.200	$-0.0667\,Hh$	5.20
7′	#7 ÷ 3.83		..	1	0.2275	0	1.23
8′	#8 ÷ 4			1	0.30	$-0.0167\,Hh$	1.30
9	#8′ − #7′			...	0.0725	$-0.0167\,Hh$	0.07
9′	... ...	.			1	$-0.230\,Hh$	...
7″	....			1		$+0.0523\,Hh$	...
6″	.....		1		.	$+0.0207\,Hh$	......
1″	...	1	.			$+0.0523\,Hh$	.....

The work of solving these four simultaneous equations could have been done more easily by taking advantage of the various unit coefficients instead of working systematically across from left to right, eliminating the unknowns in regular order. In many problems this systematic procedure is the easier method and it is followed here for sake of illustration. Each step of the work is checked in the right-hand column where is listed the total of the coefficients on the left-hand side of each equation. The operation performed on the individual coefficients results in a total that always should equal the result of the same operation performed on the total in the right-hand column.

The final figures in the right-hand column of the table equal 2 EK times

the value of θ_a, θ_b, θ_c and $(-3\,R)$. Substituting the proper values in the equation for M_{ab} gives

$$M_{ab} = 2\,EK\left(\frac{2 \times 0.0523\,Hh}{2\,EK} + \frac{0.0207\,Hh}{2\,EK}\right) = +\,0.125\,Hh\,.$$

Note that the 2 EK terms cancel.[1] Proceeding in similar fashion the other moments are found to have these values:

$$\begin{aligned} M_{ba} &= +0.0937\,Hh = M_{bc} \\ M_{be} &= -0.188\,Hh \\ M_{fa} &= -0.177\,Hh = M_{dc} \\ M_{eb} &= -0.209\,Hh. \end{aligned}$$

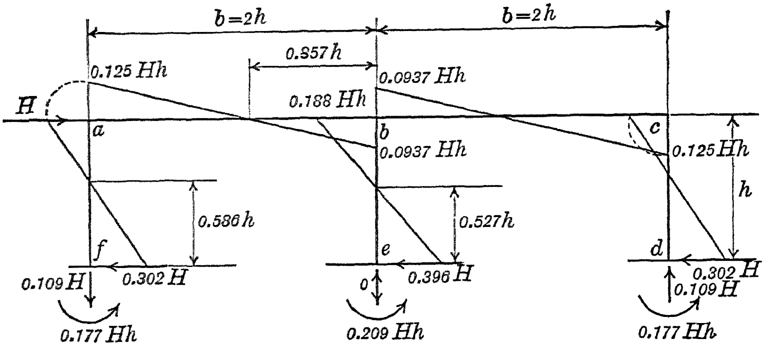


FIG. 67

The moments at the feet of the columns are negative, that is, acting upon the columns in a counter-clockwise direction. The desired reactions are shown in Fig. 67 with moment curves for the several members, the values being for the case where the girder span equals twice the column height.

[1] Since 2 EK does not enter into the result it is plain that the moments depend on the proportional stiffness of the several parts and not upon the properties of the actual material or sections. Also there is a definite relation between stress and deformation within the elastic limit. Because of these two facts it is possible to analyze complicated indeterminate structures with ease and certainty by study of the deformations of cardboard or celluloid models which preserve the same ratios of stiffness throughout as the original structures. Professor George E. Beggs of Princeton University is the originator of this method, which he describes in the Proceedings of the American Concrete Institute of 1922 and 1923. His method has been used with success and economy for research and for actual design. Professor Beggs' achievement is a great step forward in an important field as indeterminate structures are being used more and more. The difficulties of mathematical analyses are generally tremendous and their results often regarded with suspicion as based upon uncertain assumptions. This new and direct method is free from these objections.

Attention should be called to the necessity of checking the computation carefully at each stage of the work. It is very easy to make slips in setting up the equations, in preparing the table for the solution of the simultaneous equations, and in making that solution. The values of θ should not be used until they have been inserted in the original equations and found to satisfy every one of them. The best way to avoid mistakes in the calculation of reactions is to make liberal use of careful sketches.

Example 50. Calculate the moments at the ends of the slabs making up the culvert section shown in Fig. 68.

Data. For top of culvert $I = 1000''^4$ for a 1-ft. strip.
For walls of culvert $I = 800''^4$ for a 1-ft. strip.
E is constant for all members.

Solution. The only new elements presented by this problem are the transverse loads and the numerical values for the sections.

For the top and bottom $K = 1000/240$; for the sides $K = 800/120$. The ratio of these two values, called n on the figure, is 1.6.

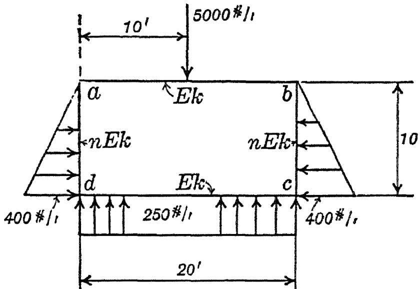


FIG. 68

For the top $C_{ab} = \frac{PL}{8} = -150{,}000$ in.-lbs., being counter-clockwise. At the other end of the member the moment is clockwise, so $C_{ba} = +150{,}000$ in.-lbs. Similarly, $C_{dc} = \frac{WL}{12} = 100{,}000$ in.-lbs.; $C_{ad} = \frac{WL}{15} = 16{,}000$ in.-lbs.; $C_{da} = \frac{WL}{10} = -24{,}000$ in.-lbs.

Certain information is given by symmetry: $\theta_a = -\theta_b$; $\theta_d = -\theta_c$. There are no deflections to consider, so $R = 0$ in all cases.

The following expressions are written for the moments in the left half of the structure, using the general slope deflection equation (43):

$$
\begin{aligned}
M_{ab} &= 2\,EK(2\,\theta_a + \theta_b - 3\,R) \pm C_{ab} \\
&= 2\,EK\theta_a - 150{,}000 \\
M_{ad} &= 2\,EK(2\,n\theta_a + n\theta_d) + 16{,}000 \\
&= 2\,EK(3.2\,\theta_a + 1.6\,\theta_d) + 16{,}000 \\
M_{da} &= 2\,EK(3.2\,\theta_d + 1.6\,\theta_a) - 24{,}000 \\
M_{dc} &= 2\,EK\theta_d + 100{,}000.
\end{aligned}
$$

The corresponding moments in the right half are equal to these numerically and opposite in sign. These expressions contain two unknowns,

θ_a and θ_d, and two equations for finding these unknowns are obtained by the condition of equilibrium existing at each joint.

$$M_{ab} + M_{ad} = 0$$
$$M_{da} + M_{dc} = 0.$$

Substituting the values found gives:

$$4.2\,\theta_a + 1.6\,\theta_d = 134{,}000 \div 2\,EK$$
$$1.6\,\theta_a + 4.2\,\theta_d = -76{,}000 \div 2\,EK.$$

Solution of these equations and of the expressions for end moment gives the required information:

$$M_{ab} = -104{,}600''\# \qquad M_{dc} = +64{,}400''\#.$$

The directions indicated by these signs agree with evident facts.

Problem 34. Solve the problem presented in Example 48, by the Method of Slope Deflection.

CHAPTER XII

BUILDING DESIGN. FLOORS WITH BEAMS AND GIRDERS

91. The rapid growth in favor of reinforced concrete as a material for building construction is due to its durability, its fire-resisting properties and its relatively low cost. A reinforced concrete frame can almost always be built more cheaply than one of structural steel which is fire-proofed. Usually it can be erected in less time following the completion of the plans than a steel structure which has to wait for the necessary shop work on the steel. "A floor a week" is a common standard for progress of erection by competent contractors when conditions are favorable.

In the usual form of construction reinforced concrete is used for the entire frame, floors, columns and footings. In tall buildings the columns are often made of structural steel encased in concrete to save the floor space that would be occupied by reinforced concrete columns. Reinforced concrete is also much used for the floor slabs in steel frame buildings.

There are three main types of reinforced concrete floors: that made with beams, or with beams and girders (see definitions of Art. 77) supporting the slab in the same fashion as is usual with steel or timber framing; lightweight floors consisting of a series of closely spaced joists or ribs supported by beams; flat slabs or girderless floors consisting of a slab supported directly by the columns, made either uniform in thickness or with increased depth about the columns. The slab, beam and girder floor is shown in Fig. 69, the flat slab in Fig. 70 and a simple type of lightweight ribbed floor, made with clay tile between the joists, in Fig. 77.

92. Floor Loads. The floors of buildings are usually designed to support a uniform load of intensity sufficient to ensure the strength required to carry safely any concentration of load that may be expected. In some instances these concentrations are of sufficient magnitude and definiteness so that they are used directly in the calculations. The following table from the Boston Building Code illustrates common requirements regarding live or movable loads in different types of structures.

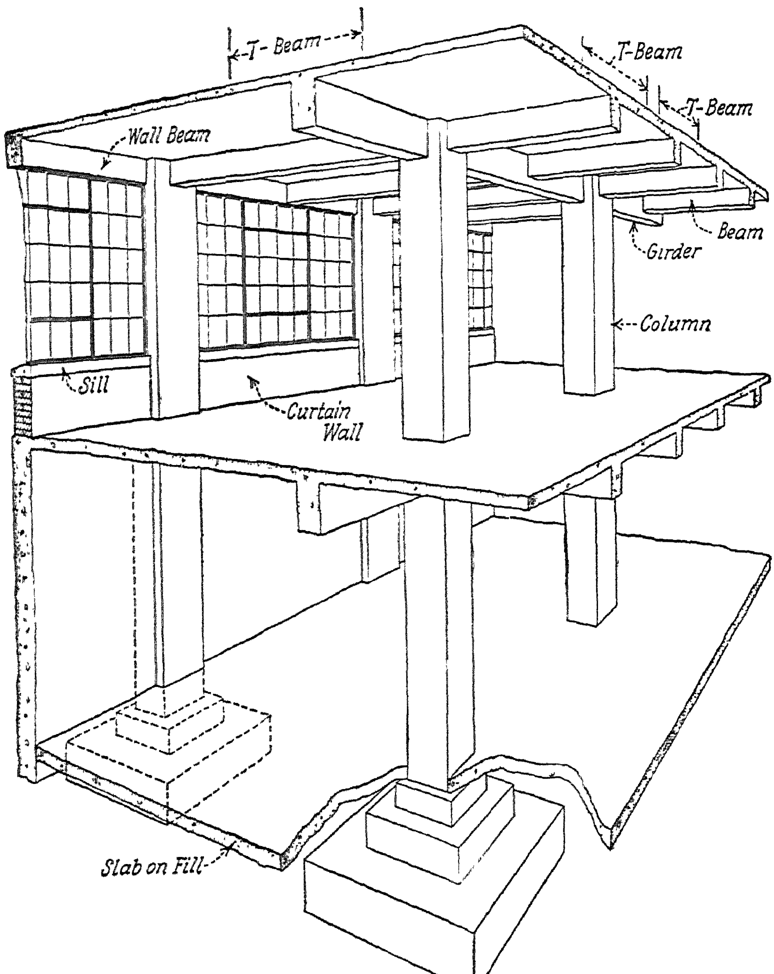


FIG. 69. — Perspective View of Beam and Girder Skeleton.[1]

[1] Reprinted by permission from "Concrete Plain and Reinforced," Taylor, Thompson and Smulski, Vol. I. Published by John Wiley & Sons, Inc.

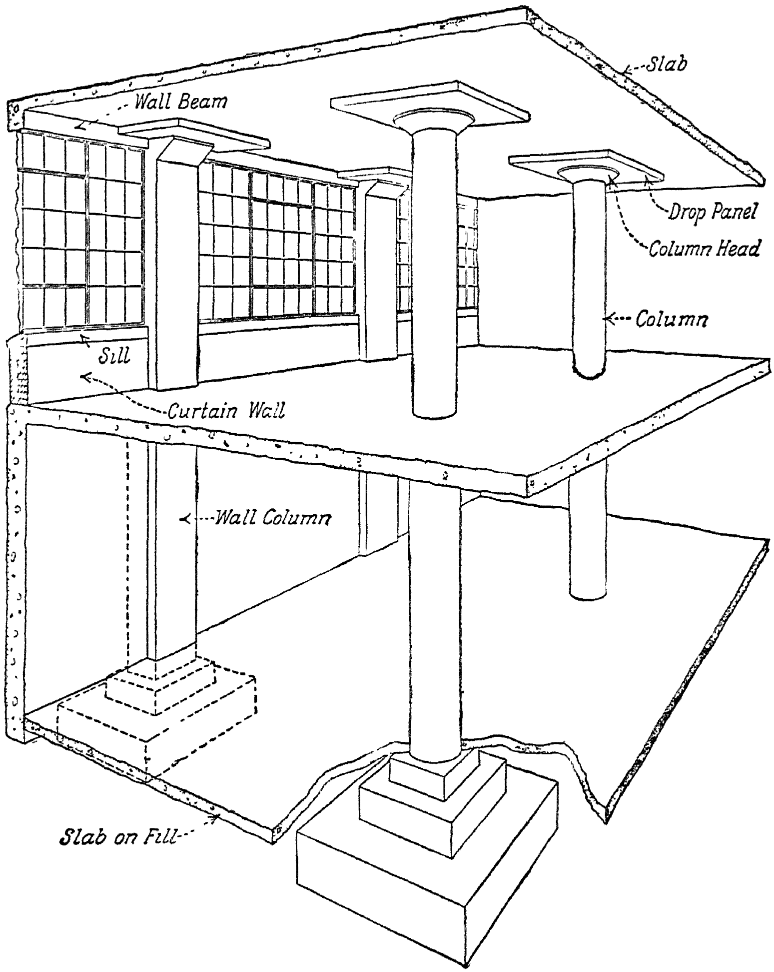


FIG. 70. — Perspective View of Flat Slab Skeleton.[1]

[1] Reprinted by permission from "Concrete Plain and Reinforced," Taylor, Thompson and Smulski, Vol. I. Published by John Wiley & Sons, Inc.

LIVE LOADS FOR BUILDINGS

A Portion of a Table in the Building Code of the City of Boston

Class of Buildings	Pounds per sq. ft.
Armories, Assembly Halls and Gymnasiums	100
Garages, Private, not more than two cars	75
Public	150
Grandstands	100
Hotels, Clubs, Hospitals, etc.	
Public portions	100
Residence portions	50
Manufacturing, Heavy	250
Light	125
Office Buildings, First floor	125
All other floors	75
Public Buildings, Public portions	100
Office portions	75
Residence Buildings, including Porches	50
Schools and Colleges; Assembly Rooms	100
Class Rooms never to be used as assembly halls	50
Sidewalks	250
(or 8000 lbs. concentrated, whichever gives the larger moment and shear.)	
Stairs, corridors and fire escapes;	
From Armories, Assembly Halls and Gymnasiums	100
Others	75
Storage, Heavy	250
Light	125
Stores, Retail	125
Wholesale	250

The dead load, that is the weight of the complete floor itself, may be calculated from the data in the following table:

WEIGHT OF STRUCTURAL MATERIALS

Brickwork, ordinary	120#/cu. ft.
" pressed brick	130
Concrete; cinder structural	108
" " floor filling	96 and less
" stone or gravel	150 or 144
Stone; granite, bluestone and marble	160 to 170; average 165
" limestone	140 to 180; average 160
Wood; yellow pine, grade I	42
" " " " II	35
" " " " green; $4\frac{1}{2}$#/ft. B.M.	54
" oak (green 60)	50
" spruce (green 34)	30
Roofing; copper sheets	$1\frac{1}{2}$#/sq. ft.
corrugated iron without sheathing	1 to 3
felt and asphalt " "	2
felt and gravel " "	8 to 10
slate, $\frac{1}{4}$ in. thick	9
common shingles	$2\frac{1}{2}$ to 3
skylights, including frames	4 to 10
sheathing, 1 in. thick	
white pine, hemlock, spruce	3
yellow pine	4
Plaster; on tile or concrete	5
suspended metal lath and plaster	10

Partitions; weights per sq. ft.

Thickness	3″	4″	5″	6″	8″	10″
clay tile	17	18		25	31	35
gypsum block	10	12	14	16		

In many buildings partitions form part of the loads and they need particular consideration where the floor load is less than 125 lbs. per sq. ft. They are usually made of gypsum, tile or brick and the data in the above table are sufficient to estimate their weight. Stud partitions are not used in concrete buildings and wood or pressed steel office partitions are too light to need consideration. When the structural design is made before partitions are located, it is common to include in the design a square foot allowance for their weight. For example: with a 12 ft. clear story height, a 6 inch tile partition plastered on both sides would weigh 420 lbs. per lin. ft. If from the use of the building it was possible that these might be located about 20 ft. on centers, this would amount to about 20 lbs. per sq. ft. In ordinary practice this load will vary from 10 to 20 lbs. per sq. ft. Brick walls should be located and treated as concentrated loads. It should be noted that in light, short span construction, the possibility of a partition being placed parallel to and between beams may be the governing factor in slab design.

93. Slabs Supported on Four Sides.[1] The slabs in the floors shown in Fig. 75 are so long in proportion to the width of a panel that they are treated as though supported on two sides only. It often happens that panels are square or nearly so and that one-way reinforcement is not adequate.

In Fig. 71 is shown a rectangular slab supported on four edges. An approximate analysis of the load distribution may be made by considering the two center strips, each 1 foot wide, extending from beam to beam, each strip being assumed to carry a uniformly distributed load. The deflection of any point of the longer strip may be expressed as $K_L w_L L^4/EI$ where K_L is a coefficient that varies with the point of deflection and w_L is the assumed load per foot on the strip. The deflections of the two strips are the same at their intersection and so for

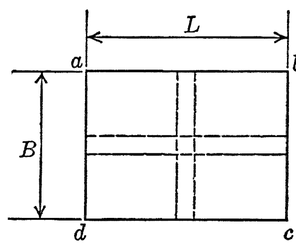


FIG. 71

[1]See paper by Westergaard and Slater "Moments and Stresses in Slabs," Proceedings of the American Concrete Institute, 1921, Art. 7, page 430. Also "Principles of Reinforced Concrete Construction," Turneaure & Maurer, 3d Edition, page 256.

that point

$$\frac{K_L w_L L^4}{EI} = \frac{K_B w_B B^4}{EI} \quad \text{where} \quad K_L = K_B.$$

$$w_L = \left(\frac{B^4}{L^4}\right) w_B \text{ and } w_B = w\left(\frac{L^4}{L^4 + B^4}\right);$$

$w_B + w_L = w =$ the total load per square foot on the slab When $L = B$ the load is divided equally between the two strips and the same reinforcement is needed in both directions; when $L = 1.5\,B$, $w_B = \frac{81}{97}\,w$. The 1916 Joint Committee gives the following equation for the distribution

$$w_B = \left(\frac{L}{B} - 0.5\right) w. \tag{45}$$

Inspection of these equations shows that the second considers the short span to be more heavily stressed than does the first, carrying all the load when the length exceeds 1.5 times the breadth, which is a universally accepted rule. The following instructions are given by the Joint Committee for applying this equation (45):

"Two-thirds of the calculated moment" (that is the moment in the width or length of the panel) "may be assumed as carried by the center half of the slab and one-third by the outside quarters." Regarding the design of the supporting beams the same report continues; "The distribution" (that is of load along the beam) "which may be expected ordinarily is — in accordance with the ordinates of a parabola having its vertex at the middle of the span." The 1924 Joint Committee report makes no mention of slabs supported along four sides.

94. Floor Surfaces. The wearing surfaces for reinforced concrete floor slabs are made of tile, brick, asphalt and wood as well as the more widely used cement or granolithic finish. A granolithic finish consists of a layer of rich mortar made with coarse sand containing many large hard particles which resist abrasion and wear. These hard particles are brought to the surface by trowelling before the mortar has set or by grinding after hardening. The thickness of this mortar layer depends upon the exposure to wear and the manner of placing. A layer as thin as $\frac{3}{4}$ inch is satisfactory when laid upon the unhardened concrete slab with which it is perfectly bonded after setting. An integral finish

accordingly can be counted upon as adding to the strength of the slab when the work is well done. Thicker courses are required when laid after the floor slab has set. A terrazzo floor is a similar type of finish made with marble or granite chips whose broad polished surfaces give a distinctive variegated appearance.

To be satisfactory, cement finishes must be applied by skilled men in accordance with the best practice. Standard specifications are given in the 1924 proceedings of the American Concrete Institute. For information regarding other types of wearing surfaces the reader is referred to " Concrete Plain and Reinforced " Vol. I, Taylor, Thompson and Smulski.

95. Slab, Beam and Girder Floor. General Features. (*a*) *Live Loads.* Generally the live loads used in design are assumed larger than those that may ever be expected to cover the entire floor area, in order to cover the effect of local concentrations of heavy weight. Many building codes recognize the waste involved in designing beams and girders for full loading and permit a reduction of live load. For example the Boston Building Code contains these paragraphs:

" Live loads may be reduced . . . as follows:

" In all buildings except armories, garages, gymnasiums, storage buildings, wholesale stores and assembly halls, for all flat slabs of over one hundred feet area, reinforced in two or more directions and for all floor beams, girders or trusses carrying over one hundred square feet of floor, ten per cent reduction."

" For the same but carrying over two hundred square feet of floor, fifteen per cent reduction."

" For the same, but carrying over three hundred square feet of floor, twenty-five per cent reduction."

" These reductions shall not be made if the member carries more than one floor and therefore has its live load reduced according to the table " given elsewhere in the code.

(*b*) *Moments in Slabs and Beams.* The 1924 Joint Committee recommends a higher positive moment in the end span and a higher negative moment at the first interior support of slabs (Art. 107 *c–d*, Appendix B) than did its predecessor, which did not differentiate between end spans and interior spans except for beams. For long spans this is a better approximation to the facts. It is also an improvement to place a minimum value on the negative moment at the outer end of the end span. The two recommendations together are very conservative both as regards beams and slabs for a positive moment of $wL^2/10$ will hardly be reached until there

is a readjustment at the outer end of the member which practically makes that a free or hinged support. An amount of steel in the top of slabs at the spandrel end sufficient to develop a negative moment of $WL^2/16$ will prevent any appreciable amount of cracking and force the readjustment to take place by rotation of the spandrel beam, thus putting that member in torsion. The resistance of the spandrel to torsion is usually assumed to be ample without calculation and if this is the case, a positive moment of $wL^2/10$ is practically impossible of attainment. A different situation however exists regarding the moment factor for beams supported by other beams, for here probably the negative moment at the outer end is small and the positive moment in the end span correspondingly large.

For short span slabs up to perhaps 6 feet or even 7 feet it has been an unwritten law with many to count upon the help of so-called arch action (the assumption that the slab acts as a flat arch with the bottom steel as tie rods) and reduce the amount of negative reinforcement by one-third or one-half from that theoretically necessary. For spans of 4 and 5 feet many floors have been built with no steel in the top. These reductions are not good practice as often unsightly cracking has followed. Top steel is required wherever tension exists, a tendency easily visualized by picturing (in exaggerated scale) the form taken by a slab or other structure under load. A cracking sometimes neglected is that tending to occur parallel with and close to girders where the slab and girder reinforcement are parallel. Cross rods are needed over the girder in the top of the slab.

A comparison of the moment factors in Arts. 107 and 110 of the Joint Committee Report (Appendix B), shows the estimate placed upon the effect of stiff supports in reducing the deflections of beams and accordingly the positive moments. The stiffness of a member is proportional to its moment of inertia and inversely proportional to its length and is measured by I/h which may be called its stiffness factor. The supporting columns in Art. 110 (*b*) (Appendix B), are stiffer in relation to the beam than in Art. 110 (*c*). So in the second case there is more deflection and more positive moment and also less restraint at the outer end and therefore less negative moment.

The maximum positive and the maximum negative moments in a continuous beam do not occur simultaneously. For maximum

positive moment the beam itself and alternate spans on either side carry a full load and the moment curve is similar to that marked *a* in Fig. 72. For this loading, with the maximum positive moment assumed as $wL^2/12$, the corresponding negative moment at the supports is $wL^2/24$, the total height of the parabolic moment curve being $wL^2/8$. For maximum negative moment at B, Fig. 72, spans AB and BC are loaded and also alternate spans on either side. The resulting moment curve is not symmetrical but this

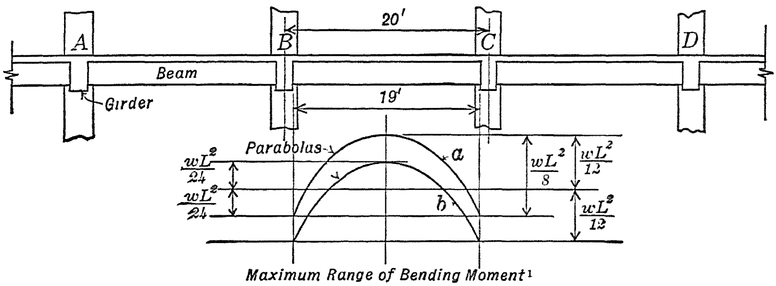


FIG. 72

may be ignored safely and a symmetrical curve such as *b*, assumed to represent the bending moment variation. Knowing the curves to be parabolas it is a simple matter to locate the points of inflection and the points where any of the reinforcing bars may be dispensed with, following the familiar graphical method used to determine the lengths of plate girder cover plates. For office practice diagrams or a table such as Table 14 are convenient for rapid work.

(*c*) *Bending Moment in Girders.* Reduction in the bending moments in continuous girders are made corresponding to those for continuous beams. See Art. 87.

(*d*) *Fireproofing.* The Joint Committee rules for protective covering of steel are quoted in Art. 27. They are more conservative than the older standards and are to be commended in general. However, in slabs the requirement of 1 inch of cover is somewhat open to criticism as the broad expanse of a slab is not highly vul-

[1] The total height of the moment curve for a uniformly loaded beam with equal negative moments at the ends is $wL^2/8$. When the end moments are unequal, their mean plus the positive moment at the center equals the same total.

nerable. Up to the time of this report (1924), $\frac{3}{4}$ inch was considered ample by most engineers who usually added 1 inch to the depth from the compression face to the steel (d) for the total thickness. To many the adding of $1\frac{1}{4}$ inches will seem a bit of academic precision. It is bound to be such unless use is made in the field of some approved form of bar support, which keeps the steel closely in the place assigned it in calculations.

(e) *Slab Reinforcement.* Three different methods of arranging the reinforcement of continuous slabs are shown in Fig. 73. The first is that preferred for slabs up to about 6 inches in thickness

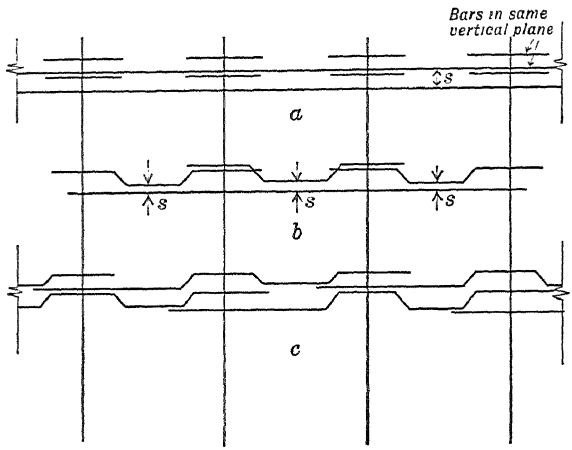


FIG. 73

where bending steel is troublesome on account of the small depth of the bends. This approval is conditioned upon the proper placing of the top steel. When the short straight bars in the top of the slab over the supports are placed during the pouring of the concrete, as is usual, they come to rest far from their assigned position unless the work has been expertly done with careful supervision to prevent dislodgment by the workmen after placing. Of the two systems of bent rods shown the last is the better. The other gives uneven and sometimes excessive spacing of the top steel. A good rule limits the spacing of slab steel to from $1\frac{1}{2}$ to $2\frac{1}{2}$ times the thickness of the slab.

Crossing the main reinforcement there are always placed spacer bars, sometimes called temperature or shrinkage reinforcement. These serve a very useful purpose in assisting to maintain

the spacing of the main rods wired to them and they also prevent the opening of cracks due to shrinkage and to temperature changes. In floor slabs ordinarily $\frac{3}{8}$ inch round rods are used, spaced at from 18 inches to 24 inches.

(*f*) *Minimum Thickness of Slabs.* It is difficult to make thin slabs with the concrete of uniform quality and with the steel reinforcement in proper position to resist the tensile stresses. In such slabs small variations in quality of concrete and in the location of the steel have very large effect. Therefore most engineers limit the thickness of slabs to a minimum of 4 inches. Usually the total thickness is made a multiple of the half inch.

(*g*) *Beams and Girders.* The design of a continuous tee beam forming part of a floor system comprises two major operations; (*a*) choosing a stem of sufficient size so that the diagonal tension and the compressive stresses in the flange at the center and in the stem at the support are within working limits, and so that there are adequate clearance and cover for the reinforcement; (*b*) choosing the reinforcement. The problem opens with a flange (the slab) of known thickness and indefinite width already provided.

Diagonal tension is measured by shear and usually the first step in design, after the moment and shear have been computed, is to find the size of stem required by the limiting shear stress. Until the publication of the preliminary report of the present Joint Committee (in 1921), this limit was always set at 6 per cent of the 28-day compressive strength of the concrete when diagonal tension reinforcement was used and at 2 per cent of the 28-day strength for no diagonal tension reinforcement. The 1924 report (Art. 190–191, etc., Appendix B), proposes the 6 per cent limit for ordinary cases and a limit of 12 per cent of the 28-day strength when special anchorage is provided for the main reinforcing rods.

The width of stem chosen is usually a whole number of inches and often an even number. Sometimes it is possible to save on formwork by making the width such that it may be built with stock widths of lumber without resawing. What is called for by the plans as an 8 inch beam width is probably often made with stock lumber to measure $7\frac{5}{8}$ inches. The majority of designers do not consider formwork in choosing the size of beam stem.

It is entirely feasible, although not usual, for the second step, to estimate the size of stem required by the limiting compressive

stresses at the supports. A limiting value of R, (M/bd^2), for use as a guide in choosing a trial section, can be determined from Plate IX or X by consideration of the desired ratio of tensile and compressive reinforcement at the support, a ratio depending upon the arrangement of rods and upon the size of the beam. A section may be chosen in a similar way by use of Tables 9 and 12.

Quite commonly the maximum positive and negative moments in a beam are the same. If the main reinforcement consists of an even number of rods of one size, the required tension area over any support may be sufficiently provided for by bending up one-

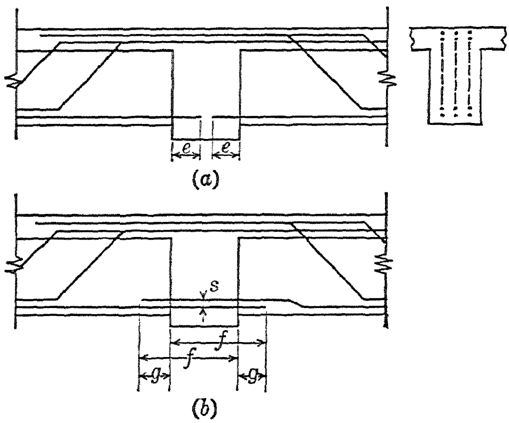


FIG. 74

half of the bars from each adjoining beam. The simplest arrangements are either the use of tension steel only, requiring a relatively large section, or the use of compression steel equal in amount to one-half or to all of the tension steel. The possibilities in this situation are illustrated by Fig. 74 *a–b*. In the first sketch the bottom rods extend so short a distance (e) beyond the face of the support that there is no opportunity to develop any considerable amount of stress, and the bars are inoperative as compression reinforcement. In the second case the lower rods are extended a sufficient distance (f) beyond the face of the support to develop the compressive stress existing at their level in the beam, and accordingly the area of the compression steel equals one-half of that of the tension steel (or $p' = \frac{1}{2}\,p$, see Plate IX). In this case the distance f may or may not be enough to cause the bars to

extend into the adjoining beam. Usually there is sufficient clearance to allow the two sets of rods to lap by each other, making the dimension s (Fig. 74) zero. It is not necessary to follow the rules of bar spacing and clearance strictly for the short distance of this overlapping. The third arrangement is to extend the bars from one beam a sufficient distance (g) into the adjoining beam to be effective as compression reinforcement there. The distance g must be at least equal to that required to develop the compressive stress in the bar at the face of the support. Usually this arrangement demands two layers of steel, making $s = 2$ inches (center to center of rods). On Plates IX and X the values for R for $p' = \frac{1}{2}\,p$ and $p' = p$ are directly indicated and may be easily estimated for other ratios.

If the main reinforcement of a beam consists of straight bars only, each set of rods must extend entirely across the region of positive or negative moment for which it is used. If a bar is cut off in the zone of stress the end should be bent up or hooked beyond the theoretical point of cutting off so that it may take on the same strain as the surrounding concrete and be equally stressed with the longer rods alongside. A short bar should be placed inside of and across the hook to reinforce the concrete against splitting.

Having chosen trial dimensions for the stem the next step is to calculate the area of steel required by the maximum positive moment and choose the bars. The steel area can be found by the familiar relation, total tension (moment divided by lever arm) divided by unit tensile stress, or by tables. The lever arm of the resisting couple may be approximated as from 0.90 to 0.92 of the depth (d) to the steel. In selecting rods it is necessary to pay attention to their spacing for proper clearance and sometimes to bond stress.

The Joint Committee sets no limit to the bond stress when adequate anchorage is provided. (See Arts. 136, 137, 139, 140 Appendix B and discussion in Art. 74.) Their equation (35) can be rewritten for a single square bar with side D thus:

$$f_sD^2 = Qu \times 4\ Dy + u \times 4\ Dx$$

$$Qy + x = \frac{f_s}{4\ u}\ D.$$

The same relation holds for round rods and is recognized as Equation (8), developed in Art. 54.

In the case of negative reinforcement over the support, as in the slab shown

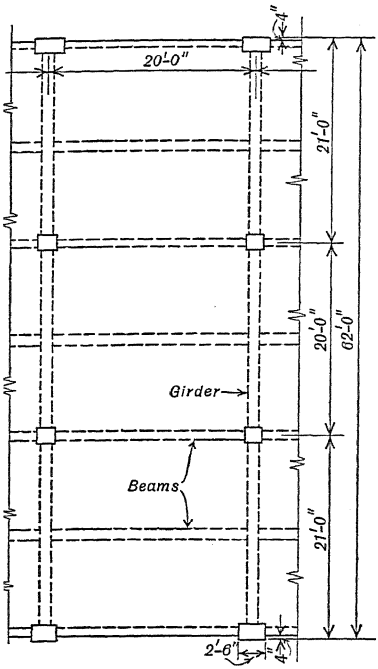
on Computation Sheet BF2, Art. 97, the section of maximum stress is at the face of the support and the anchorage begins at the point of inflection, about 0.20 to 0.25 of the span length from the support. (Table 14.) For these values Q would equal 0.80 to 0.75 and Qy would be 0.16 to 0.19 of the span. So if the length of embedment required to develop the maximum stress in the rod exceeds this fraction of the span the bar must extend beyond the point of inflection a distance equal to the deficiency. The usual practice of extending these top bars to either the quarter or third points of the span often makes use of the Joint Committee equation unnecessary.

When the steel area is calculated as above suggested a separate investigation must be made of the stresses in the concrete at the sections of maximum positive and negative moment. It is customary to allow a higher limit in compression at the support than at the center of the span for two reasons. The negative moment decreases very rapidly and the highly stressed length is very short indeed. The negative moment section is also less critical than that at mid span since cracking here results only in increased positive moment. However, the usual factor of safety for the support is even less than indicated by the fiber stresses employed (see Art. 188, Appendix B), since the bottom of the beam which is the compression face is liable to great damage in case of fire.

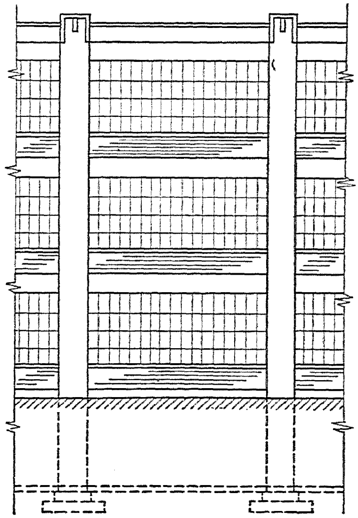
96. Data for Design Problem. (Computation Sheet BF1.) Figure 75 shows a typical bay of a three-story industrial building with beam and girder floors. It is assumed that this lay-out has been determined by preliminary computations for economy (see Chapter XVIII) and conditions of use. The data necessary for design are listed on Computation Sheet BF1 and should always be shown in a similar manner at the beginning of any set of computations.

This is a relatively simple problem so far as application of theory is concerned. It is greatly complicated, however, by the fact that competent designers use many short cuts based on experience. The student of the subject is apt to assume from some of the short cuts he may see used that reinforced concrete design is largely a matter of guess work. Too many times, perhaps, it is but the student must remember that the work of the competent designer is based on much experience and much checking of assumptions and short cuts by exact methods.

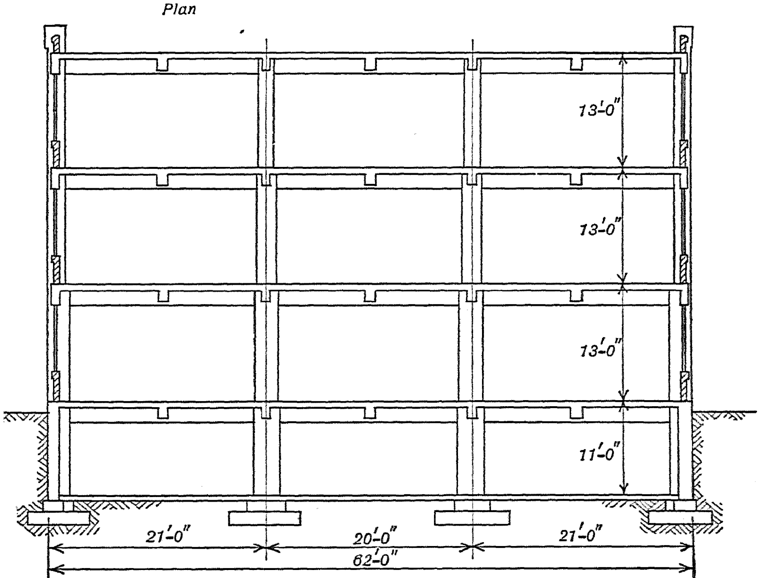
Two different solutions are shown for this problem, one using curves (Plates VI to X) and the other using tables (Tables 9 to 12,



Plan



Elevation



Section Through Building

FIG. 75

Appendix F). Both methods are equally correct and it is simply a question of speed and convenience which is used.

Inspection of the computations for this problem shows that in every instance the first step in designing a member was to obtain the span and loads, making a sketch if necessary. No problem can be solved until it is first clearly stated.

97. Design by Use of Curves. *(a) Slab.* (Computation Sheets BF1–BF2.) Reference to the typical floor plan of Fig. 75 shows that this floor slab is a rectangular beam about 19 feet wide and continuous over a series of supports about 10 feet apart. Since the width is greater than 1.5 times the span no main reinforcement is required the long way of the panel. For convenience a strip of slab 1 foot wide is designed and the same area of reinforcement placed in each foot the whole width. The Joint Committee specifies that the span of continuous slabs and beams used in computing the shear and moment shall be taken as the clear span (Art. 106, Appendix B), which may be taken as 1 foot less than the distance center to center of supports for a first trial computation.

In this floor there are many more interior spans than exterior and so it was decided to make the thickness as closely as possible that required for balanced design for the smaller moment of the interior span and use whatever extra steel might be needed to satisfy the conditions in the end panel. The end panels would be more economically designed with a greater slab thickness and less steel but this greater thickness would result in a less economical design for the interior panels. Which plan is the better in any case can be told only by a cost comparison.

The steel area was calculated by Equation (16) ($M = f_s A_s jd$) using the average value of j recommended by the 1916 Joint Committee and using the moment in inch-pounds acting upon a strip of slab one inch wide, which is numerically equal to the moment in foot-pounds acting on a strip one foot wide. The area found, 0.026 square inch, is that required per inch of width of slab and the spacing of the $\frac{1}{2}$ inch round rods was obtained by dividing the cross-sectional area of one bar by the required area per inch ($0.196 \div 0.026 = 7\frac{1}{2}$ inches). The resulting figure, the number of inches of width that one bar can reinforce, is the required spacing for that size rod. The steel area for the end span was determined by following the procedure of Example 32*b*, page 110.

COMPUTATIONS FOR SLAB, BEAM AND GIRDER FLOOR

Sheet **BF1**

Slab-Beam-Girder Floor *Data Sheet.*

Sketches on Fig. 75

Live Load: $125\#/\square'$

Floors: *1 in. granolithic*

Materials:

Steel: *Intermediate grade: deformed bars.*

Concrete: *Max. size aggregate: 1 in.*

Ultimate strength at 28 days: $2000\#/\square''$

Specifications: *1924 Joint Committee except as noted.*

Stresses:

$f_s = 16{,}000\#/\square''$

$f_c = 0.325 \times 2000 = 650\#/\square''$ *bending.*

$= 650 \times 1.15 = 750$ " *at supports.*

$u = 0.05 \times 2000 = 100$ "

$v = 0.02 \times 2000 = 40$ " *no web reinforcement.*

$= 0.06 \times 2000 = 120$ " *with web reinforcement.*

Constants: $16{,}000 - 650$ $n = 15$

$R = 108$

$p = 0.0077$

$k = 0.379$

$j = 0.874$

Remark: *Figures in parentheses refer to articles in the Joint Committee Report: Appendix B.*

Computation for Floor Slab

Allowable $f_c = 650$ $f_s = 16{,}000$ $n = 15$ $v = 120$ $u = 100$

		Live Load	= 125
Clear Span = 9′-0″	*Assume 12″ beams.*	*Slab 5″*	= 63
Bearing Span = 10′-0″		*1″ Grano. Fin.*	= 13
Panel Width		*Beam*	=
			$201\#/\square'$

$$M = \frac{wL^2}{12} = 201 \times \overline{9.0}^2 \times \tfrac{1}{12} = 1360'\#/' \text{ strip: } \textit{Interior span.} \qquad [107c]$$

$$V = \frac{wL}{2} = 201 \times 9.0 \times \tfrac{1}{2} = 905\,\#$$

Thickness

$$d = \sqrt{\frac{M}{bR}} = \sqrt{\frac{1360 \times 12}{12 \times 108}} = 3.55''$$

$$+\ 1.25''$$

$$\overline{4.80}$$

Use 5″ Slab ←

$d = 3.75''$

The bond stress computed for the top rods over the interior supports was somewhat higher than the allowable without special anchorage for the rods. It was considered that this was supplied by following the common custom of extending these bars to the quarter point of the clear span, as indicated by the discussion of Art. 95 above.

In this design the same value of d was taken for both positive and negative steel. Since there is a fireproof floor finish over the structural slab it would be proper, with accurate steel placing ensured, to place the top steel within $\frac{1}{2}$ inch or $\frac{3}{4}$ inch of the slab surface, making d equal to $4\frac{1}{4}$ inches or 4 inches. This would decrease the steel area required over the beams and so increase the computed spacing.

The sketch on Sheet BF2 shows the final design clearly. It is essential that the results of any computation be indicated definitely so that the detailer working from the sheet may make no error in transferring the design to his drawing.

(*b*) *Beams.* (Computation Sheets BF2–BF4). The beams in this floor system are parallel to the face of the building and extend continuously from one end of the structure to the other. The end spans are assumed to be equal in length to the interior spans. The task is to design a suitable interior beam of a size that, with additional reinforcement, will be satisfactory also for the more highly stressed end span. The superimposed loads to be carried are those brought to the beam by a width of slab equal to the beam spacing. The beams here designed are those carried by the girders midway between columns, with larger moment factors than required for those supported by the columns. Both sets of beams would be made of the same size and usually, although not necessarily, reinforced the same.

In estimating the weight of the stem of this beam a common guide was followed; the depth in inches was taken equal to the span in feet and the breadth made about one-half the depth.

A trial size of stem was obtained as required by shear (diagonal tension) and by the compressive stress at the support, using for simplicity one-half as much compression as tension reinforcement. (See Art. 95 (*g*), above.) Two of the several arrangements of bars that provide the necessary area are shown. The 4-rod combination can be placed in one layer with proper clearances. (See Art. 82*c*, page 158.) The 6-bar combination requires two layers and

COMPUTATIONS FOR SLAB, BEAM AND GIRDER FLOOR

Sheet BF2

Floor Slab: Steel

Interior span: $A_s = \dfrac{M}{f_s jd} = \dfrac{1360''\#/''}{16{,}000 \times \frac{7}{8} \times 3\frac{3}{4}} = 0.0259\square''/''$ $\quad \frac{1}{2}''\phi$ @ $7\frac{1}{2}''$o.c. ← *for Int'r.*

Exterior span: $R = \dfrac{M}{bd^2} = \dfrac{1360 \times 1.2}{\overline{3.75}^2} = 116$ [107c]

$\therefore p = 0.0098 \quad A_s = 0.0098 \times 3\frac{3}{4}$

(*Plate VI*) $\quad = 0.0368\square''/''$ $\quad \frac{5}{8}''\phi$ @ $8''$ o.c. ←*for Ext'r.*

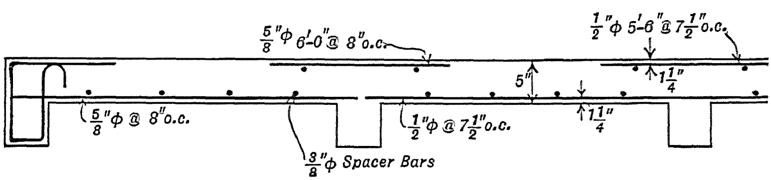
Bond Stress at Supports.

$$v = \frac{V}{bjd} = \frac{905}{12 \times \frac{7}{8} \times 3\frac{3}{4}} = 23\#/\square''$$

For $\frac{1}{2}''\phi$, $\quad u = \dfrac{vb}{\Sigma_0} = \dfrac{23 \times 7.5}{\pi \times \frac{1}{2}} = 110\#/\square''$ $\quad \therefore$ space at $\dfrac{100}{110} \times 7\frac{1}{2} = 7''$ $(6.8'')$

or anchor bars.

For $\frac{5}{8}''\phi$, $\quad u = \dfrac{23 \times 8}{\pi \times \frac{5}{8}} = 94$ $\quad$ *Anchorage:* $L = \dfrac{16{,}000}{4 \times 100} \times \frac{1}{2} = 20''$.



Computation for Floor Beam

Allowable $f_c = 650 \quad f_s = 16{,}000 \quad n = 15 \quad v = 120 \quad u = 100$

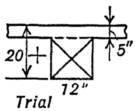
750 at supports

Clear Span = 19'-0"	Assume 12" girders.	Live Load	= 125
Bearing Span = 20'-0"		Slab 5"	= 63
Panel Width = 10'-0"		1" Fin.	= 13
		Beam	=
			201

Loads: Live + dead, from slab

$201 \times 10 = 2010$

Stem $\quad 200$ [213 actually 12" × 22" beam]



$M = \dfrac{wL^2}{12} = 2210 \times \overline{19}^2 \times \frac{1}{12} = 66{,}500'\#$ $\quad$ *Interior Spans.* [107c]

$V = \dfrac{wL}{2} = \;$ " $\times 19 \times \frac{1}{2} = 21{,}000\#$

a 1 inch deeper beam. It was decided to use 4 bars as being simpler and somewhat more economical. On the other hand the 6-rod design employs lighter bars which are more easily bent and handled, and the use of 3 instead of 2 bent rods would make the beam stiffer and better reinforced for diagonal tension.

The investigation of the compression in the concrete at the section of maximum positive moment $\left(M = \frac{wL^2}{10}\right)$ was made for an assumed end span requiring 1.2 times as much steel as an interior span. The procedure was that of Example 34, page 112.

Similarly the investigation for compression at the support was first made for an end span following the argument of Example 39, page 115. The sketches on Sheet BF4 show plainly the steel arrangement assumed. It was possible to locate the steel 2 inches from the top of the beam making d = 20 inches at the support as against 19.5 inches at the center of the beam, a gain very desirable in order to compensate for the lower value of j at the support. The assumption of equal areas for both positive and negative tension reinforcement disregards the very appreciable difference in the value of j at the two sections.

As a result of this investigation it appears that at the first interior support the straight rods from each adjacent beam must extend into the other to act as compression reinforcement. (See Fig. 74.) At the interior supports the straight bars in each beam are sufficient without other aid.

It is questionable whether there is justification for great precision in the computations for the reinforcement at the support of this continuous tee beam. The slab assists the steel in resisting the tension over the support up to the point of very severe overloading. It is not possible to take direct account of this added element of strength nor is it proper to do so since its action is uncertain and variable. However it would seem permissible for spans of equal length with equal positive and negative moments, to assume that the same area of reinforcement is required over the supports as at mid span without further computation other than required to determine the amount of compression reinforcement.

It is to be noted that a considerable excess of steel is provided at each interior support. This is because it was judged better to make all beams alike with the heavy pair of rods bent, rather than unlike with alternate pairs of heavy and light bars bent, or alike

COMPUTATIONS FOR SLAB, BEAM AND GIRDER FLOOR — Sheet BF3

Floor Beam: Continued

Stem: Shear:

$$b'd = \frac{V}{jv} = \frac{21{,}000}{\frac{7}{8} \times 120} = 200\square'' \qquad \begin{aligned} b' &= 10'' & d &= 20.0'' \\ &= 12'' & &= 16.7'' \end{aligned}$$

Support:

$$\text{Try } p' = \tfrac{1}{2}\, p. \quad \text{For } 16{,}000 - 750 \qquad \left.\begin{aligned} & \\ \text{Try } d'/d &= \frac{2.5}{d} = 0.125 \end{aligned}\right\} \begin{aligned} R &= 175 \quad (\text{Plate IX}) \\ p &= 0.0126 \end{aligned}$$

$$b'd^2 = \frac{M}{R} = \frac{66{,}500 \times 12}{175} = 4550 \qquad \begin{aligned} b' &= 10'' & d &= 21.4'' \\ & 12'' & & 19.5'' \end{aligned}$$

$$\text{Try } b' = 12'' \quad d = 19.5'' \quad v = 103\#/\square''$$

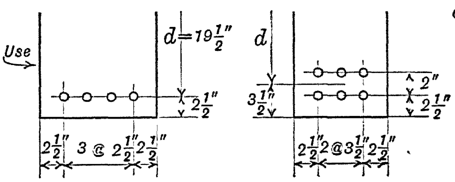
Steel.

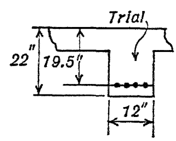
$$A = \frac{M}{f_s jd} = \frac{66{,}500 \times 12}{16{,}000 \times \cancel{0.90} \times 19.5} = \overset{2.78}{\cancel{2.84}}\square'' \qquad \left.\begin{aligned} 2\text{-}1''\phi &= 1.57 \\ 2\text{-}\tfrac{7}{8}''\phi &= 1.20 \\ & \overline{2.77\square''} \end{aligned}\right\} \leftarrow$$

0.92 actual — See below.

or

$$\begin{aligned} 4\text{-}\tfrac{3}{4}''\phi &= 1.76 \\ 2\text{-}\tfrac{7}{8}''\phi &= 1.20 \\ & \overline{2.96\square''} \end{aligned}$$





Minimum Spacing: $2\frac{1}{2}$ *diameters (page 158)*

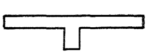
$$2.5 \times \tfrac{1}{2}\,(1'' + \tfrac{7}{8}'') = 2.34''$$

Bond: $u = \dfrac{vb}{\Sigma O} = \dfrac{103 \times 12}{2\,\pi(1 + \frac{7}{8})} = 105 > 100.$ *Excess not enough to demand special anchorage.*

Check of f_c. *@ Mid Beam — End Span (+) M*

$$M = wL^2/10 \qquad\qquad \text{Plate VII}$$

$$b\begin{cases} \dfrac{L}{4} = 57'' \; [115] \\ 16 \times 5 + 12 = 92'' \\ \text{Bm. Spcg.} = 120'' \end{cases} \qquad \left.\begin{aligned} t/d &= \frac{5}{19.5} = 0.26 \\ p &= \frac{2.84 \times 1.2}{57 \times 19.5} = 0.0031 \end{aligned}\right\} \begin{aligned} j &= 0.92 \\ \frac{f_s}{f_c} &= 43 \end{aligned}$$



$$f_c = \frac{16{,}000}{43} = 370\#/\square''$$

with 1–1 inch round and 1–$\frac{7}{8}$ inch round, bent, in each beam. The increased simplicity was held to justify the extra cost.

(*c*) *Girders.* (Computation Sheets BF5–BF6.) The girders carry the concentrated loads of the beam reactions (the end shears), and their own uniform dead weight, plus that of the live load coming directly upon them. It is not convenient nor necessary to compute the girder loading as thus described. It is simpler and sufficiently accurate to take for the concentrated load brought to the girder by a beam the total live and dead load per foot on the beam multiplied by the girder spacing or total beam length. The girder dead weight, then, is that of the stem below the slab.

It is desirable that the girder be at least 3 inches deeper than the beams as this makes it possible to place the beam reinforcement without interfering with the steel in the girder. The computation of the size of the stem made necessary by shear and compression at the support follows the argument made familiar in the design of the beam.

A convenient method of studying a beam or girder continuous over several supports is illustrated in the sketch on Sheet BF6 which was constructed as follows. Three vertical lines were drawn to represent the wall column and two interior columns. On account of symmetry it was not necessary to draw all three spans of this girder. Next the several moment factors were recorded, together with the value of the maximum positive moment in the end span, and following this, the value chosen for d for the positive steel. It will be noted that the moment factor chosen for positive moment in the interior span was that recommended by the 1916 Joint Committee which is larger than that specified by the 1924 report. (Art. 110 (*a*), Appendix B.)

The equation $A_s = M/f_sjd$ (16) is so familiar that it was not thought necessary to do more than indicate its use by recording the value of j (0.90) and the area of steel (6.56 square inches) resulting. Next the reinforcement was chosen and noted; 4–1 inch and 2–1$\frac{1}{8}$ inch square bars. It was considered wise next to investigate the maximum fiber stress in the concrete due to positive moment, and using Plate VII as before, that stress was found to be low, 570 pounds per square inch. The revised value of j made no change in the choice of rods. Next the steel area required for the interior span was found from that required for the end span,

COMPUTATIONS FOR SLAB, BEAM AND GIRDER FLOOR

Sheet BF4

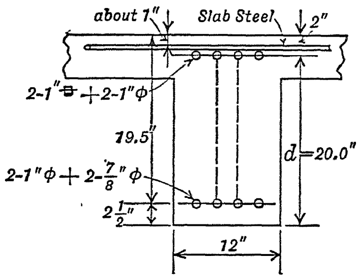
Floor Beam: Stem: Continued

Check of f_c

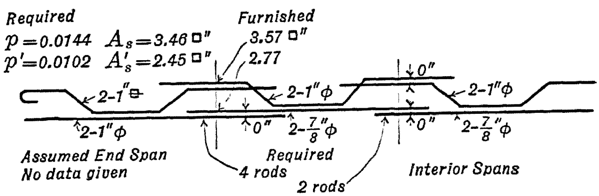
@ 1st Interior Support $M = -wL^2/10$

In end span $A_s = 2.84 \times 1.2 = 3.41\square''$

Assume $2\text{-}1''\phi - 2\text{-}1''\square = 3.57\square''$



$$\frac{M}{b'd^2} = \frac{66{,}500 \times 12 \times 1.2}{12 \times \overline{20.0}^2} = 200 \quad d'/d = 2.5/20 = 0.125. \Bigg\} \text{ Use Plate IX}$$



@ Other Interior Supports.

$$\frac{M}{b'd^2} = \frac{10}{12} \times 200 = 167 \quad d'/d = 0.125 \Bigg\} \text{ Plate IX}$$

Required:

$p = 0.0121 \quad A_s = 2.91\square''$

$p' = 0.0053 \quad A_s' = 1.27\square''$

Furnished: $3.14\square''$

$4\ \cancel{2}\text{-}1''\phi + \cancel{2\text{-}\tfrac{7}{8}\phi} = \cancel{2.77}\square''$

$2\text{-}\tfrac{7}{8}\phi = 1.20$

O.K. Exact analysis not needed.

the two areas being proportional to the maximum positive moment in the two girders and these moments varying directly with the moment factors and the spans (considering concentrated loads only).

With all the main bars chosen, it was next possible to show them on the sketch complete except for the details at the supports. The sketch at the bottom of Computation Sheet BF6 shows the cross-section at the outside of the interior column and was drawn showing at first only the bars indicated by open squares with the beam steel crossing over the top. This sketch showed that in order to clear the beam steel it was necessary to make d for the negative moment girder reinforcement $22\frac{1}{2}$ inches instead of 23 inches. The first step in the investigation of the steel area required at the support gave so high a value of R that it was evident that two layers of compression reinforcement would be needed.

The simplest way of providing this area was thought to be to allow the straight rods in each girder to extend into the supporting column a sufficient distance to develop the required compression by bond, and to provide a second layer of short rods. The area of these extra short rods was computed on the assumption that the high negative moment exists in equal value on both faces of the column. Since the straight bars in the bottom of the interior girder are smaller than those in the exterior span they were used in computing the area of the extra short bars. As a matter of fact the moment on the inside face of the column is less than that on the exterior face since the stiff column assists the girder in carrying its load. The design was completed by the investigation of the bond stress which was found to be low.

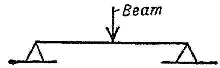
(*d*) *Beam Details* (Computation Sheet BF7). The main features of the design of the beam and girder having been determined, it was necessary next to decide upon the details, the lengths and points of bending of the principal bars and the design of the diagonal tension reinforcement. This was done for the beam without the use of some of the aids that have been developed and used hitherto, in accordance with the basic purpose of this text to lead the student to a state of complete independence of tables and formulas.

First the beam outline was drawn to scale and then a parabola (*ab*) was constructed with vertex at the center of the span to repre-

COMPUTATIONS FOR SLAB, BEAM AND GIRDER FLOOR

Sheet BF5

Computation for Floor Girder.



Allowable $f_c = 650$ $\quad f_s = 16{,}000 \quad n = 15 \quad v = 120 \quad u = 100$

750 at supports. *Live Load* = 125

Clear Span = 18′-6″ *int'r Span* · 18′-9″ *End Span* *Slab* 5″ = 63

Bearing = 20′-0″ *Int'r Span* 1″ *Fin.* = 13

Panel Width = 20′-0″ *Beam* =

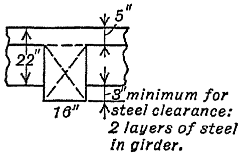
201

Loads: — *Beam:* $20 \times 2223 = 44{,}460\#$ *See Sheet BF2*

Stem: $18.75 \times 330 = 6{,}200$

$2\,)\overline{50{,}700\#}$

$V = 25{,}300\#$



M, *End Span;* $\dfrac{PL}{4} \times \dfrac{8}{10}$:

$\frac{1}{4} \times 44{,}500 \times 18.75 \times \frac{8}{10} = 167{,}000'\#$

$330 \times \overline{18.75}^2 \times \frac{1}{10} = 11{,}600$

$M = 178{,}600'\#$

Stem. *Shear.* $b'd = \dfrac{V}{jv} = \dfrac{25{,}300}{\frac{7}{8} \times 120} = 241\,\square''$ $\quad b' = 12'' \quad d = 20.1''$

Support: $\left.\begin{array}{l} \text{Try } p' = \frac{1}{2}p = 0.0065 \\ d'/d = \dfrac{2.5''}{d} = 0.10\pm \\ \quad\quad\quad\quad 1 \text{ layer} \end{array}\right\}$ $R = 180$ (*Plate IX*)

$b'd^2 = \dfrac{M}{R} = \dfrac{178{,}600 \times 12}{180} = 11{,}900 \quad b' = 16''$

$d = 27.3''$

Too large

$\left.\begin{array}{l} \text{Try } p' = p = 0.0168 \\ d'/d = \dfrac{3.5}{d} = 0.15\pm \\ \quad\quad\quad\quad 2 \text{ layers.} \end{array}\right\}$ $R = 228$ (*Plate IX*)

$b'd^2 = \dfrac{180}{228} \times 11{,}900 = 9390 \quad b' = 14'' \quad d = 25.9''$

16	24.2
17	23.5
18	22.8

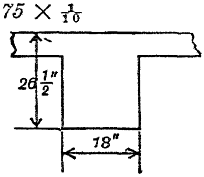
$\left.\begin{array}{l} \text{Try } b' = 18'' \\ \quad\quad d = 23'' \end{array}\right\}$

Total depth $= 26\frac{1}{2}''$

Stem weight $= 405\#/' = 7550\#$ $\left\}\begin{array}{l} \Delta V = \frac{1}{2} \times 1400 = 700\# \\ \Delta M = 1400 \times 18.75 \times \frac{1}{10} \\ \quad\quad = 2600'\# \end{array}\right.$

Revised $V = 26{,}000\#$

$M = 181{,}200'\#$



Unit shear $= \dfrac{26{,}000}{18 \times \frac{7}{8} \times 23} = 72\#/\square''$

$A_s = 0.0168 \times 18 \times 23 = 6.95\,\square''$ *approx.*

See $p = p'$ *above.*

sent the curve of reinforcement area required.[1] This parabola was first considered to correspond to curve a in Fig. 72. (See discussion in Art. 95 (*b*).) An axis (*de*) at $\frac{2}{3}$ the total height from the vertex represents the axis of the beam when the positive moment equals $wL^2/12$. Then *ae* represents 2.77 square inches, the total area furnished at mid span, assumed equal to that there required, and the distance to the curve from *de* at any point represents the steel area required at that point.

For a maximum negative moment of $wL^2/12$ the axis is transferred to *fg*, locating the point of inflection 47 inches from the face of the support. For this case the parabola is taken to represent curve b in Fig. 72. The points of bending down the steel were determined from triangle *hjk* drawn below the parabola and corresponding to *fgb* in the upper figure, with *jk* taken as a straight line for simplicity. The vertical distance from *hj* to line *jk* at any point represents the steel area required in the top of the beam at that point.

The limits of the points of bend having been determined as thus briefly indicated, the bent steel was sketched in with a 45 degree bend located within the required range.

The computations had shown the 2–$\frac{7}{8}$-inch straight bars to be necessary for compression reinforcement at the support. They are there stressed to about 7800 pounds per square inch. The exact determination of this stress is not important. Triangle *mno* represents the stress curve for the concrete in the stem at the support assuming the maximum stress of 750 pounds per square inch to be realized, and assuming the neutral axis to lie at a distance of 0.4 d from the compression face, the bottom of the beam. The length of embedment (18 inches) is the distance the straight bars should extend into the support in order to develop their stress at the face of the support.

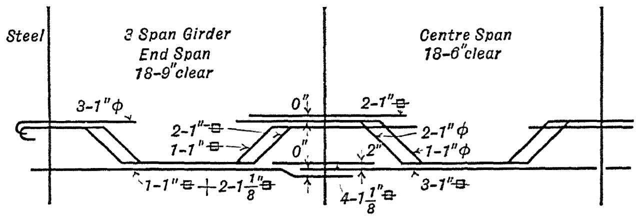
The bent bars should extend over the support a short distance beyond the farthest point of inflection in the adjoining span, perhaps 3 inches to 6 inches more than the 56 inches $(47 + \frac{18}{2})$ distance in this case. It is customary, however, to extend these bars

[1] A simple construction method for a parabola is indicated in the sketch on the computation sheet. The half span and the total height of the curve (*ac* and *bc*) were divided into an equal number of equal parts (6 in this case). The intersection of any vertical through a division point on *ac* with a line drawn from *a* to the corresponding division point on *bc* is a point on the curve.

COMPUTATIONS FOR SLAB, BEAM AND GIRDER FLOOR

Sheet BF6

Stem: Steel



$$M.\ Factor = -\frac{wL^2}{16} \qquad +\frac{wL^2}{10} \qquad -\frac{wL^2}{10} \qquad +\frac{wL^2}{12} \qquad -\frac{wL^2}{10}$$

$= 181{,}200'\#$

$d = \ldots\ldots\ldots\ldots\ldots\ldots 23'' \ldots\ldots 22\tfrac{1}{2}'' \ldots\ldots 23''$

$j = \ldots\ldots \cancel{0.90}\ \ 0.91 \ldots\ldots$

$A_s = \dfrac{M}{f_s j d} = \cancel{6.56}\square''\ \ 6.50\square''$

$\left\{\begin{array}{ll} 2\text{-}1\tfrac{1}{8}\square & 2.52\square'' \\ 4\text{-}1''\square & 4.00 \\ & 6.52\square'' \end{array}\right.$

Check of f_c: *Plate VII*

$b = L/4 = 56'' \pm$

$j = 0.91$, $\dfrac{f_s}{f_c} = 28$; $p = \dfrac{6.52}{56 \times 23} = 0.0051$; $t/d = 5/23 = 0.22$

$f_c = \dfrac{16{,}000}{28} = 570\#/\square'' < 650$

$R = \dfrac{M}{b'd^2} = 239$

Plate IX

$\dfrac{d'}{d} = \dfrac{3.5}{22.5} = 0.16$ for 2 layers

$\ldots\ 0.91$

$6.50 \times \dfrac{10}{12} \times \dfrac{18.5}{18.75} = 5.35\square''$

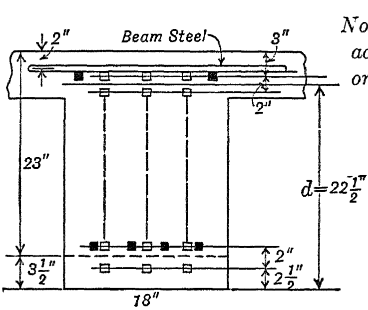
$3\text{-}1''\square + 3\text{-}1''\phi = 5.36\square''$

	Required:	Furnished
$p = 0.0176$	$A_s = 7.12\square''$	$7.36\square''$
$p' = 0.0195$	$A_s' = 7.89\square''$	$8.03\square''$

Furnished. Top: $3\text{-}1''\phi + 3\text{-}1''\square = 5.36\square''$
add $2\text{-}1''\square$ $\quad 2.00$
$\quad 7.36\square''$

Note: Straight bars act in compression only in own span.

Bottom: $3\text{-}1''\square \quad 3.00\square''$
add $4\text{-}1\tfrac{1}{8}\square \quad 5.03$
$\quad 8.03\square''$



Extra bars at support shown in black.

Bond:

$$u = \frac{vb}{\Sigma_0} = \frac{72 \times 18}{3\pi + 5 \times 4} = 44\#/\square''\ \ O.K.$$

at least to the quarter point and this gives a greater length. Both straight and bent bars were made a multiple of 6 inches in total length.

No discussion of the method of stirrup proportioning is necessary in view of the consideration already given that subject (Art. 53). The only new feature to be noted is the Joint Committee requirement that provision be made for one-fourth as much shear at the center as at the end. This of course is to care for the effect of incomplete loadings which would cause larger shear and therefore larger diagonal tension at any section than given by full loading. For purposes of illustration the bent rods are considered as making stirrups unnecessary in the length of the bend. However, the authors prefer the practice that uses full stirrup reinforcement without regard to the bent rods.

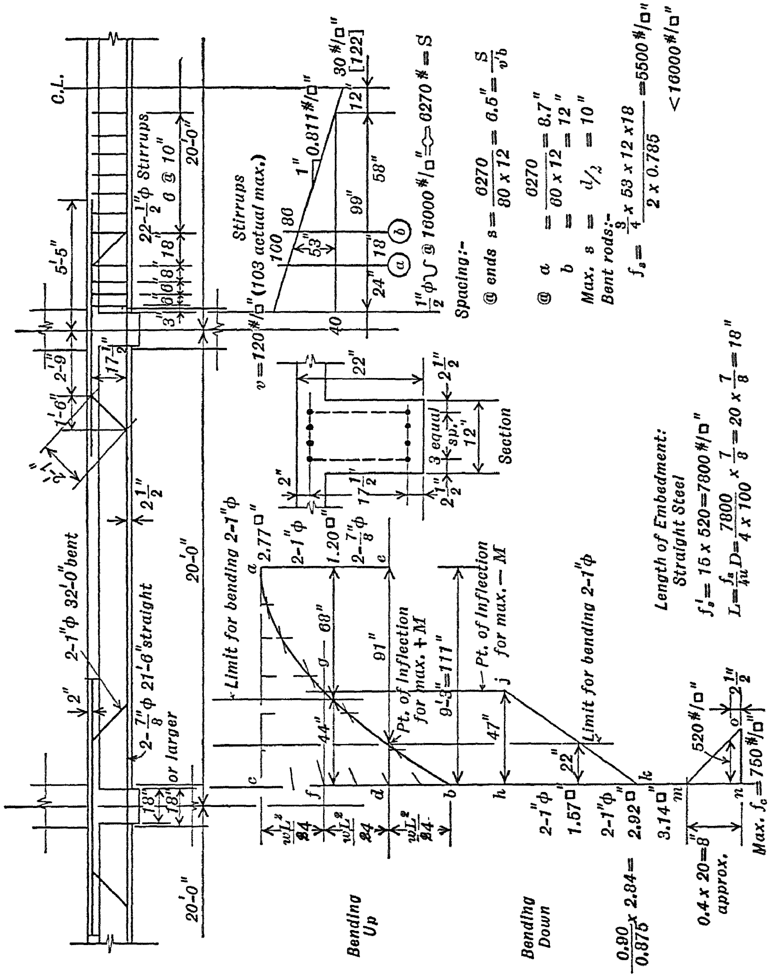
(*e*) *Girder Details* (Computation Sheet BF8). The general features of this sheet are the same as those of sheet BF7 for the beam. For the girder the chief load is the concentrated weight of the beam and so the moment curve is taken as made up of straight lines, neglecting the curvature introduced by the small distributed weight of the girder stem. The relative division of the total moment into positive at the center and negative at the support is assumed to be the same as for a uniformly loaded beam, and on this basis the points of inflection were located as shown. The triangle *abc* represents the curve of required steel areas for maximum positive moment; the triangle *def* for maximum negative moment. These curves are for an interior span with equal positive and negative moments. On the outside face of the interior column the moment is 20 per cent larger than that for an interior span, and the area of steel furnished is correspondingly greater. This larger area of steel was used (triangle *deg*) in determining the points of bending down bars, giving somewhat greater lengths over the support than required for the interior span. Another new feature, here introduced simply for illustration, is the bending up of the bent rods one at a time. Conservative practice would not allow this on account of the lack of symmetry introduced.

The length of the 4–$1\frac{1}{8}$ inch bars used in the bottom of the girder at the column was found as follows. For maximum negative moment the value of M/bd^2, called R, varies from 239 at the support to zero at the point of inflection, 74 inches away. Assuming straight line variation in between, 33 inches from the support

COMPUTATIONS FOR SLAB, BEAM AND GIRDER FLOOR

Sheet BF7

Beam Details:



$R = 134$ (for 16,000–750) and no compression steel is required at that point. Approximately half way out, 17 inches from the face of the column, one-half of the total compression reinforcement, represented approximately by the lower layer of rods, ceases to be adequate and the second layer, the 4–$1\frac{1}{8}$ inch bars, becomes necessary. The total length of these rods should be somewhat more than $2 \times 17 + 18 = 52$ in.

In designing the stirrups the variation of shear due to the uniform load was disregarded. As in the case of the beam the shear was taken as somewhat more than the actual maximum to provide for possible overload.

98. Design by Tables (Computation Sheets GF1–GF3). These sheets show another design of this same floor by the use of tables. The problem was approached independently and no attempt made to work out an identical solution. The slab for the interior span was picked from Table 9 as that providing a moment of resistance nearest to and higher than the actual. The steel area is proportional to the moment. It is not sufficiently precise to prorate the steel for the end span, using the same thickness as for the interior since the steel stress falls far below that assumed. So the steel area for the end span was found as before by use of Plate VI. If the end spans made up one-half or more of the total floor area the primary design would be made for them and the area of steel reduced in the interior spans as is done with the girder.

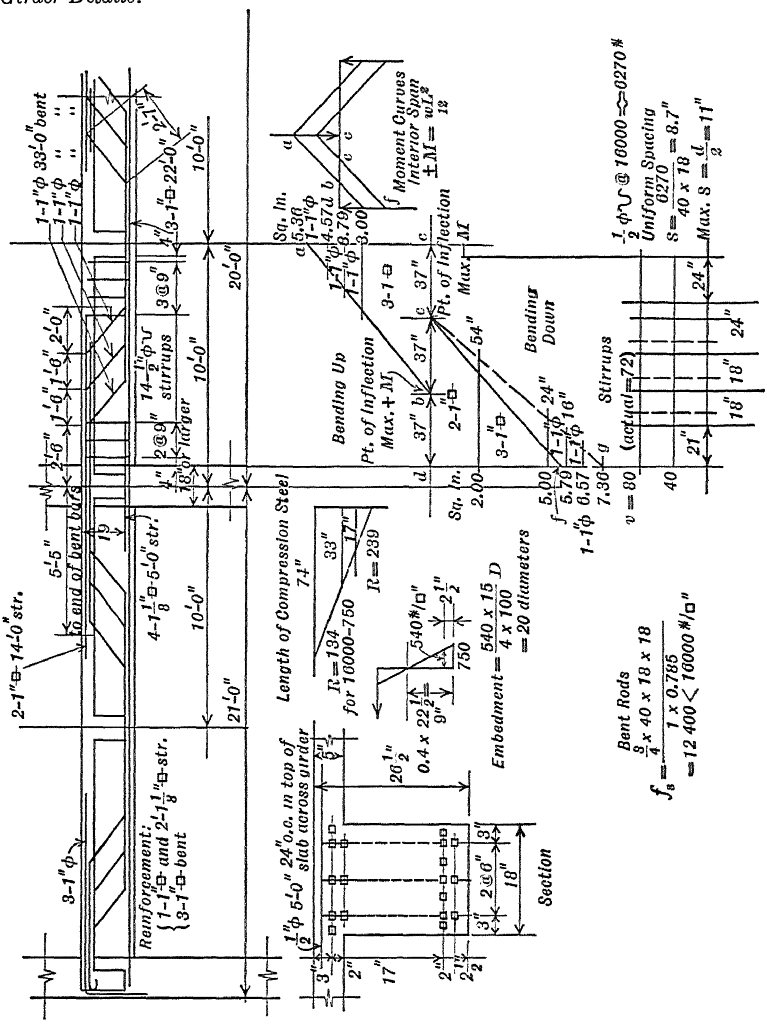
It is assumed that there are many more interior than end span beams in this building and so the design was made for the former. The load computation is self explanatory. The ratio of actual moment to the allowable moment given in Table 11 is the width of flange needed and it is obvious by inspection that it is less than the allowable. The steel area given in Table 11 is that required to balance one foot width of flange, so that the total steel required is the tabular area multiplied by the required width of flange, expressed, as previously noted, by the ratio of actual to tabular moment.

At the support, the allowable bending moment of the stem with tension reinforcement only was computed (44,500 foot-pounds) and subtracted from the actual moment, leaving the amount to be cared for by compression steel. Table 12 gives the resisting moment supplied by adding 1 square inch of compressive steel

COMPUTATIONS FOR SLAB, BEAM AND GIRDER FLOOR

Sheet BF8

Girder Details:



at depths of 2 inches and 3 inches; in this case 11,930 foot-pounds with the rods 2 inches from the face of the beam. The required compression steel area is proportional to the moment. To balance this compression steel more tension steel must be added and this was found by dividing the moment by the arm times the stress. To this was added the tension area for the simple beam previously found, giving the total tension steel area. In deciding upon the bending of bars it should be remembered that extra steel can be added more easily at the top in the slab than in the contracted width of the bottom.

For the end span the positive steel and the increased moment can be prorated directly from the typical. The steel at the support was designed as before. The steel in the top of the beam at columns was dropped 1 inch to give right of way to the steel of the more heavily loaded girder.

In the typical span, from Table 14, $M = \frac{wL^2}{12}$, one rod of six can be bent up 0.17 l or 3 feet 3 inches approximately from the center line and 3 rods at 0.29 l or 5 feet 6 inches. At the top the inflection point is 0.21 l or 4 feet 2 inches out. So one third of the steel can be bent down at 50/3 or 17 inches out from the face of the column and another one-sixth, 2 feet 1 inch out. To cover these requirements (neglecting the loop bars in this computation) the top steel from the adjacent beams must extend 4 feet 2 inches + 6 inches (one-half the support) or 4 feet 8 inches from the center line, and 5 feet 0 inches is used. This is shown on Fig. 76. One-third of the top steel, 2 rods, can be bent down at 17 inches + 6 inches from the center line, and another bar 8 inches further out. A distance of 17 inches from the support for the first point of bend is rather large for this depth beam (about $\frac{2}{3}$ d is common practice) so including the two $\frac{1}{2}$-inch loop bars in the computation, the two $\frac{3}{4}$-inch rods are $\frac{3}{10}$ of the total area and can be bent down 15 inches from the face of the support. Adding $\frac{1}{2}$ the width of the support (as assumed), gives 21 inches. The outer rod is

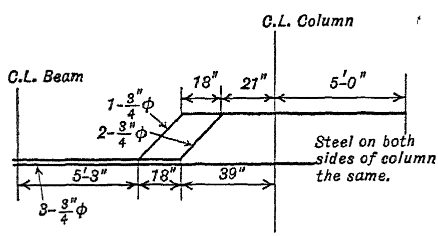


Fig. 76

COMPUTATIONS FOR SLAB, BEAM AND GIRDER FLOOR
Using Tables

Sheet GF1

Computation for Floor Slab

Allowable $f_c = 650 \quad f_s = 16{,}000 \quad n = 15 \quad v = 120 \quad u = 100$

		Live Load	= 125
Clear Span	= 9′-0″	Slab 5″	= 63
Bearing Span	= 10′-0″	1″ Fin.	= 13
Panel Width		Beam	=
			201

$$M = \frac{201 \times 9^2}{12} = 1360'\# \text{ (Interior Span)}$$

Table 9: $d = 3\tfrac{3}{4}'' — M = 1512'\# — A_s = 0.35\square''$

$$A_s = \frac{1360}{1512} \times 0.35 = 0.31 \qquad \tfrac{1}{2}''\phi\text{-}7\tfrac{1}{2}''\text{o.c.} = 0.31 \qquad 5''\text{ Slab}$$

End Span $\quad M = \dfrac{12}{10} \times 1360 = 1632'\#$

Tables would require $d = 4''$; if we maintain $d = 3\tfrac{3}{4}''$

$$\left.\begin{aligned} R &= \frac{1632 \times 12}{(3\tfrac{3}{4})^2 \times 12} = 116 \end{aligned}\right\} \text{ Plate VI}$$

$$p = 0.0098$$

$$A_s = 0.0098 \times 3\tfrac{3}{4} = 0.0368\square''/''$$

Use $\tfrac{5}{8}''\phi$ @ 8″ o.c. 5″ Slab

Bond stress same as in previous example.

Computation for Typical Beam

Allowable $f_c = 650 \quad f_s = 16{,}000 \quad n = 15 \quad v = 120 \quad u = 100$

	750 at Supports	Live Load	= 125
Clear Span	= 19′-0″ [assumed]	Slab 5″	= 63
Bearing	= 20′-0″	1″ Fin.	= 13
Panel Width	= 10′-0″	Beam	=
			201

$w = 201 \times 10 = 2010$

Stem $\quad 201$

$$2210 \times 19 = 42{,}000\# \qquad V = 21{,}000$$

$$b'd = \frac{V}{jv} = \frac{21{,}000}{\tfrac{7}{8} \times 120} = 200\square'' \qquad \text{Try } b' = 10''\ d = 20''$$

Int. Span — $M = 42{,}000 \times \dfrac{19}{12} = 66{,}500'\#$

From Table 11: 5″ Tee, $d = 20'' — M = 39{,}000'\# \quad A_s = 1.63\square''$

$$A_s = \frac{66{,}500}{39{,}000} \times 1.63 = 2.78\square'' \qquad 2\text{-}\tfrac{7}{8}''\phi + 4\text{-}\tfrac{3}{4}''\phi = 2.97\square''$$

Assume $\quad d = 21'' — M = 42{,}100 — A = 1.67$

$t = 5''$

$$A_s = \frac{66{,}500}{42{,}100} \times 1.67 = 2.64 \qquad 6\text{-}\tfrac{3}{4}''\phi = 2.65 \quad \text{[Use this]}$$

bent down over the lower bend in the first pair, which is common practice. Checking the bottom steel, the first rod is bent up at 10 feet less (21 inches + 18 inches + 18 inches) = 5 feet 3 inches from the center line as against 3 feet 3 inches possible: the two bent rods are bent up 6 feet 9 inches as against 5 feet 6 inches allowable. Other supports are figured similarly. It is usual in ordinary beams to detail the next to end span as typical although the inflection point in accordance with the table is 0.26 l instead of 0.21 l. With practice all this can be done as the detail is sketched by the use of the table with all necessary computations done mentally or on the slide rule.

Stirrups in office practice are taken from a table or diagram. Without these the required spacing should be computed at three or four points and the spacing between these points adjusted mentally. The stirrups are here placed without considering bent bars.

The girder of this building has a moment of $\frac{WL}{10}$ in two out of three spans so it was designed directly for this moment. The moment of a simple beam with the concentrated center load, $\frac{WL}{4}$, is reduced in the same ratio as for a uniformly loaded beam, to $\frac{WL}{5}$. Unit shear is always low in a beam loaded at the center so that the stem width was governed by other considerations.

The depth was assumed 1 inch greater than the beam so that the bottom layer of beam steel will slip in between the layers of girder steel. This is perfectly practicable although it is more convenient to have two or more inches increased depth, as in the first solution, if head room is available. The determination of positive steel was as before. At the support a concrete section 1 foot wide was seen from the tabular value of 64,600 foot-pounds to be able to carry only about one-third of the moment. From experience it was recognized that this would call for an excessive amount of compression steel so a 16 inch breadth was assumed. This worked out by the same method as used for the beam with reasonable steel values. The steel for the center span was prorated from the end span and it was found possible to get enough compressive steel at the bottom from the bottom layers of the two beams; two extra bars were added at the top. Location of the

COMPUTATIONS FOR SLAB, BEAM AND GIRDER FLOOR

Sheet GF2

Typical Beam: Continued

At Support.

From Table 10:

$d = 20''$ $\qquad M = 53{,}400 \qquad A_s = 2.32$ (*for* $b = 12''$)

$$10''\ \text{Stem}\ \begin{cases} M = 0.83 \times 53{,}400 = 44{,}500'\# \\ A_s = 0.83 \times 2.32 = 1.93\square'' \end{cases}$$

$66{,}500 - 44{,}500 = 22{,}000'\#$ *to be carried by compr. steel.*

Table 12: $1\square''$ *Compr. Steel will carry* $11{,}930'\#$

$$A_s' = \frac{22{,}000}{11{,}930} = 1.85\square''$$

$6\text{-}\frac{3}{4}''\phi$ *O.K.*

Tension Steel:

From above	1.93
$+\dfrac{22{,}000 \times 12}{16{,}000\,(20 - 2)} =$	0.92
	2.85

Bend $3\text{-}\frac{3}{4}''\phi$ *each side and use* $2\text{-}\frac{1}{2}''\phi$ *loop bars* $= 3.05\square''$ (*Art. 140c, page 338.*)

Bm. $10'' \times 24''$

End Span

$$A_s = \frac{12}{10} \times 2.64 = 3.17 \qquad 4\text{-}\tfrac{7}{8}''\phi + 2\text{-}\tfrac{3}{4}''\phi = 3.29\square''$$

$$M = 1.2 \times 66{,}500 = 80{,}000'\#$$

At Support $\quad 10''$ *Stem* $\quad M = 44{,}500 \qquad A_s = 1.93$

$80{,}000 - 44{,}500 = 35{,}500$ *to be carried by compr. steel*

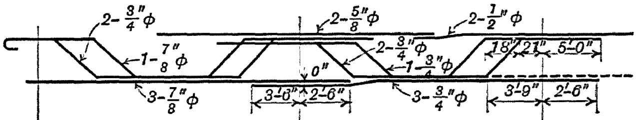
$$A_s' = \frac{35{,}500}{11{,}930} = 2.98\square''$$

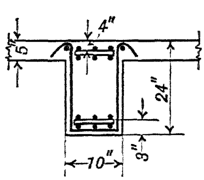
as for typical support

Tension Steel:—

From above	1.93
$+\dfrac{35{,}500 \times 12}{16{,}000\,(20 - 2)} =$	1.48
	$3.41\square''$

$$\begin{cases} \text{Use Compr. Steel} & 3\text{-}\frac{3}{4}''\phi + 3\text{-}\frac{7}{8}''\phi = 3.13 \\ \text{Use Tension Steel} & 5\text{-}\frac{3}{4}''\phi + 1\text{-}\frac{7}{8}''\phi + 2\text{-}\frac{5}{8}''\phi = 3.41\square'' \end{cases}$$





$$v = \frac{21{,}000}{\frac{7}{8} \times 21 \times 10} = 115; \quad \tfrac{3}{8}''\phi.\text{U}$$

$$\text{Min. } s = \frac{2 \times 0.11 \times 16{,}000}{\frac{2}{3} \times 115 \times 10} = 4.6''$$

Space 3, 4, 4, 4, 5, 5, 5, 6, 6, 6, 8, 8, 8, 11

bending points and design of shear reinforcement would be made as in the previous solution or by using Table 14.

99. Light Weight Floors. Solid concrete slabs, such as have been described in the preceding articles, are heavy and uneco-

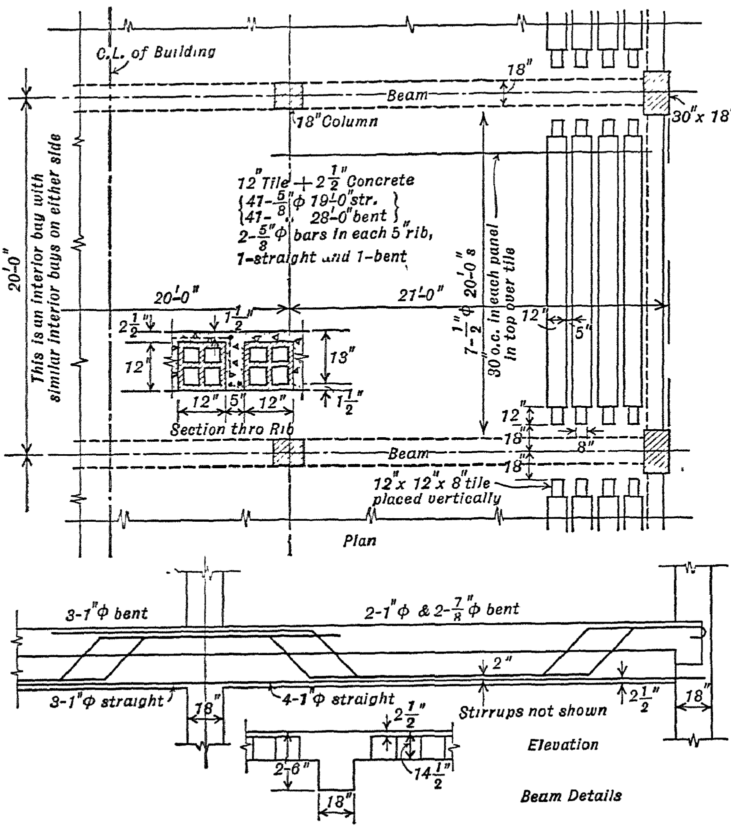


FIG. 77

nomical for long spans and light loads. Their excessive weight is due to the large mass of concrete in the tension part which is much greater than is required to resist the diagonal tension and shearing stresses. In light weight floors this difficulty is met by forming the floor slab of a series of tee shaped joists, the space between the stems being filled by hollow tile of clay, gypsum or steel, or else left empty by the removal of the forms. The simplest type and

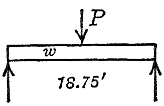
COMPUTATIONS FOR SLAB, BEAM AND GIRDER FLOOR

Sheet GF3

Computation for Girder

Allowable $f_c = 650$ $\quad f_s = 16{,}000$ $\quad n = 15$ $\quad v = 120$ $\quad u = 100$

Clear Span 18.75 (*Assumed*)	*Live Load* = 125
Bearing = 20′-0″	*Slab* 5″ = 63
Panel Width = 10′-0″ *Concentrated*	*Fin. & Plaster* = 13
	Beam = 20
	221



$$w = 300 \times 18.75 = 5{,}600\#$$
$$P = 10 \times 20 \times 221 = \underline{44{,}200\#}$$
$$49{,}800\#$$
$$V = 24{,}900\#$$

End Span

$$\left.\begin{aligned} M &= 44{,}200 \times \frac{18.75}{5} = 166{,}000 \\ &+ 5600 \times \frac{18.75}{10} = 10{,}500 \end{aligned}\right\} = 176{,}500'\#$$

Table 11: $t = 5''$ $\quad d = 22''$ $\quad M = 45{,}200$ $\quad A = 1.71$

$$b = \frac{176{,}500}{45{,}200} = 3.9' \qquad A_s = 3.9 \times 1.71 = 6.68\square''$$
$$6\text{-}1''\phi + 2\text{-}1''\square = 6.71$$

At Support $f_c = 750$

Table 10: $d = 22''$ $\quad M = 64{,}600$ $\quad A_s = 2.55$

Try 16″ Stem $\quad M = 1.33 \times 64{,}600 = 86{,}100'\#;\ A_s = 1.33 \times 2.55 = 3.40\square'$

$176{,}500 - 86{,}100 = 90{,}400;\ d' = 2''$ $\quad 1\square''$ *adds* $M = 13{,}650'\#$

Table 12 $\qquad A_s' = \dfrac{90{,}400}{13{,}650} = 6.62\square''$

Tension Steel: *From above* $\quad 3.40$

$$+\frac{90{,}400 \times 12}{20 \times 16{,}000} = \underline{3.38}$$
$$6.78\square''$$

Center Span: $A_s = \dfrac{10}{12} \times 6.68 = 5.56$ $\qquad 4\text{-}1''\phi + 4\text{-}\tfrac{7}{8}''\phi = 5.55$

At support: *Bottom* $6\text{-}1''\phi + 2\text{-}1''\square = 6.71$ *Required* 6.62

[$2\text{-}1''\square + 2\text{-}1''\phi$ *from end span*, $4\text{-}1''\phi$ *from c. span*]

Top: $4\text{-}1''\phi$ (*End*) $+ 4\text{-}\tfrac{7}{8}''\phi$ (*c.*) $+ 2\text{-}\tfrac{7}{8}''\phi = 6.75$

Required 6.78

$$v = \frac{24{,}900}{\frac{7}{8} \times 16 \times 22} = 81\#/\square'' \qquad s = \frac{2 \times 0.196 \times 16{,}000}{16 \times 81 \times \frac{2}{3}} = 7.25''$$

$\tfrac{1}{2}''\phi$U — *spaced* $7''(< \tfrac{1}{2}\,d)$

in many respects the best is that shown in Fig. 77 made with clay tile. Such floors are economical for light loads and long spans. The thin flanges (never less than 2 inches thick) of the tee beams, known as topping, with the aid of the tile serve to transmit the loads to the ribs. A heavy wearing surface is always used but even with this aid it is obvious that this type of construction is not suited to heavy concentrated loads. Generally the ceilings are plastered, metal lath being necessary where steel forms are used. Sometimes the ceiling is left unplastered showing the ribs with no fillers between.

The individual joists in light weight construction are tee beams and their design follows the usual rules. It is not desirable to use stirrups and so the shearing stress is kept low. In Fig. 77 the stems of the joists are shown as 5 inches wide for the greater part of their length and 9 inches wide toward the ends, thus furnishing increased section for resisting diagonal tension. The usual standard and also the minimum practicable width of rib is 4 inches. In this design the tiles stop 18 inches from the center line of the supporting beams, thus providing a 36 inch flange for the beam and changing the joist section from the tee shape to rectangular at the supports. In computing resistance to diagonal tension the shell of the tile may be considered as increasing the thickness of the web by one-half their own thickness (Art. 124, Joint Committee, Appendix B), usually taken as 1 inch for computations. The tile should not be assumed to add to the resisting section otherwise.

The following table of the weight of tile is required as data for designing hollow tile floors.

Weight of Hollow Burned Clay Tile
(All tile are 12 inches × 12 inches square in plan)

Depth of Tile	Weight	Depth of Tile	Weight
2 ins.	16 lbs.	8 ins.	32 lbs.
3	16	9	36
4	18	10	38
5	21	12	42
6	24	15	50
7	29	..	..

Problem 35. Investigate the stresses in the floor shown in Fig. 77. The design shown was made as a substitute for the slab, beam and girder floor previously discussed in this chapter and all necessary data may be found there.

Suggestions and Discussion. Investigation of Joists. The load per foot of length carried by a joist is 17/12 of the total load on each square foot of floor. Taking both positive and negative moment as $wL^2/12$, the bending moment is 10,000 ft.-lbs.; the end shear is 3260 lbs. It is suggested that curves of shear and moment be drawn in order to facilitate determining the values of these functions at any section. Take the shear at the center as one quarter of that at the end.

The unit shear at the edge of the flange of the supporting beam is 17 lbs. per sq. in.; maximum intensity in the 9 in. rib is 27 and in the 5 in. rib 41 lbs. per sq. in. (d = 13 in.). The last value is so close to the 40 lb./sq. in. limit for a beam without diagonal tension reinforcement that the excess may be ignored.

The maximum stress in the concrete at mid span is about 375 lbs./sq. in., and that in the steel 16,800 lbs./sq. in.

At the support the section is rectangular (b = 17 in. and d = 13 in.) with tension reinforcement consisting of 2–$\frac{5}{8}$ in. rounds bent up from the adjoining ribs. The straight $\frac{5}{8}$ in. rounds are not long enough to serve as compression reinforcement nor are they required for that purpose. Here R = 42 and inspection of Plate VI shows the fiber stresses to be low. Exact determination is not essential.

The 9 in. rib is entirely in the negative moment region and accordingly is also an inverted rectangular beam. Here there is compression steel available if desired. The maximum stresses are in the neighborhood of 14,000–450 lbs./sq. in. Similarly for the end of the 5 in. rib, Plate VI shows the stresses to be around 10,000–400 lbs./sq. in.

The maximum bond stress is 75 lbs./sq. in. This is a detail that often causes difficulty in this type of construction.

Investigation of Beam. The load per foot on the beam is 5480 lbs. This figure makes allowance for the fact that the 3-ft. length over the stem weighs about 230 lbs. more than it would were it of tile and concrete. The maximum unit shear is 125 lbs./sq. in. which is not excessive. (d = 26$\frac{1}{2}$ in.)

On entering Plate VII with the proper values to investigate the tee beam section at the point of maximum positive moment in the end span it becomes plain that the neutral axis is in the flange and the beam is in reality equivalent to a rectangular section with b = 36 inches and d = 26$\frac{1}{2}$. Using Plate VI the stress in the steel is found to be about 16,500 pounds per square inch for both end and interior spans and the concrete stress about 600 pounds per square inch in the end span and 540 pounds per square inch in the interior span.

100. Roof Framing. The stems of the beams and girders supporting the roof of this building should be made the same sizes as those of the floor members as this makes it possible to use the floor forms again in constructing the roof. The slab can be made thinner, 4 inches being about the minimum advisable. The steel

in the roof slab and its supports would not be the same as in the floor but would be lighter as required by the loads.

Problem 36. Prepare an alternate design for the floor system studied in this chapter using a 6 ft. 8 in. spacing for the beams instead of 10 ft. 0 in. Data as before except: f_s = 18,000 lbs./sq. in. f_c = 800 lbs./in.; 900 lbs./sq. in. at supports. Total depth of girder is limited by clearance to 24 in.

Ans. Slab; 4 in. thick; reinforcement in end span, $\frac{3}{8}$ in. rounds at 7 in. o.c.; in interior spans, $\frac{3}{8}$ in. rounds at 8 in. o.c.

Beams, width 8 in., total depth 20 in., d = 16.5 in., at support, $17\frac{1}{2}$ in.; reinforcement 2–$\frac{3}{4}$ in. rounds straight, 1–$\frac{3}{4}$ in. and 1–$\frac{7}{8}$ in. round bent.

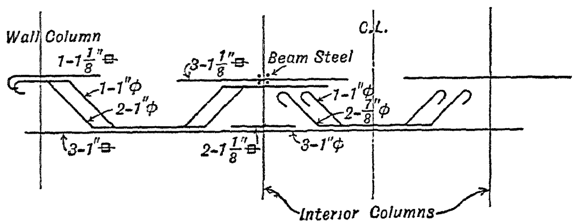


FIG. 78

Girders, width 16 in., total depth 24 in., d = 20.5 in. reinforcement, see Fig. 78. The arrangement here shown was chosen to enable the bars to be assembled in units outside the beam forms and set in place with a minimum of interference between the various units. These units will be complete except for the $1\frac{1}{8}$ in. squares through the columns which are to be placed separately after the beam and girder units are in place. Had the bent bars in the interior span been extended through the columns it would have been difficult to place the beam units.

101. Earthquake-Proof Construction. The ordinary skeleton construction illustrated by the building designed in these chapters does not offer as large resistance to earthquake shock as might be desired. Where such shocks are of rare occurrence, as in the eastern part of the United States, the engineer practically never considers their possibility in proportioning his structures. In view of the possibility of severe earthquakes in nearly all localities, their neglect in design is questionable, particularly since it is simple and inexpensive to reinforce buildings in such a way as to make them strongly resistant to such shocks.

The earth waves that cause damage are chiefly the horizontal movements, the vertical motion affecting only the foundations of relatively high and narrow buildings. The horizontal movement may take place in any direction and may produce a dynamic

effect equivalent to the application of a horizontal force of $\frac{1}{10}$ of the weight of the structure. It has been recommended[1] that provision be made for a shear in any story of a building equal to $\frac{1}{15}$ to $\frac{1}{20}$ of the weight above, placing reliance upon the reserve of strength in the frame to resist any greater effect.

These horizontal forces may be carried to the foundations in the same manner as wind loads but preferably by a properly distributed system of reinforced concrete walls, designed as vertical beams. This will provide sufficient rigidity, and without it stone facing, glass, curtain walls of brick and tile and other brittle material in the building will suffer heavy damage in an earthquake. A rigid frame obtains its name from the rigid construction at the junctions of the members and actually is a flexible structure. So it is not sufficient to make the frame strong enough in itself without walls. Since walls are always used in buildings there is little added cost caused by using reinforced concrete for a sufficient number of them to provide for earthquake forces.

Certain details of reinforcement are also important; all bars should be thoroughly anchored so that the structure will be securely tied together; in vertical columns with longitudinal steel and ties the hooping should be closely spaced for some distance above and below the floors; at the main corners of buildings diagonal bands of floor slab reinforcement should run at least a dozen feet back from the corner column; diagonal bars should be placed across the corners of all openings in concrete walls. None of these details will add much to the cost of the building but in the event of a severe shock they will add greatly to its chance of survival with little damage.

102. Deflection and Camber. Concrete beams are very stiff and it is only in unusual cases that it becomes necessary to compute their deflections under load. Occasionally, the builder gives long span beams a slight camber (that is, makes them higher in the middle than at the ends), in order to prevent them from sagging below the level of their supports, which is unsightly. In ordinary construction the bottoms of beams and slab forms are made horizontal.

Various formulas have been proposed for computing deflection.

[1] "Essentials of Earthquake-proof Construction," H. M. Hadley, a pamphlet published by the Portland Cement Association. Also see 1924 Proceedings, American Concrete Institute.

The simplest procedure is that recommended by Professors Turneaure and Maurer[1] who use the formulas and constants derived for homogeneous beams, using the moment of inertia of the transformed concrete section including the tension concrete, and taking n as 8 or 10. This means that the value of the modulus of elasticity for concrete is taken at approximately its actual value. The full concrete section is taken since the cracks in the tension face of the beam do not have much influence on the amount of deflection.

Example 51. An end supported rectangular beam carries a total uniformly distributed load of 10,000 lbs. The span is 16 ft. and there is no restraint at the ends. What is the deflection at the center? $n = 8$, $b = 10$ in., $d = 15$ in., $A_s = 1.76$ sq. in.

Solution. The neutral axis is found to lie 0.60 in. below the mid-depth of the beam, 8.1 in. below the top. The moment of inertia of the transformed section about the center line is 3610 and about the neutral axis 3550 in.^{4} The deflection is

$$d = \frac{5}{384} \cdot \frac{wL^4}{EI} = \frac{5 \times 10{,}000 \times \overline{16}^3 \times \overline{12}^3}{384 \times 3{,}750{,}000 \times 3550} = 0.07 \text{ inch.}$$

Problem 37. What is the deflection of the roof beam shown in Fig. 88, page 260. Take $n = 10$. *Ans.* 0.20 inch.

[1] "Principles of Reinforced Concrete Construction," Chapter VI, 3d Edition. Example 51 is from that text.

CHAPTER XIII

BUILDING DESIGN. FLAT SLAB FLOORS

103. The flat slab, first called the "Mushroom Floor," by the originator, Mr. C. A. P. Turner, is a type of construction distinctive to concrete. These slabs have no supporting beams except at the margins but rest directly on columns which are usually built with enlarged heads, called capitals. Often a portion of the slab about the column capital, called a drop panel, or plinth, is made thicker than the rest of the floor. Because of their economy and other advantages flat slabs have largely replaced beam and girder construction in buildings adapted to their use.

Flat slab floors are suitable for use in buildings at least two bays and preferably three bays wide, where the column spacing can be made fairly regular with panels from 17 feet to 30 feet each way and live loads of 100 pounds per square foot or more. Forms are much less expensive than in beam and girder construction which offsets savings in steel and concrete possible in the older type of design. The great saving, however, is in building height. A commercial building has a required clear story height, which added to the floor thickness gives the floor to floor height. A flat slab will be one to two feet less in over-all thickness, effecting a large saving in columns, walls and partitions.

The flat ceiling of the girderless floor offers several advantages because of the absence of beams; the easier layout of sprinklers and of any other piping or shafting supported under the ceiling; easier artificial lighting; better day time lighting with windows that extend to the ceiling; and better ventilation because of the absence of pockets in the ceiling.

Since corners are the most vulnerable parts of concrete masses exposed to fire, flat slab construction suffers far less in fire than do beam and girder buildings.

The flat slab type of building is primarily adapted to industrial use — factories, warehouses and garages — but because of low cost it is sometimes used very satisfactorily for stores, hotels and office buildings. The main drawback to these latter uses is the

difficulty of satisfactory architectural treatment of interiors; however there are now numerous buildings in which these difficulties have been reasonably well overcome.

104. Systems. There are four common systems of reinforcing flat slabs: the two-way, the four-way, the circumferential and the three-way. The first three have columns at the corners of rectangular panels, while the last has the columns arranged at the apices of equilateral triangles.

The two-way system has reinforcement parallel to the column center lines both ways, the steel in the half of the panel centered on the column being heavier than the intermediate bands between columns. The four-way system replaces the intermediate bands of the two-way between columns with two lines of reinforcement parallel to the diagonals of the panels. The circumferential (Smulski or S.M.I. System) uses hoops and radial rods centered on the columns and the intersection of panel diagonals. The three-way system has bands parallel to the sides of triangular panels.

All flat slabs were originally patented systems. The fundamental patent has expired however and the two and four-way systems are now designed and built without payment of royalty. The circumferential and the three-way types as well as some other special types, most of which are variations of the ordinary two-way system, are still covered by patent and subject to royalty.

The four-way was the original system of Mr. Turner. As originally built, it had the disadvantage of four layers of steel over the columns which reduced the effective depth and made concreting difficult. The two-way steel arrangement does not come so near paralleling all lines of stress with rods as does the four-way, but it is simple to design and construct and seems in every way satisfactory. It is probably the most used system today.

The circumferential system and the three-way are somewhat more complicated in details and are designed by the patentees. The Smulski circumferential system probably arranges steel to take stress more directly than any other and often effects a saving in the weight of steel required.

105. Stresses in Flat Slab. Formulas and rules for flat slab design are largely empirical although theoretical considerations have entered into their making. The usual rules, such as those

of the Joint Committee, are sufficiently complete to cover the common conditions of floor panels of uniform size. They do not give information that is sufficient for the analysis of floors with large irregularities. The tyro should not attempt the solution of such problems which are properly the field of the specialist. The theoretical analysis[1] of a non-homogeneous continuous flat plate supported on a series of circular column capitals may now be considered well established but only a few have mastered its mathematical intricacies.

The shape taken by a continuous, loaded flat slab, supported on points, is shown by the heavy lines of Fig. 79, study of which shows where tension steel is required. Such a slab may be analyzed approximately by considering it to be first, a beam spanning from AD to BC and second, as a beam spanning from AB to CD. By supplying the steel required in each assumed beam the slab is safely reinforced in all directions.

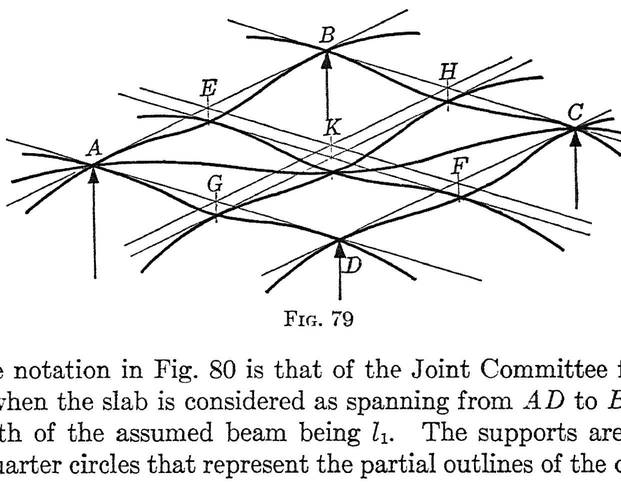


FIG. 79

The notation in Fig. 80 is that of the Joint Committee for the case when the slab is considered as spanning from AD to BC, the breadth of the assumed beam being l_1. The supports are along the quarter circles that represent the partial outlines of the column capitals and the span may be considered as the distance between the centroids of the supporting arcs, $l - \frac{2}{3}c$. The total height of the parabolic moment curve of a uniformly loaded fixed-ended beam is $\frac{wL^2}{8}$, two-thirds of this being negative moment. Simi-

[1] "Moments and Stresses in Slabs," Westergaard and Slater, Proceedings, American Concrete Institute, 1921: "Die Theorie elastischer Gewebe und ihre Anwendung auf die Berechnung biegsamer Platten" by Dr. Ing. H. Marcus: "Elastische Platten" by Dr. Ing. A. Nádai.

larly, the theoretical height of the moment curve,[1] for a uniformly loaded flat slab in the direction l, is

$$M_0 = \tfrac{1}{8}\,(wl_1)\,(l - \tfrac{2}{3}\,c)^2 \tag{46}$$

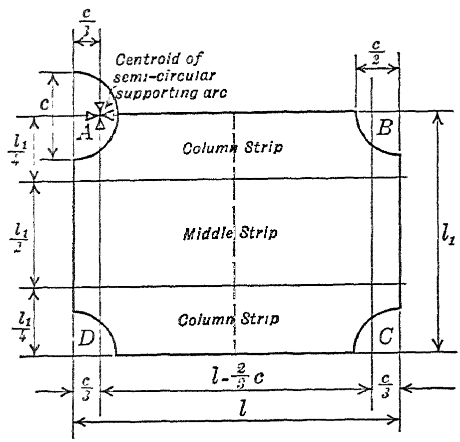


FIG. 80

where wl_1 is the load in pounds per foot of length of beam. The corresponding formula of the Joint Committee (No. 36, Art. 142, App. B; this formula is identical with Equation (46) above except for the different coefficient), gives a value which is 72 per cent of the theoretical. This lower value is to be used in proportioning the tension steel. Tests and theoretical studies show that the concrete carries tension to a very considerable degree in flat slabs and so this reduction is justified so far as the steel is concerned. The full moment should be used in studying concrete stress.

In Art. 144 of their report the Joint Committee gives a table for the distribution of this total moment, 60 per cent to 65 per cent of it being taken as negative. The moment is not distributed uniformly across the breadth of the beam (l_1) but is larger in the strips over the columns than between columns. For convenience the slab is assumed to be divided into strips as shown in Fig. 80. The critical section for positive moment is indicated by the dotted line; that for negative moment follows the outlines of the capitals and the center line of columns connecting them.

[1] "Statical Limitations upon the Steel Requirements in Reinforced Concrete Flat Slab Floors," by John R. Nichols, Trans. Am. Soc. C. E., 1914.

By interchanging l and l_1 in Equation (46), the expression applies to the moment for the slab considered as a beam spanning in the l_1 direction.

The moments in rectangular panels are as easily found as are those in square panels, as is evident from the previous discussion. When the departure from the square is too great, and when other irregularities occur, the usual methods become too approximate and more exact analysis is necessary. The limit of length to breadth for these formulas is usually taken at about 4 to 3.

The shearing stress at the periphery of drop or column capital is not a matter of much concern on account of the relative high strength of concrete in shear. A short distance from these peripheral sections large diagonal tension stresses develop which must be kept low as no diagonal tension reinforcement is provided in flat slabs. The shear on sections, distance d from the edges of drop and capital and parallel to those edges, is taken as a measure of this diagonal tension.

Once the moment and shear have been determined, the problem of flat slab design investigation and design becomes simply a repetition of processes already made familiar in these pages. About the only description needed regarding the computations in the example that follows relates to interpretation of the Joint Committee rules.

106. Design of Flat Slab Floor. (Computation Sheets FS1–FS7.) The computations herewith give the complete design for a floor to replace the slab, beam and girder floor of the previous chapter, using the same loads and stresses. For the most part the calculation should be easy to follow by having reference to the 1924 Joint Committee specifications, reference to which is made by figures in brackets on the computation sheets.

(*a*) *Trial Dimensions.* (Computation Sheet FS1.) The usual diameter of column capital is from 0.20 to 0.25 of the span, 0.225 being the most common.

The formulas for slab thickness are of two sorts: an arbitrary fraction of the span to prevent undue deflection and a second limit to prevent excessive concrete stresses. The expression $t_2 = 0.02\, l\sqrt{w'} + 1$ inch (No. 38, Joint Committee, Appendix B) can be derived as follows: assume stresses of 16,000 – 650, $n = 15$, $c = 0.225\, l$, $l_1 = l$, $d = t_2 - 1$ inch; moment of $\frac{1}{8}\,(w'l_1)(l -$

$\frac{2}{3}\ c)^2$ with 20 per cent of that as the amount at the critical section for positive moment in the column strip. Then, bending moment equals moment of resistance = Rbd^2

$$0.20 \times \tfrac{1}{8}\,(w'l)\,(0.85\,l)^2\,(12) = 108 \times \frac{12\,l}{2} \times (t_2 - 1)^2$$

the solution of which gives the equation as quoted. The other equation (No. 37, Joint Committee) for thickness of the drop is obtained in a similar manner, making allowance for arch action and other factors that tend to increase the concrete stresses.

There is considerable variation in the limit put upon the minimum size of columns to be used with flat slabs and that adopted, $C = \frac{l}{15}$, is representative. This limitation is necessary because of the high bending stresses that may be induced in columns in this type of construction.

(*b*) *Slab Thickness.* (Computation Sheet FS2.) The discussion jumps over the calculation of shearing stresses and moments on this sheet to the check of required slab thickness. The critical section for negative moment is through the drop of an interior column parallel to the wall of the building, and has the dimensions (when revised) shown in Fig. 81. The difference in the widths of the two column strips and the drop is not great and the tension steel outside the drop is fully effective for this section as shown.

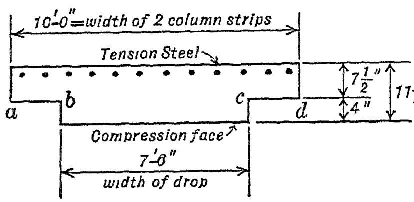


FIG. 81

The concrete along the faces *ab* and *cd* is practically at the neutral axis and carries little if any stress. So the width was taken as that of the drop.

The moment was increased by 39 per cent to give the full theoretical value for the section with a factor of 0.125 in place of 0.09. The factor 1.15, as explained by reference to the Joint Committee report, gives the increase of the moment at the first interior support over that at a typical interior support.

In this connection the formulas given by the Joint Committee for this operation are interesting. Their formula 41 (Art. 156, App. B), is derived as

COMPUTATIONS FOR FLAT SLAB FLOOR

Sheet FS1

Flat Slab Floor: Two Way Reinforcement; with Drop. *Data:*

Layout shown on Fig. 75 and Computation Sheet FS7

Live Load, 125 lbs./sq. ft.

Floor Finish, 1 in. granolithic

Materials: Steel. Structural Grade — Deformed Bars.

Concrete: Ultimate Strength at 28 days 2000 lbs./sq. in.

Maximum size aggregate: 1½ in.

Specifications:

1924 Joint Committee except as noted.

Stresses: $f_c = 0.325 \times 2000 = 650$ *lbs./sq. in.* $n = 15$

$f_c = 1.15 \times 650 = 750$ *lbs./sq. in. at supports*

$f_s = 16{,}000$ *lbs./sq. in.*

Shear in Beams $\left\{ \begin{array}{l} v = 0.02 \times 2000 = 40 \text{ lbs./sq. in.} \\ \quad \text{without diagonal tension reinforcement} \\ = 0.06 \times 2000 = 120 \text{ lbs./sq. in.} \\ \quad \text{with diagonal tension reinforcement} \end{array} \right.$ $u \left\{ \begin{array}{l} = 0.05 \times 2000 \\ = 100 \text{ lbs./sq. in.} \end{array} \right.$

Trial Dimensions

Note: Numbers in brackets refer to articles in Jt. Comm. Report.

Span: $l = l_1 = 20'\text{-}0''$ *c.c. columns.*

Cap: $c = 0.2\,l = 4'\text{-}0''$

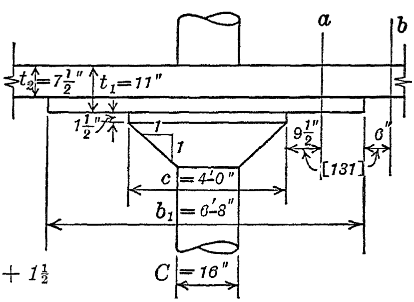
Drop: $b_1 \geqq \frac{1}{3}\,l = 6'\text{-}8''$ *square* [146]

Slab: $t_2 \geqq \frac{1}{32}\,l = 7\frac{1}{2}$ *in.* [145]

$\geqq 0.02\,l\,\sqrt{w'} + 1$ *in.* [145]

$\geqq 0.02 \times 20\,\sqrt{232} + 1$

$\geqq 7.1$ *in.*



$$t_1 \geqq 0.038\left(1 - 1.44\frac{c}{l}\right) l \sqrt{Rw'\frac{l_1}{b_1}} + 1\tfrac{1}{2}$$

[145]

$$\geqq 0.038\left(1 - \frac{1.44 \times 4}{20}\right) 20 \sqrt{\frac{232}{2} \times 3} + 1.5$$

$\geqq 11.6$ *in.*

$\leqq 1.5\,t_2 \leqq 1.5 \times 7.5 = 11.25$ [146]

Minimum Column Diameter: $C \geqq \frac{1}{15}\,l \geqq 16$ *in.*

Maximum Cap Diameter: $c \leqq 3\,C \leqq 4'\text{-}0''$

Loads:		
	Live	*125 lbs./sq. ft.*
	Finish	*13*
	7½" Slab	*94*
	$w' =$	*232 lbs./sq. ft.*

---

follows (see Art. 60 for formulas used): In general $M = Rbd^2 = \frac{1}{2} f_c kjbd^2$; and

$$f_c = \frac{2\,M}{jkbd^2}.$$

Here $M = RM_0$ and $b = \frac{l_1}{2}$ and $jk = 0.67\sqrt[3]{pn}$, approximately.[1]

Therefore $f_c = \dfrac{4\,RM_0}{0.67\sqrt[3]{pn}l_1d^2}$ which is the same as the Joint Committee Formula 41 except that 6 is there used for 4. That is, the Joint Committee is saying that the concrete stress in this location is to be taken as 50% greater than that computed in the usual manner from the reduced bending moment used for proportioning the steel. A similar analysis of their Formula 40 shows that the increase in figured concrete stress at the drop is 33% for a column capital with a diameter equal to 0.20 of the column spacing.

In this design the slab thickness was determined by the higher moments in the exterior panels. A more economical floor would have resulted had the column spacing been changed so as to equalize the moments in interior and end panels.

(c) *Reinforcement for Interior Panel.* (Computation Sheet FS3.) The negative steel area required in this slab is provided by bending up bars from the bottom as shown in the sketches on this computation sheet. The Joint Committee requires that at least four-tenths of the positive reinforcement be thus bent up and that at least one-third be left straight. This makes necessary the use of additional short bars in the top over the column head.

(d) *Wall Panel.* (Computation Sheets FS3–FS5.) The first computation sheets show a design for this floor with a small spandrel beam that is assumed to carry its own weight and that of the curtain wall only. It is necessary to use a drop at the wall column in this situation, in addition to the usual bracket. A wall column 18 inches by 30 inches was assumed with an 18-inch bracket. The computations follow the same course as at an interior column and show the shearing stresses to be satisfactory.

Where the interior and the exterior spans are so closely alike as in this case it would not be common to determine the actual spans as was done on Sheet FS4, which illustrates the procedure necessary for unequal spans.

In the computation of steel areas the only figure that needs explanation is the factor 0.80 in the calculation of the negative moments at the wall column and at the spandrel. The Joint

[1] See footnote to Art. 104, etc., Appendix B.

COMPUTATIONS FOR FLAT SLAB FLOOR — Continued

Sheet FS2

Shearing Stresses

See Art. 131, Jt. Comm. Rept.

Section a (Sketch: Sheet FS1)

$$v = \frac{232\ (20 \times 20 - \frac{\pi}{4} \times \overline{5.58}^2)}{(\pi \times 5.58 \times 12)\ (\frac{7}{8})\ (9.5)} = \frac{87{,}200\#}{1750\square''}$$
$$= 50\#/\square''$$
$$< 0.02 \times 2000\ (1 + \tfrac{4}{10}) = 56\#/\square'' \qquad O.K.$$

Section b

$$v = \frac{232\ (20 \times 20 - \overline{7.67}^2) = 79{,}100\#}{(4 \times 7.67 \times 12)\ (\frac{7}{8})\ (6)} = 41\#/\square''$$
$$< 40 \left(1 + \frac{6.67}{10}\right) = 67\#/\square'' \qquad O.K$$
$$0.03\, f_c' = 60\#/\square''. \quad [131]$$

Moments

$M_0 = 0.09\ (232 \times 20)\ (20 - \frac{2}{3} \times 4)^2 = 126{,}000'\#$

[*Formula 36, Art. 142, Jt. Comm. Rept.*]

[*Rearranged to show its meaning:* $M_0 = awL^2$]

Distribution: — [144]

2-Column Strips:	$-M$:	50%	63,000'#
	$+M$:	20	25,200
Mid Strip	$-M$:	15	18,900
	$+M$:	15	18,900
		100%	126,000'#

Check of Slab Thickness

Drop:

$$d = \sqrt{\frac{M}{bR}} = \sqrt{\frac{63{,}000 \times 12 \times 1.15 \times 1.39}{80 \times 134}} = d$$

[See 147] — width of drop

d required at 1st interior support

$R = 134$ *for 16,000 − 750*

$$\left\{\begin{array}{l}\text{For 90'' drop}\\ \text{required}\end{array}\right\} d \begin{array}{l} = 10.6 \text{ in.} > 9.5'' \text{ trial} \\ = 10.0 \text{ in.} = 10.0'' \text{ used}\end{array} \Bigg\} \therefore \text{Make drop } 7'\text{-}6'' \text{ sq., } 11\tfrac{1}{2}'' \text{ thick}$$

Slab:

$$d = \sqrt{\frac{25{,}200 \times 12 \times 1.15 \times 1.39}{120 \times 108}}$$
$$= 6.1 \text{ in.} \quad > 6 \text{ in. used.}$$

$R = 108$ *for 16,000−650*

Committee does not make a definite recommendation as to the amount of bending moment to be assumed at these sections and so recourse was had to the best practice for this design. Mr. Sanford E. Thompson[1] and others recommend that this moment be taken as 0.80 of the negative moment at the same section of an interior panel.

A reading of the sections noted in the Joint Committee report will explain the various details of bar lengths on Sheet FS7.

(*e*) *Wall Beam.* (Computation Sheets FS5–FS6.) Sometimes with light curtain walls between columns it is possible to omit wall beams in flat slab construction but usually they are necessary. A light beam such as that shown on Sheet FS5 is often assumed to deflect equally with the slab under the weight of the curtain wall above it and thus to receive no load from the slab. This is apparently permitted by the Joint Committee and is followed in this design, which presents no peculiar features otherwise. A deeper beam that extends either above or below the slab plainly stiffens the thinner member and so is compelled to carry some load from the slab, an amount usually judged to be from 20 per cent to 25 per cent of the panel load. The design on Sheet FS5 follows this rule. With this stiff spandrel it is proper to reduce the moments in the wall panel by the amount produced by the load assumed taken by the beam.

The simplest sort of wall beam to build is one below the slab but it is an undesirable type often since it interferes with illumination. A low upturned beam, integral with the slab, is difficult to construct on account of the formwork and because of the difficulty of pouring the concrete at the same time as that of the slab. A high beam which is also used as the curtain wall is usually poured after the slab and with the columns of the story above. The positive bending moment is often taken as $wL^2/16$ instead of the larger figure on the computation sheets. Since the upright spandrel is cast after the slab it is essential that the positive and negative reinforcement be sufficient to carry at least the slab dead load. This will prevent unpleasant cracking if the forms are removed before the beam is in place.

With the deep wall beam the shear around the bracket (Section *a*, Computation Sheet FS3) is reduced one-half and a drop panel is not needed. Also, the column strip running parallel to the wall

[1] "Concrete Plain and Reinforced," page 335, Vol. I.

COMPUTATIONS FOR FLAT SLAB FLOOR — Continued

Sheet FS3

Steel. Interior Panel

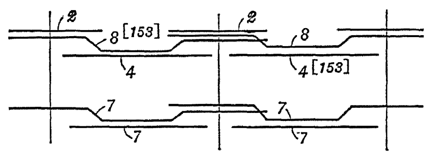
2 Col. Strips: $-M$: $A_s = \dfrac{63{,}000 \times 12}{16{,}000 \times \frac{7}{8} \times 10} = 5.40\square''$ $18\text{-}\frac{5}{8}''\phi = 5.51\square''$

$+M$: $A_s = \dfrac{25{,}200 \times 12}{16{,}000 \times \frac{7}{8} \times 6} = 3.60\square''$ $12\text{-}\frac{5}{8}''\phi = 3.68\square''$

Mid Strip: $-M$: $\dfrac{15}{20} \times 3.60 = 2.70\square''$ $14\text{-}\frac{1}{2}''\phi = 2.74\square''$

$+M$: $\dfrac{15}{20} \times 3.60 = 2.70\square''$ $14\text{-}\frac{1}{2}''\phi = 2.74\square''$

2-Column Strips all $\frac{5}{8}''\phi$ rods @ about 10″ o.c.
Mid Strips all $\frac{1}{2}''\phi$ rods @ about $8\frac{1}{2}''$ o.c.



Wall Panel. Bracket: Try 18″

Shear [131]

Section a.

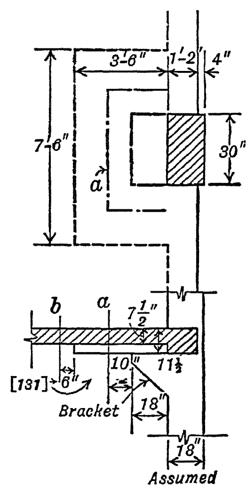
Total Shear =

$$232\left(\frac{20 \times 20}{2} - \frac{(30+20)(18+10)}{144}\right) = 44{,}200\#$$

$$v = \frac{44{,}200}{(30 + 20 + 2 \times 28)\left(\frac{7}{8}\right)(10)} = \frac{V}{bjd}$$

$$= 48\#/\square''$$

$$< 40\left(1 + \frac{2.5}{10}\right) = 50\#/\square'' \quad \text{O.K.}$$



Section b.

$$\text{Total Shear} = 232\left(\frac{20 \times 20}{2} - 4.0 \times 8.5\right)$$

$$= 38{,}500\#$$

$$v = \frac{38{,}500}{(102 + 2 \times 48)\left(\frac{7}{8}\right)(6)}$$

$$= 37\#/\square''$$

$$< 40\left(1 + \frac{7.5}{10}\right) = 70\#/\square'' \quad \text{O.K.}$$

$$< 60\#/\square''.$$

beam carries much less load than in the former arrangement. Custom provides for the same reinforcement as in the middle strip.

A beam receiving a load from one side only, as here, is subjected to considerable torsion, in this case estimated as 0.80 of the negative moment on an interior middle strip. Allowance was made for the slightly increased span in the wall panel. The torsional shear sets up diagonal tension stresses in the inside face of the beam which increase the usual inclined tension stresses. This may be seen by inspection of Fig. 82, which shows the variation of torsional shear intensity on a homogeneous rectangular section. The greatest intensity of this stress is at c and is approximately equal to[1]

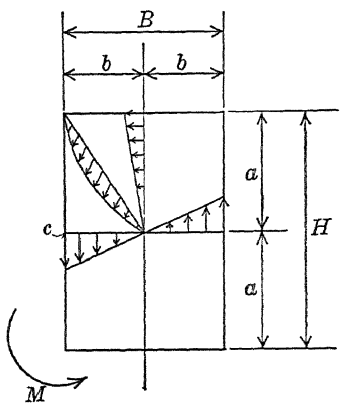


FIG. 82

$$v_t = \frac{M(15\,a + 9\,b)}{40\,a^2b^2}.$$

This equation has been rearranged in terms of the total breadth and height (B, H) of the section and used on Sheet FS6 for a very approximate solution of this reinforced concrete beam. The shear so found was treated exactly as though it were the usual vertical shear, except that it was considered over only one-half the width and as varying from a maximum to zero in the half width. The inside legs of the stirrups only are stressed in tension by the torsion and so the outside legs are available to carry the normal diagonal tension and to act as hangers to carry the slab load which is hung from the bottom of the beam.

This is a very approximate treatment of a complex problem. So little experimental work has been done that no recommendations can be made as to allowable stresses. The most that can be said is that this suggested solution furnishes a rough guide. Many thin spandrels with heavy slab loads have bulged and cracked badly under the torsion. This is one of the problems that should receive the attention of investigators.

[1] Applied Mechanics, Vol. II, Fuller & Johnson, page 382. Also see Concrete Plain and Reinforced, Vol. I, Taylor, Thompson and Smulski, page 94.

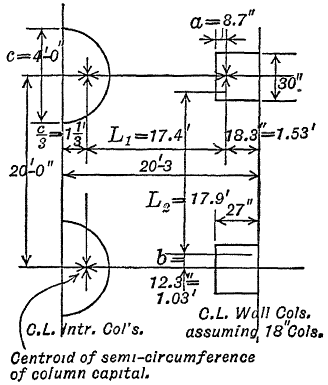
COMPUTATIONS FOR FLAT SLAB FLOOR — Continued

Sheet FS4

Wall Panel Spans
For interior panels

$$\text{bearing span} = \left(l - \frac{2c}{3}\right)$$
$$= 20 - 2 \times \tfrac{4}{3}$$
$$= 17.33'$$

For wall panels (see sketch) →
perpendicular to wall
$L_1 = 17.4'$
parallel to wall
$L_2 = 17.9'$



Dimension a locates centroid of bracket edge $a = \dfrac{2 \times 27 \times 13.5}{30 + 2 \times 27} = 8.7''$

Dimension b locates centroid of half bracket edge $b = \dfrac{27 \times 15 + 15 \times 7.5}{27 + 15} = 12.3''$

Steel in Exterior Panels. Perpendicular to wall.
Perpendicular to wall. *2 Column Strips*

At 1st Interior Column $-M: \; A_s = 5.40 \left(\dfrac{17.4}{17.33}\right)^2 (1.15) = 6.26\square''$ ← [147] $\left\{ \begin{array}{l} 8\text{-}\tfrac{5}{8}''\phi = 2.45 \\ 9\text{-}\tfrac{5}{8}''\phi = 2.75 \\ 4\text{-}\tfrac{5}{8}''\phi = \underline{1.22} \\ \qquad\qquad 6.42\square'' \end{array}\right.$

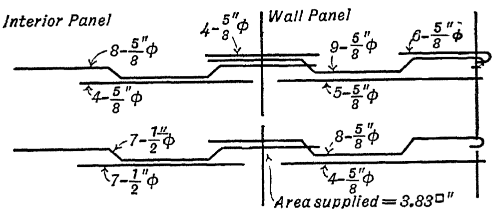
$+M: \; A_s = 3.60 \quad (do)^2 \quad (do) \quad = 4.18\square''$ $\qquad 14\text{-}\tfrac{5}{8}''\phi = 4.28\square''$

At Wall Column $-M: \; A_s = 5.40 \quad (do)^2 \quad (0.80) = 4.36\square''$ ← *cf.* [149] $\left\{ \begin{array}{l} 9\text{-}\tfrac{5}{8}''\phi = 2.75 \\ 6\text{-}\tfrac{5}{8}''\phi = \underline{1.84} \\ \qquad\qquad 4.59\square'' \end{array}\right.$

Mid Strip ← [147]

$-M = +M: \; A_s = 2.70 \left(\dfrac{17.4}{17.33}\right)^2 (1.30) = 3.53\square''$ $\qquad 12\text{-}\tfrac{5}{8}''\phi = 3.67\square''$

at wall $-M: \; A_s = 2.70 \quad (do)^2 \quad (0.80) = 2.16\square''$ $\qquad 8\text{-}\tfrac{5}{8}''\phi = 2.45\square''$



} *2 Column Strips*

} *Mid Strip*

Wall Panel: Steel parallel to wall.
Column Strip at wall: [148]

$$-A = 5.40 \left(\frac{17.9}{17.33}\right)^2 \times \tfrac{1}{2} = 2.88\square''$$
$$+A = 3.60 \left(\;\text{“}\; \right)^2 \times \tfrac{1}{2} = 1.92\square''$$

$8\text{-}\tfrac{5}{8}''\phi = 2.45\square''$
$2\text{-}\tfrac{5}{8}''\phi = \underline{0.61}$ *top bars*
$\qquad 3.06\square''$

Use → $\left\{ \begin{array}{l} 7\text{-}\tfrac{5}{8}''\phi \text{ 3 straight 4 bent} \\ 2\text{-}\tfrac{5}{8}''\phi \text{ top bars} \end{array}\right.$

Temperature reinforcement is very important in this type of beam.

107. Irregularities in Flat Slabs. Nearly every large flat slab building has irregularities that present design problems of considerable complexity. It is not possible in this text to cover all the possibilities but two common difficulties will be noted: openings in the floors and varying outside bays. An opening of less than 2 feet on a side outside the drop panel can usually be taken care of by simply spreading the reinforcing to make place for it. In the drop panel even a small opening should be avoided, but if one is necessary, the concrete stress should be checked at the reduced section. In roofs, skylights are often needed. If they are less than half the panel width in maximum dimension it is sometimes possible, by centering them in the panel, to cut out the mid-section steel which would pass through the opening. This is possible because the skylight area carries little load. This should not be done in adjacent panels, and wherever it is done, extra rods should be provided for the negative mid-section moment to replace the bent rods which are omitted. Where extra loads are applied around the edge of openings, which may often be the case with elevator or other shafts having a partition around them, beams should be used. As soon as a beam is put in, however, it takes slab load owing to its greater stiffness, and often requires girders to carry its reactions to the columns. Beams are therefore a distinct disadvantage. Some designers take care of irregular openings by considering the column strips as beam systems supporting a two-way slab at right angles to them, represented by the central part of the mid-section. If a typical panel is analyzed on this basis, it will be found that the moment coefficients reduced for full continuity, using the center to center distance (l) for the span, are on the safe side.

It sometimes happens that a flat slab building must cover an irregular lot. In such cases it is usually best to make the interior panels rectangular and take care of the variation in overall dimensions by end panels of varying span. The steel for such spans can be easily computed, but when the outside span is less than half the main span it should be noted that the entire short span may have negative moment. It is also necessary in detailing such cases to be sure that extra steel is used over the first interior support to replace that lost by reducing the bent steel in the end

COMPUTATIONS FOR FLAT SLAB FLOOR — Continued

Sheet FS5

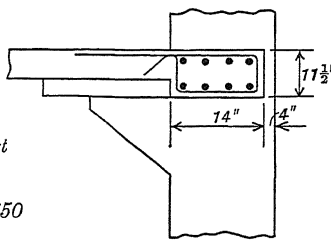
Spandrel or Wall Beam.

Loads: *Span: 17'-6" clear*

Wall: 3'-0" × 9"	270#	
Sash	30	[*No Slab load· see § 148*]
Beam	160	

$$\pm M = 460 \times 17.5^2 \times \tfrac{1}{12} = 11{,}800'\#$$
$$V = 460 \times 17.5 \times \tfrac{1}{2} = 4{,}000\#$$

Trial section shown: —



Shear: $v = \dfrac{4000}{14 \times \frac{7}{8} \times 9} = 36\#/\square'' < 40$ O.K.

Steel: *Check of* f_c

$$R = \frac{M}{bd^2} = \frac{11{,}800 \times 12}{14 \times 9 \times 9} = 125 \text{ at support}$$

From Plate VI, $p = 0.0091$ *for 16,000 — 750*

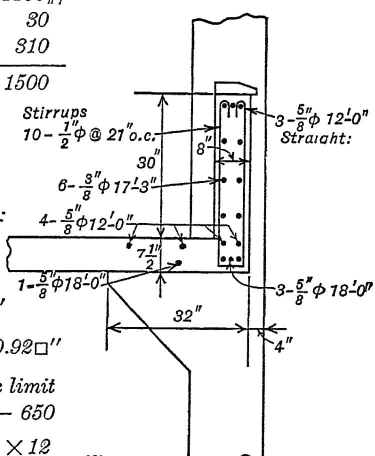
$$A_s = 0.0091 \times 14 \times 9 = 1.15\square''$$
$$4\text{-}\tfrac{5}{8}''\phi = 1.22\square''$$

$4\text{-}\frac{5}{8}''\phi$ *12'-0".* $4\frac{5}{8}''\phi$ *19'-0".* *Use* $8\text{-}\frac{3}{8}''\phi$ *Stirrups @ 24" o.c.*

Spandrel. Alternate Design without Drop at Wall Column

Loads: *Clear Span = 17'-6"*

Live + Dead from Slab: $232 \times \frac{20}{4}$	= 1160#/'
Sash	30
Beam	310
	1500



Shear: $V = 1500 \times \dfrac{17.5}{2} = 13{,}100\#$

$$v = \frac{13{,}100}{8 \times \frac{7}{8} \times 35} = 54\#/\square''$$

Moment & Steel: *Approximately:*

$$M = 1500 \times \frac{17.5^2}{12} = 38{,}300'\#$$
$$A_s = \frac{38{,}300 \times 12}{16{,}000 \times \frac{7}{8} \times 35} = 0.94\square''$$
$$3\text{-}\tfrac{5}{8}''\phi = 0.92\square''$$
$$p = \frac{0.92}{8 \times 35} = 0.0033 < 0.0077 \text{ the limit}$$

for 16,000 — 650

Alternate Solution: $R = \dfrac{M}{bd^2} = \dfrac{38{,}300 \times 12}{8 \times 35^2} = 47$

Exact Method: ∴ *Required* $p = 0.0033$ (*from Plate VI*)

$$A_s = 0.0033 \times 8 \times 35 = 0.93\square''$$

Use $3\text{-}\frac{5}{8}''\phi$

panel. Sometimes the irregularity of outside spans is increased by an architectural treatment which requires a spacing of exterior columns varying from the typical. A slight lateral deflection of typical bands can be made but if the eccentricity of the exterior columns is more than about 10 per cent of the span the columns can be neglected and the slab rested on a stiff spandrel considered as a simple support. In special cases of flat slabs any designer not thoroughly experienced in this class of construction should work on a very conservative basis.

COMPUTATIONS FOR FLAT SLAB FLOOR — Continued

Sheet FS6

Wall Beam. Continued.

Slab: Dead Load: — Without upper portion of beam: $b = 32''$

$$f_s = 20{,}000\#/\square'' \qquad f_c = 800\#/\square''$$

$$M = 107 \times \tfrac{20}{4} \times \overline{17.5}^2 \times \tfrac{1}{12} = 13{,}700'\#$$

$$R = \frac{M}{bd^2} = \frac{13{,}700 \times 12}{32 \times \overline{6}^2} = 143 \qquad \textit{Required } p = 0.0100$$

Furnished by slab steel —		$A_s = 0.01 \times 6 \times 32 = 1.92\square''$	
$\frac{1}{2}''\phi$ @ $8\frac{1}{2}''$ o.c.	3 bars	0.59	
$\frac{5}{8}''\phi$ Top Bars	1 bar	0.31	0.90
Required extra			$1.02\square''$
Use $4\text{-}\frac{5}{8}''\phi$		=	1.20

Slab Steel: Parallel to Wall Beam:

Same steel and spacing as for interior middle strip up to edge of beam. i.e.

alternate straight & bent: $\frac{1}{2}''\phi$ @ $8\frac{1}{2}''$ *o.c.*

Stirrups:

Torsion: $$v_t = \left(3 + \frac{1.8}{H/B}\right)\frac{M}{B^2H} = \frac{3.38\,M}{8^2 \times 37.5} = 0.0014\,M \qquad \frac{H}{B} = \frac{37.5}{8} = 4.7$$

$$= 0.0014 \times 15{,}200 \times 12 \times \tfrac{1}{2} \qquad 2\,M = 0.80 \times 18{,}900\left(\frac{17.4}{17.33}\right)^2$$

$$= 128\#/\square'' \text{ on inside of beam.} \qquad = 15{,}200'\#$$

Stirrups required: Use $\frac{1}{2}''\phi$ *stirrups:* $2 \times 0.196 \times 16{,}000 = 6270\#$

$$N = \frac{\frac{1}{2}\left(\frac{128}{2}\right)\left(\frac{8}{2}\right)\left(\frac{17.5 \times 12}{2}\right)}{\frac{1}{2} \times 6270} = 4.3 \text{ each end.}$$

tension inside only

Diagonal Tension: Negligible account extra rods in outside for torsion.

Slab load: $$N = \frac{1160 \times 17.5}{6270} = 3.3 \text{ in whole length.}$$

Also furnished by outside stirrup legs

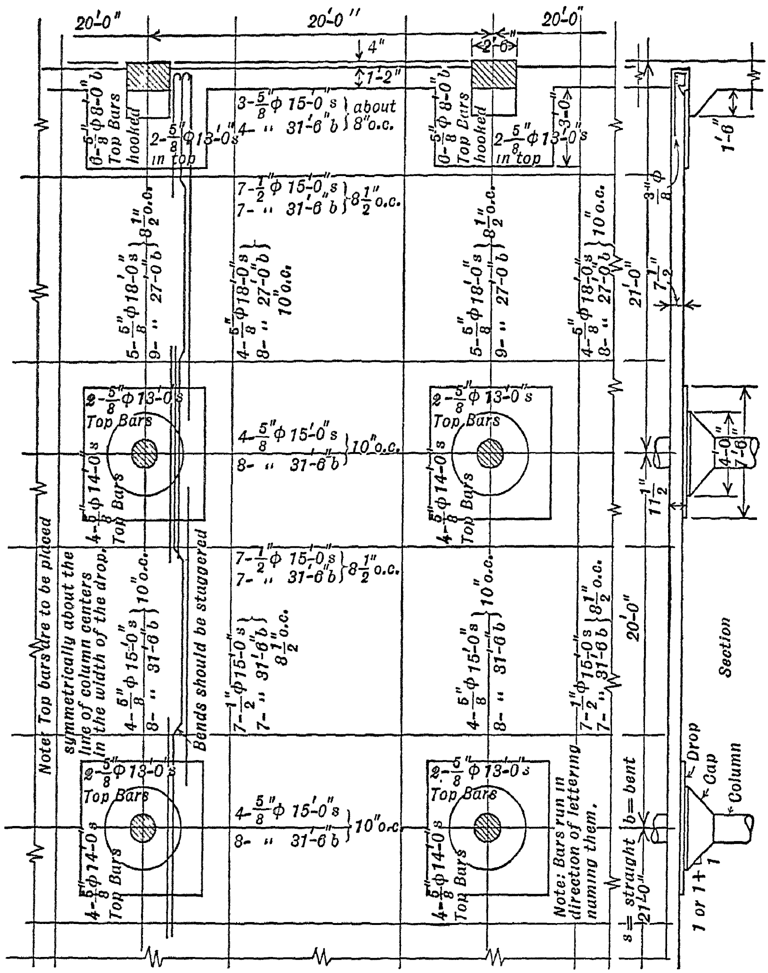
Use $10\text{-}\frac{1}{2}''\phi$ @ $21''$ *o.c.*

Temperature Steel:

$A_s = 0.003 \times 8 \times 30 = 0.72\square''$ *Use* $6\text{-}\frac{3}{8}''\phi = 0.66\square''$

COMPUTATIONS FOR FLAT SLAB FLOOR — Continued

Sheet **FS7**



CHAPTER XIV

BUILDING DESIGN. COLUMNS

108. In a tall reinforced concrete building (eight stories or more), there is place for all the usual types of concrete columns:

In the top story carrying the light load of the roof, columns reinforced with longitudinal steel only, secured against misplacement and buckling by lateral ties or binders, usually made of $\frac{1}{4}$-inch round steel, placed 8 inches on centers;

In the stories below with heavier loads and with likelihood of unequal loading causing bending, columns reinforced with longitudinal rods placed within a closely spaced helix or spiral of heavy wire, the whole protected from fire by about 2 inches of concrete;

In the lower stories where the heavy loads result in extremely large and heavy reinforced concrete sections, either composite columns with a structural steel or cast iron core surrounded by a hooped concrete section, or a structural steel column with concrete fireproofing;

Below the basement floor, transferring the column loads to the footings, short heavy pedestals (or piers) of plain concrete without reinforcement.

Practice is not yet standardized regarding composite columns and structural steel sections with heavy fireproofing and the reader should consult the larger handbooks on this subject for information concerning them.[1] Their action and consequently their design differs materially from that of the true reinforced concrete column.

109. Concrete Pedestals Without Reinforcement. On account of its lack of toughness and tensile resistance plain concrete is not suited for any but very short columns. Good practice limits the height to three or four times the least lateral dimension. In order that there be no severe bending stresses due to the cantilever action of the part of the pedestal extending beyond the column supported by it this distance should not be greater than one-half the depth.

[1] There is an excellent and comprehensive chapter on columns in "Concrete Plain and Reinforced," Taylor, Thompson and Smulski, Vol. I.

A difficulty often met with is the transfer of the load from the highly stressed column to the pedestal. The bearing stress under a column may be increased over the usual value since the mass of concrete of the pedestal outside the loaded area acts like the spiral reinforcement of a spiral column to prevent lateral deformation. This problem is considered briefly in the next chapter (Art. 121).

110. Reinforced Concrete Columns. It has already been pointed out that the column reinforced with longitudinal steel secured by widely spaced binders is not so dependable as one with a spiral enclosing the main reinforcement. The 1924 Joint Committee is more conservative than its predecessor in regard to this type of column and reduces both the working stress and the maximum amount of reinforcement that may be used from the limits set by the 1916 report. This is shown on Plate XI (page 396). However the later report permits the whole column section to be counted on as carrying load which is contrary to previous practice. This is perfectly proper structurally since the whole section acts as a unit at all stages of loading. With spiral-reinforced columns it is not proper to count on the concrete outside the closely spaced hooping as carrying load for this shell spalls off at high stresses and separates from the core. An objection to the change where columns are exposed to fire hazard is that the concrete to a depth of an inch or more is liable to severe disintegration in case of fire.

The large majority of reinforced concrete columns are relatively short and do not require a modification of unit stress for varying slenderness ratios as do steel columns. When this ratio exceeds 40, that is, when the unsupported length exceeds about 10 times the least lateral dimension of the core, the Joint Committee requires a column formula to be used. This formula and other data for design are shown in considerable detail on Plate XI.

In Fig. 83 are shown curves for the allowable average unit stress on the core of spirally reinforced columns as specified by the 1916 and by the 1924 Joint Committees. For values of the steel ratio below 2.5 per cent the new report is much more conservative than the old. The reason for this is evident by consideration of the argument presented by Mr. F. R. McMillan in his paper " A Study of Column Test Data," which appears in the 1921 Proceedings of the American Concrete Institute. Mr. McMillan pointed out the indisputable fact that concrete columns shrink on setting and place an initial compression in the steel and tension in the con-

crete. In addition there is a continued yielding of concrete under long and continued load, a permanent set, which increases this effect. He found from a study of test data that the total value of the deformation is about $m = 0.0005$, making the corresponding

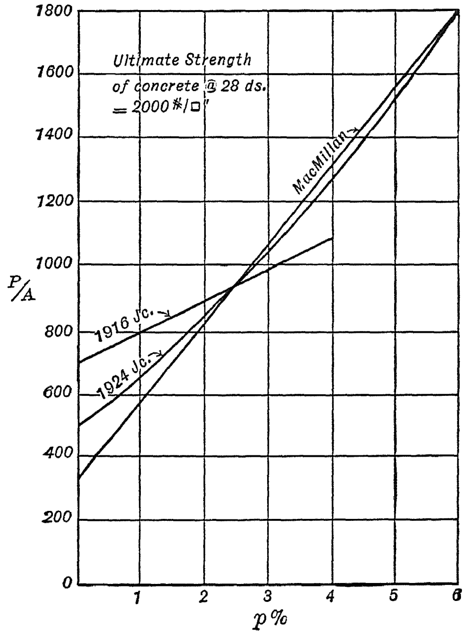


FIG. 83

initial stress in the steel ($E = 30{,}000{,}000$) about 15,000 pounds per square inch. Using the usual notation the total load on the column then is

$$P = EmpA + f_cA(1 + (n - 1)p).$$

The unit stress in the steel is

$$f_s = nf_c + (Em = 15{,}000).$$

The value of n, based on the total strain of the concrete, is very large. For the very common value of 1000 pounds per square inch for the average core stress, the unit stress in the steel varies as shown in Fig. 84. For the smaller steel ratios this stress is very large, even when the usual value of n is used in the calculation. Accordingly, Mr. McMillan puts his formula into this working

form for concrete with a strength of 2000 pounds per square inch at 28 days;

$$\frac{P}{A} = 333(1 + 29\,p) + 15{,}000\,p$$

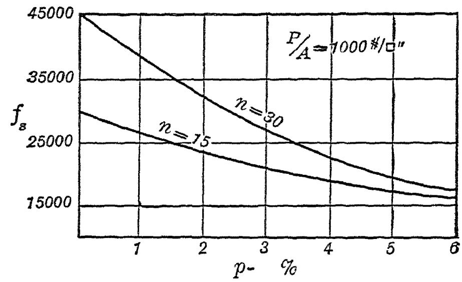


FIG. 84

which is plotted in Fig. 83 for comparison with those of the two Joint Committees. By restricting the columns with a small amount of reinforcement to a low average stress the unit stress in the steel is kept much lower by the 1924 Joint Committee than by that of 1916, something very desirable according to Mr. McMillan's evidence. Some engineers have been so much impressed by these considerations that they favor the consistent use of large steel ratios. The usual practice is to use the smallest permissible amount of steel for economy.

111. Column Loads. In a concrete building the method of determining column loads by summing the reactions of the beams framing into the column is incorrect, because beam loads are based on clear spans. The better way in all column design is to find the floor area tributary to each column and from this compute the dead and live floor load for the column. The dead loads of the beams, partitions and the column itself are then added in as separate items.

Floor live loads are reduced for column design in certain classes of buildings. In an office building or hotel for example, the floors are designed to accommodate relatively heavy loads, such as safes and files, over small areas. These heavy loads will not cover large areas and so the load is reduced on certain beams carrying several hundred square feet of floor as noted in Chapter XII. When more than one floor is considered there will be even fewer of these heavy

loads in proportion to the tributary area, or the average will be lower, so increasing deductions are applied to columns as they carry more floors. These deductions do not apply to buildings of the warehouse type which are likely to be loaded to the design load over the entire area. Conservative practice is illustrated by the following quotation from the report of the Building Code Committee to the United States Department of Commerce (Nov. 1, 1924)[1]:

" Except in buildings for storage purposes the following reductions in assumed total floor live loads are permissible in designing all columns, piers or walls, foundations, trusses and girders.

Reduction of total live loads carried	*Per cent*
Carrying one floor	0
Carrying two floors	10
Carrying three floors	20
Carrying four floors	30
Carrying five floors	40
Carrying six floors	45
Carrying seven or more floors	50

" For determining the area of footings the full dead load plus the live loads, with reductions figured as permitted above, shall be taken; except that in buildings for human occupancy, — a further reduction of one-half the live load as permitted above may be used." In this report the live loads are somewhat lower than is usual.

112. Bending Moment in Columns. Certain columns are subjected to an easily ascertainable moment from crane brackets or other eccentric loads but more commonly the moment in columns is due to their being part of a rigid frame.

In Chapter XI two methods for the calculation of moments in rigid frames were presented, Slope Deflection and Least Work, the first named being generally the more easy of application. Both methods become too cumbersome to be practical when the frame analyzed is at all complex. The most practicable method for structures with members of variable cross-section is that of Professor Beggs (footnote, page 188). An interesting approximate method based upon that of slope deflection has been developed

[1] " Minimum Live Loads Allowable for Use in Design of Buildings." For sale by Superintendent of Documents, Washington, D. C. Price 10 cents.

by Professor Richart[1] by which he has prepared Fig. 85 which gives the results obtained by analysis of the frame shown in Fig. 86 where all beams are alike and all columns are alike in all stories. The loading giving maximum moment in the column below A and

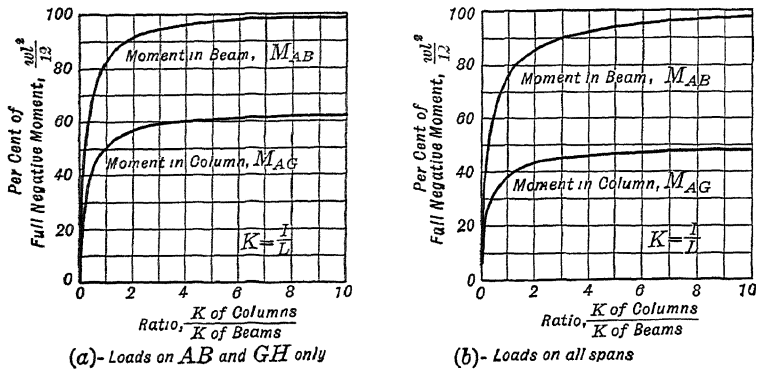


FIG. 85

above G is that shown in the figure, more than 90 per cent of the effect being due to the loads on AB and GH, a combination easily likely to happen. The loading combination shown in Fig. 86 is improbable and need not be provided for.

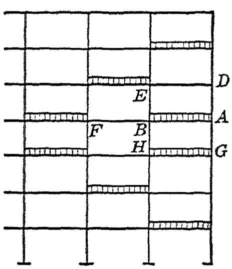


FIG. 86

The value of the ratio $\frac{K \text{ of columns}}{K \text{ of beams}}$ usually exceeds 2 and it is evident from Fig. 85 that usually more than 90 per cent of full fixed end moment is realized at the end of the beam framing into a wall column and that 60 per cent of the same $\frac{wL^2}{12}$ is the moment to be expected in a column above or below a beam.

By the slope deflection equation the moment at the end of a piece mn subjected to end moments, but not to lateral loads, without end deflection, may be expressed as

$$M_{mn} = 2\,E\,K(2\,\theta_m + \theta_n) = N\,E\,K\,\theta_m$$

[1] "A Study of Bending Moment in Columns," Proceedings of the American Concrete Institute, 1924. The cuts and most of the material in this article are directly from that paper.

with θ_n expressed in terms of θ_m. Here N may be called the restraint factor and K the stiffness factor.

In the paper referred to Professor Richart gives easily verified formulas for finding the beam moment M_{AB} (Figs. 85–86). This moment equals the sum of those in the columns just above and just below the floor, their relative values being in proportion to the NK factors for the columns. Here N, the restraint factor, is very uncertain in value, and the stiffness factor, K, very definite. It is advantageous to use the following empirical relations for finding the division of the beam moment between the columns;

$$M_{AD} = -M_{AB}\left(\frac{4 + r}{4 + 4\,r}\right)$$

and

$$M_{AG} = -M_{AB}\left(\frac{3\,r}{4 + 4\,r}\right)$$

where

$$r = \frac{K\text{ (Column above)}}{K\text{ (Column below)}} \qquad \text{(Compare Fig. 86)}$$

In Fig. 87 are shown the results when the column section is different in different stories. The curves for M_{AG} in this diagram with $r = 0$ and $r = 1$ are the same as the two full line curves in Fig. 85*a*. The lower dotted line in Fig. 87 shows a reasonable maximum moment to be expected in interior columns at sections such as those at top and bottom of column BH in Fig. 86. The 1924 Joint Committee rules (Art. 110 *b–c*, Appendix B) for the beam moment, M_{AB}, are shown graphically in Fig. 87 by the heavy dotted line which lies well above Richart's curve.

In flat slab buildings the column capitals make the columns of non-uniform cross-section and modify the results obtained for members of constant section. Professor Richart concluded, however, that the moments at the top and at the bottom of the uniform portion of columns with capitals are not usually appreciably larger than would exist at the extremities of the column were no capital used.

The curves of Fig. 87 may be used to estimate the moments in the columns of flat slab structures and in frames containing girders by replacing the $wL^2/12$ term by its equivalent in each case. By the Joint Committee ruling this would amount to from

60 per cent to 65 per cent of the total moment in an interior panel (Art. 144, Table VI, Appendix B), an average of about 0.056 $wl_1 (l - \frac{2}{3} c)^2$. For a girder, this term is the moment occurring at the end of a fixed ended girder with the given loading. "If the loads are symmetrical about mid-span, the 'fixed-beam' moment may be expressed as the average ordinate of the moment diagram for a simple beam of like span and loading."

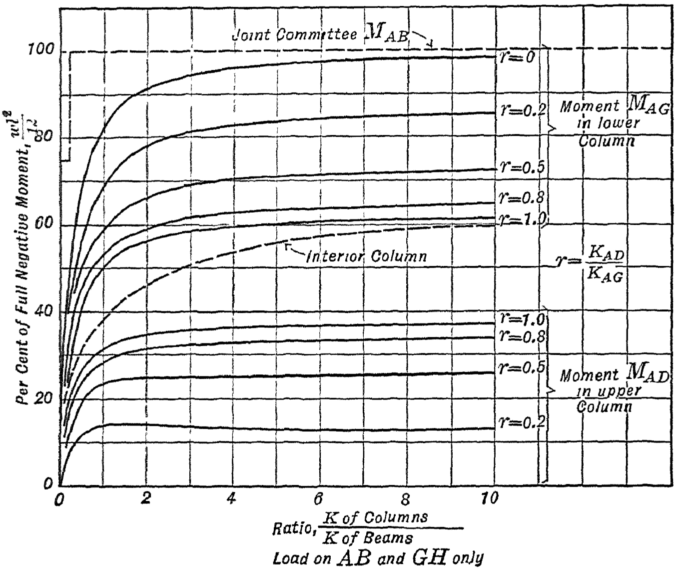


FIG. 87

For computations the value of the moment of inertia of the full concrete section, without the steel, is usually sufficiently accurate, since only the relative values are needed. The flange of a tee beam in a rigid frame is useful only in the positive moment portion of its length. Professor Richart recommends that the moment of inertia of such a beam be taken for the full concrete section without steel, taking the flange width the same as the maximum that may be used in stress analysis. The width of flat slab acting as a member of the rigid frame should be taken as one-half the panel length perpendicular to the span considered.

"In review it may be said that while bending moments are not commonly considered in the design of reinforced concrete columns, it is known that such moments exist and frequently

cause stresses of considerable importance. Provision for such stresses which are practically certain to occur with ordinary conditions of loading seem logical and proper. At the same time it seems justifiable to use higher working stresses . . ."

There is much difference of opinion as to the proper amount of this increase. The Joint Committee rules are very definite; an increase of 50 per cent for tied columns and 20 per cent for spiral columns (see Art. 167, Appendix B). Taylor, Thompson and Smulski[1] recognize the fact that the stress is partly due to direct loading and partly to bending. When the element of direct load is the principal one and there is either no tension or only negligible tension on the section they recommend an increase of 40 per cent in the allowable unit stress for axial compression; when considerable tension is developed in the section, they place the limit at that for bending, $0.4\, f_c'$ being the value they prefer. The 1916 Joint Committee allowed no increase in stress.

The frame of a building in an exposed situation should be designed to resist the horizontal force of the wind, with a 50 per cent increase in allowable stress. A wind pressure of 10 to 30 lbs. per sq. ft. of exposed vertical surface is commonly assumed, varying with the height of the building and the exposure. Most building laws specify the amount of wind load. An approximate analysis of a rigid frame with horizontal loads may be made by the ordinary methods of statics if it is assumed that the points of inflection of all columns and beams are at mid-length (that is, that these members are hinged at the center), and that the shear in any story is divided among the columns in proportion to one-half the total length of the beams framing into each column. For equal column spacing this means that an exterior column carries one-half as much shear as an interior column, all of which assist equally in carrying the total shear.

113. Columns with Long Span Beams. A problem that often arises is illustrated in Fig. 88 which shows a heavy beam of long span, carrying a roof. The beam shown was designed for a moment of $wL^2/8$ and the columns were assumed to carry direct load only. This is the usual solution and it is interesting to investigate the resulting stresses. The slope deflection equation shows that a negative moment of $\frac{1}{36}\left(\frac{wL^2}{8}\right)$ is developed in this case by the

[1] "Concrete Plain and Reinforced," Vol. I, page 463.

resistance of the column. It is evident that no great change can be made in the amount of beam reinforcement. A certain number of the rods should be bent up and carried along in the top at the ends. The maximum stress in the concrete of the column, utilizing the full section with no allowance for fireproofing, is 730 pounds per square inch which is rather higher than should be permitted. The amount of tension developed is very small and the limit of stress should be about 1.4 times that for direct compression, about 630 pounds per square inch.

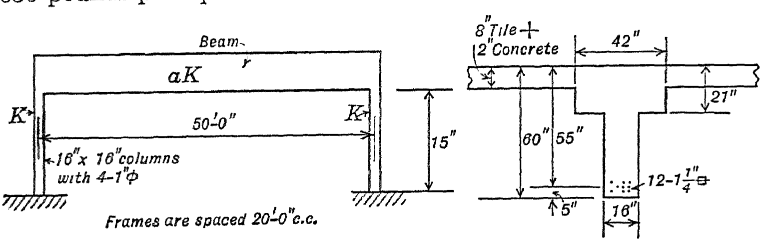


FIG. 88

In order to break the continuity of beam and column the vertical rods in the outside face of a top story column sometimes are carried only 3 inches or so into the beam, the inside rods running clear to the top as usual. Undoubtedly this has the desired effect so far as preventing the beam and columns acting as a rigid frame, which was the basis on which the above analysis was made, but it does not prevent the column from being loaded eccentrically. Using the value of θ in the slope deflection solution above, the width of crack in the outside face of the column at the bottom of the beam may be estimated at about $\frac{1}{32}$ inch. If a crack anything like this actually opens there is a large movement of the bearing surface towards the inside face of the column and an eccentricity of loading as great as that computed for full continuity of members is not unlikely. So in any case the column is subjected to a rather considerable moment and its reserve strength for the resisting of horizontal forces and unusual loading conditions is much less than assumed by the manner of design. It should be noted that in Fig. 88 the details of construction are not shown. Actually a building such as this would be built with longitudinal beams at the columns to act as bracing struts. Bracing is a very important feature in all structures and its lack has caused many serious failures. This matter of stability must be kept in mind in every design.

114. Example of Column Design. (Computation Sheets C1–C6.) The theory of column design is discussed in Chapter VII. The actual design of typical columns for the building previously considered is here introduced. The arrangement of column computations as shown on Sheet C1, or something very similar, is best practice. The sketch at the left shows the necessary dimensional data; the loading and specification are given at the top of the page. The dead and live loads are carried down separately as is necessary where a varying live load reduction is used, although no reduction is made here. Kips, or units of 1000 pounds, are very convenient in column computations. Round columns are usually made even inches in diameter on account of forms. The $\frac{h}{R}$ limit of 40 is not often exceeded in building work. The computations for size and steel are self explanatory. In the upper story the requirement of $f_c = 0.225\ f_c'$ as well as the neglect of fireproofing are taken from the 1916 Joint Committee. The stress as used to determine size is that found from $P/A = f_c\,[1 + (n - 1)\,p]$ which is the basis of the curves on Plate XI.

On Sheets C2–C3 is shown for illustration a computation for combined bending and direct stress on this column although it is not common practice to do this for interior columns unless the live loads are 300 pounds per square foot or more.

Sheet C4 shows computation for an exterior column. The width of exterior columns is usually constant for appearance, the exact dimension being that required for the sash. With the use of a reasonable minimum thickness the upper stories are usually lightly stressed. The ruled computation sheet is similar in form to the interior column. Plate XIV on page 399 is used for computing combined stress. Supplementary sheets which follow the main computation were used to figure moments. The methods are those already explained.

115. Special Columns. Individual columns in a building differing from any others can often be designed by comparison with the typical, so far as moment conditions are concerned. That is to say inspection will often show a ratio of loading or span between special and typical columns which will be safe and close enough for practical purposes. The direct load can and should always be computed for each column. Corner columns have bending about two planes usually 90 degrees apart. It is usually

easier to compute the stresses about the two axes separately and add them than to work with resultants. Architectural and construction conditions usually make corner columns larger than typical wall columns and they have lighter loads, so they are seldom a difficult feature unless long spans induce exceptionally large moments.

Note: Sheet C1 — Spiral reinforcement is measured by the ratio (p') of its volume to that of the enclosed concrete; $p' = \frac{a\pi D}{\frac{1}{4}\pi D^2 b}$ where a is the area of the wire used, D is the core (spiral) diameter and b is the pitch. For $p' = 1\%$, the common minimum used in this computation, the size (area) of wire required for any diameter is $a = Db/400$. The spiral standards now in force are given in the Engineering News-Record of January 6, 1927.

The weights of column capitals used on Sheet C1 are found on Plate XVI, page 401. In checking the clear height of columns below the drop note that the capitals terminate in a 2 in. vertical edge.

COMPUTATIONS FOR INTERIOR COLUMN

Sheet C1

Typical Interior Column for Flat Slab Building.
Specifications: 1924 Joint Committee except as noted.
Concrete: Ultimate Strengths @ 28 ds. { 2000#/□" 1:6 Mix, f_c=450 n=15; 3000 " 1:3 " n=10
Tributary Area: 20×20 = 400 □'

Story	Loads: 1000# units		Dead	Live	Total	Computations.	Design.
3.	Live	40#/□'		16.0		Minimum diameter: $\frac{20\times12}{15}$ = 16"	Round Column.
9'-8½"	Slab	75 6"				Effective " = 13": area = 133 □"	
11'-2½"	Roofg.	6 81	32.4			(following 1916 Jt. Comm.)	1-6 Mix.
12'-0"	Drop	6'8" sq. 3"	2.0			$\frac{P}{A} = \frac{54,000}{133}$ = 406 < 513 #/□"	16" Diam.
	Cap	4'0" Diam.	1.2			Plate XI: p = 1%	5-$\frac{5}{8}$" φ
	Col.	16" "	2.4			A_s = 133 × 0.01 = 1.33 □" 5-$\frac{5}{8}$"φ = 1.5 □"	$\frac{1}{4}$" φ Ties
			38.0				12" diam.
					54.0	$\frac{L}{r} = \frac{9.7\times12}{1/4\times16}$ = 29 < 40	8" o.c.
2.	Live	125#/□'		50.0		Trial p = 1% A = $\frac{154000\pm}{785}$ = 197 □"	1-3 Mix
9'-9"	Slab	94 7½"				From Pl. XI Try 16" Core: 201 □"	20" Diam.
11'-1"	Fin. 1"	13 107	42.8			$\frac{P}{A} = \frac{153500}{201}$ = 765 < 785 Use p = 1%	7-$\frac{5}{8}$" φ
12'-0"	Drop		2.0			A_s = 0.01 × 201 = 2.01 □"	
			44.8			7-$\frac{5}{8}$"φ = 2.14 □"	
	Cap	4'0" Diam	1.0			Spiral: use p = 1%	$\frac{3}{8}$"φ Spiral
	Col.	20" "	3.7			for $\frac{3}{8}$"φ pitch = $\frac{400\times0.11}{16}$ = 2.7"	16" diam.
			49.5				
		Totals	87.5	66.0	153.5	maximum " = $\frac{16}{6}$ = 2.67"	2½" pitch
1.	Live	125#/□'		50.0		Trial: p = 1%: A = $\frac{255000\pm}{785}$ = 325 □"	1-3 Mix
9'-11"	S+F+D		44.8			Try 20" core: 314 □"	24" Diam.
11'-1"	Cap	4'0" Diam	0.9			$\frac{P}{A} = \frac{254400}{314}$ = 810 #/□" p = 1.15%	6-$\frac{7}{8}$" φ
12'-0"	Col.	24" "	5.2			A_s = 3.61 □"	
			50.9			6-$\frac{7}{8}$"φ = 3.61 □"	$\frac{3}{8}$"φ Spiral
						Spiral: p = 1%	20" diam
		Totals	138.4	116.0	254.4	for $\frac{3}{8}$"φ pitch = $\frac{400\times0.11}{20}$ = 2.2"	2" pitch.
B.	Live	125#/□'		50.0		Trial: p = 1% A = $\frac{355000\pm}{785}$ = 452 □"	1-3 Mix.
8'-1"	S+F+D		44.8			Try 24" core: 452 □"	28" Diam.
9'-1"	Cap	4'0" Diam	0.7				
10'-0"	Col.	28" "	5.8			$\frac{P}{A} = \frac{355700}{452}$ = 787 □" p = 1%	6-1" φ
			51.3			A_s = 4.52 □"	
						6-1"φ = 4.71 □"	$\frac{1}{2}$" Spiral
						Spiral: p = 1%	24" diam.
		Totals	189.7	166.0	355.7	for $\frac{1}{2}$"φ pitch = $\frac{400\times0.196}{24}$ = 3.3"	3" pitch.

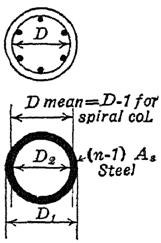
COMPUTATIONS FOR INTERIOR COLUMN

Sheet C2

Investigation of Stresses due to Moment. Data

Data: $A_t = \text{Transformed Area} = \dfrac{\pi D^2}{4} + (n-1)A_s$

$$I = \frac{\pi D^4}{64} + I_{steel}\text{—assumed to be a ring with diameter} = D - 1''$$



$$= \frac{\pi D^2}{4} \cdot \frac{D^2}{16} + \left[\frac{\pi(D_1^4 - D_2^4)}{64} = \frac{\pi(D_1^2 - D_2^2)(D_1^2 + D_2^2)}{4 \times 8 \times 2}\right]$$

$$\text{Area of ring} = \frac{\pi(D_1^2 - D_2^2)}{4} = (n-1)A_s$$

$$\text{Since } D_1 = D_2 = D_{mean}, \text{ closely: } \frac{D_1^2 + D_2^2}{2} = D^2_{mean}$$

$$\therefore I = \frac{\pi D^2}{4} \cdot \frac{D^2}{16} + (n-1)A_s \cdot \frac{(D - 1'')^2}{8}$$

Transformed Section and Moment of Inertia

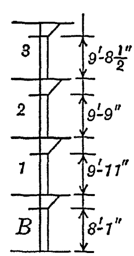
Story	D	$\frac{\pi}{4}D^2$	$(n-1)A_s$	A	$\frac{\pi D^4}{64}$	I_{steel}	I
3	16	201	* 21	222	3216	315‡	3531
2	16	201	† 19	220	3216	535	3751
1	20	314	32	346	7850	1448	9298
B	24	452	43	495	16300	2860	19160

* $n = 15$ † $n = 10$

‡ For this section $I = I_{core} + (n-1)A_s \cdot \dfrac{(D-5)^2}{8}$

Distribution of Moments (*page 267*) $\left[\dfrac{\left(\frac{I}{h}\right)\text{col. above}}{\left(\frac{I}{h}\right)\text{col. above} + \left(\frac{I}{h}\right)\text{col. below}}\right]$

3d Story	$\dfrac{I}{h} = \dfrac{3531}{116.5}$	$= 30$	0.47
2nd "	$= \dfrac{3751}{117}$	$= 32$	0.29
1st "	$= \dfrac{9298}{119}$	$= 78$	0.28
Basement	$= \dfrac{19{,}160}{97}$	$= 197$	



Unsupported Lengths

COMPUTATIONS FOR INTERIOR COLUMN

Sheet C3

M = *Total moment at any floor*

$= 0.04\,wl(l - \frac{2}{3}c)^2$ [*Am. Conc. Institute 1920*]

$= 0.04\,wl^3\left(1 - \frac{2}{3}\frac{c}{l}\right)^2$ *Assume* $\frac{c}{l} = 0.225$, *the average value*

$= 0.029\,Wl$

$M = \frac{1}{40}\,Wl = 0.025\,Wl$ *recommended by Sanford E. Thompson,*
member of 1924 Joint Committee

$= \frac{1}{40} \times 125 \times \overline{20}^3 = 25{,}000'\#$
Use this value

At 3d Floor $M_{above} = 25{,}000 \times 0.47 = 11{,}750'\#$

$M_{below} = 13{,}250$

3d Story Col. $f_c = \dfrac{P}{A} \pm \dfrac{Mc}{I} = \dfrac{54{,}000}{222} \pm \dfrac{11{,}750 \times 12 \times 8}{3531}$

$= 244 \pm 320$

$= 564\#/\square''$ *compression* $< 600\#/\square''$ [*Art. 167b*]

$= 76\#/\square''$ *tension*

$\therefore f_s < 15 \times 76 < 16{,}000\#/\square''$

(*Above solution is approximate.*) *O.K.*

2nd Story Col. $f_c = \dfrac{150{,}000 \text{ approx.}}{220} \pm \dfrac{13{,}250 \times 12 \times 8}{3751}$

$= 682 \pm 340$

$= 1020\#/\square''$ *comp.*

$> (1.2 \times 720 = 865\#/\square'')$ [*Art. 167a*]

$\therefore$ *By J. C. column overstressed.*

Taylor, Thompson & Smulski, in "Concrete, Plain & R'f'd," Vol. 1, for spiral cols., recommend $f_c = 0.35\,f_c' = 0.35 \times 3000 = 1050\#/\square''$. *They agree with 1916 Joint Comm. in this. For bending without tension they allow an increase of 40% in* f_c. *See pages 421 and 463.*

$\therefore$ *Col. assumed O.K.*

At 2nd Floor: $M_{above} = 25{,}000 \times 0.29 = 7{,}250'\#$

$M_{below} = 17{,}750'\#$

2nd Story Col. $f_c = \dfrac{153{,}500}{220} \pm \dfrac{7250 \times 12 \times 8}{3750}$

= *lower stress than above*

1st Story Col. $f_c = \dfrac{250{,}000}{346} \pm \dfrac{17{,}750 \times 12 \times 10}{9298}$

$= 723 \pm 229$

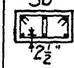
$= 950\#/\square''$ *max. comp.* *O.K.*

By inspection — conditions at 1st floor are seen to be O.K.

COMPUTATIONS FOR EXTERIOR COLUMN

Sheet C4

Typical Exterior Column for Flat Slab Building.
Specifications: 1916 Joint Committee except as noted.
Concrete: Ultimate Strength @ 28 ds. 2000#/□" n = 15
Tributary Area: 20 X 10 = 200□' 1-6 Mix.

Story	Loads: 1000# Units.					Computations.	Design
			Dead	Live	Total		
3	Live Slab Roofg Spd'l Brk't Col. " 	40#/□' 75 6" 6 81 10'x3'6" Upper 5' Lower 10' Totals	 16.2 7.7 0.5 2.8 27.2 5.6 32.8	8.0 8.0	 40.8	$M = \frac{1}{40} \times 121 \times 20^3 = 24{,}200'\#$ $A_s = \frac{M}{f_s jd} = \frac{24200 \times 12}{16000 \times \frac{7}{8} \times 15.5} = 1.34\square''$ in tension face. $\frac{P}{A} = \frac{40800}{15 \times 27} = 100\#/\square'' < 480$ for $p = \frac{1}{2}\%$ $A_s = 0.005 \times 15 \times 27 = 2.0\square''$ Try $6\text{-}\frac{3}{4}''\phi = 2.64\square''$	 30" 18" 2½" $6\text{-}\frac{3}{4}\text{-}\phi$ $\frac{1}{4}''\phi$ Ties 8" o.c.
2	Live Slab Fin. 1" Spd'l Brk't Col. " 	125#/□' 94 7½" 13 107 10'x3'-7½" 2' 10' Totals	 21.4 7.9 0.5 1.1 63.7 5.6 69.3	25.0 33.0	 102.3	$M = \frac{1}{40} \times 232 \times 20^3 = 46400'\#$ total Divided equally above and below. Investigate above section: Pl. XIV. $\frac{M}{Pd} = \frac{23200 \times 12}{32800 \times 13} = 0.65$ } $\frac{P}{bhf_c} = 0.22$ $p = \frac{2.64}{13 \times 25} = 0.0081$ $f_c = \frac{32800}{13 \times 25 \times 0.22} = 460\#/\square''$ $f_s = 18 f_c = 8300\#/\square''$ By inspection other sections are OK.	Same
1.	Live St F + Col. " 	125#/□' Sp. + B 2' 10' Totals	 29.8 1.1 100.2 5.6 105.8	25.0 58.0	 163.8	$M = 23200'\#$ as above. By inspection $f_s < 8300$ above. Max. f_c - approximately. $\frac{M}{Pd} = \frac{23200 \times 12}{163800 \times 13} = 0.13$ $p = 0.008$ $f_c = \frac{163800}{13 \times 25 \times 0.68} = 740 > 1.4 \times 450 > 630$ More precise solution needed: See data sheet.	Same.
B	Live St Ft Col. . 	125#/□' Sp + B 2' 8' Totals	 29.8 1.1 136.7 4.5 141.2	25.0 83.0	 224.2	Trial calculations on data sheet. Below floor $f_c = \frac{P}{A} \pm \frac{Mc}{I}$ $= \frac{219700}{471} \pm \frac{22200 \times 12 \times 7.5}{10400}$ $= 462 \pm 192$ $= 638 > 630$ O.K.	Same with 6-1"ϕ

COMPUTATIONS FOR EXTERIOR COLUMN

Sheet C5

Data Sheet *Minimum Thickness*

Thickness of Column. See Vol. 1, "Concrete, Plain & Reinforced," Taylor, Thompson & Smulski, page 313, etc.

$$d_1 = 1.15\sqrt[3]{\frac{hL_1}{bL}} \cdot d$$

$$= 1.15\sqrt[3]{\frac{12 \times 20.25}{2.5 \times 20}}\left[6 + \frac{3}{3}\right]$$

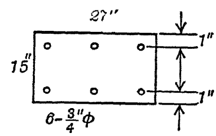
$$= 14''$$

18″ used

Here h = *story height*
L_1 = *span* $\perp$ *wall*
L = *span* $\|$ *wall*
b = *column width*
d = *slab thickness* (t_2)
or $= t_2 + \frac{1}{3}(t_1 - t_2)$

1st Story Column

Stress below 2nd Floor



$$M = 0.5 \times 46{,}400 = 23{,}200'\#$$

$$P = 158{,}200\#$$

$$f_c = \frac{158{,}200}{442} \pm \frac{23{,}200 \times 12 \times 7.5}{9160}$$

$$= 358 \pm 228$$

$$= 586\#/\square'' < 630\#/\square''$$

O.K.

$$A \quad 27 \times 15 = 405$$
$$6 \times 0.44 \times 14 = \underline{\ 37\ }$$
$$442\square''$$

$$I/27 \times \overline{15}^3 \times \tfrac{1}{12} = 7600$$
$$37 \times \overline{6.5}^2 = \underline{1560}$$
$$9160 \text{ in.}^4$$

Distribution of Moments:

$$M_{ab} = 2\,E\,\frac{I}{L}\,(2\,\theta_a + \theta_b) \text{ by Slope Deflection equation}$$

$$\text{For} \quad \theta_a = \theta_b \quad M = 6\left[E\,\frac{I}{L}\,\theta_a\right]$$

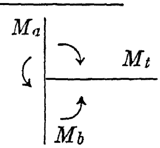
$$\theta_a = -\theta b \qquad = 2\left[\quad \text{``} \quad\right]$$

$$\theta_b = 0 \qquad = 4\left[\quad \text{``} \quad\right] \qquad \text{i.e. Fixt end at } b.$$

$$\theta_b = -\frac{\theta_a}{2} \qquad = 3\left[\quad \text{``} \quad\right] \qquad \text{i.e. Free end at } b.$$

$$M = NEK\theta \text{ in general where } N = 2 \text{ to } 6$$

$$\text{Then } M_a = \frac{N_aK_a}{N_aK_a + N_bK_b} \cdot M_t \quad \text{in general.}$$



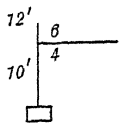
COMPUTATIONS FOR EXTERIOR COLUMN

Sheet C6

Data Sheet

Moment Distribution at 1st Floor.

Assume same section above and below —



$$\frac{N_aK_a}{N_aK_a + N_bK_b} = \frac{\frac{6\,I}{12}}{\frac{6\,I}{12} + \frac{4\,I}{10}} = \frac{6}{10.8} = 0.56$$

$M_a = 0.56 \times 46{,}400 = 26{,}200'\#$

$M_b = \quad 20{,}200'\#$

Stresses at 1st Floor

$$\text{Above} \quad f_c = \frac{163{,}800}{442} \pm \frac{26{,}200 \times 12 \times 7.5}{9160} = 370 \pm 258 = 628 \quad \text{O.K.}$$

$$\text{Below} \quad f_c = \frac{219{,}700}{442} \pm \frac{20{,}200 \times 12 \times 7.5}{9160} = 497 \pm 199 = 696\#/\square'' > 630$$

Try same section with 6-1″φ

$A = 405 + 14 \times 6 \times 0.785 = 405 + 66 = 471\square''$

$I = 7600 + 66 \times \overline{6.5}^2 = 10{,}400''^4$

$I/h = 10{,}400 \div 120 = 86.5$ *for basement*

$= 9160 \div 144 = 63.6$ *for 1st story*

Moment Distribution with 6-1″φ in Basement Column.

$$\frac{N_aK_a}{N_aK_a + N_bK_b} = \frac{6 \times 63.6}{6 \times 63.6 + 4 \times 86.5} = \frac{382}{728} = 0.53$$

$M_a = 0.53 \times 46{,}400 = 24{,}600'\#$

$M_b = \quad 22{,}000'\#$

CHAPTER XV

BUILDING DESIGN. FOUNDATIONS

116. Nearly all engineering structures consist of two parts, the superstructure above and the substructure below the level of the ground. The foundation is that portion of the substructure whose function is the distribution of the load to the earth. In order to reduce the bearing pressure to proper limits it is necessary to spread out or enlarge that part of the foundation which is in immediate contact with the earth. The spread part of any foundation unit is called a footing.

Concrete is in universal use for the foundations of all types of superstructures. It is the function of this chapter to consider briefly the application of the simple principles of reinforced concrete design already outlined to the relatively massive members used as the supports of buildings. Consideration of the carrying capacity of different soils and its determination is beyond the scope of this text.

117. Foundations of Buildings. The foundations of a building consist usually of spread footings beneath the interior columns and either local footings or continuous wall footings beneath the exterior columns, the loads being transmitted to the soil at a level only a short distance below the basement floor. When the bearing stratum lies so far below the usual footing level that the cost of excavating open pits is prohibitive, it becomes necessary either to use piles which rest upon the harder underlying material and act as columns, or to drive shafts or caissons, 3 feet or more in diameter depending upon the depth and manner of sinking, which are enlarged at the bottom and filled with concrete, forming a supporting pier. When the supporting stratum of soil is of low bearing capacity for a considerable depth it often becomes necessary to increase this capacity and also increase the area of distribution of the loads by means of piles. The piles are capped by footings which are similar in every way to those resting directly on the earth. Sometimes on soft soil a single bearing surface is employed for the whole structure forming a floor or raft upon which

all the basement walls and columns rest. The design of such a raft foundation is the same as that of a beam and girder or a flat slab floor. The bottom of a footing should always be below the level affected by frost action, a distance which varies with the locality.

118. Proportioning Footing Area. All soils are compressible and so all structures settle to a greater or less degree unless they are founded on solid rock. The allowable bearing pressure on a soil is set at such a value that settlement will be limited to a reasonable amount, a small fraction of an inch or several inches, according to the locality, the nature of the soil and the type of structure.

The settlement of all the different parts of a building must be equal so far as possible in order that cracking of walls and plaster and structural damage to the frame be avoided. This becomes a matter of increasing importance and difficulty with the poorer soils where the settlement to be expected and consequently the variation in settlement are greater. By far the larger part of all settlement is caused by the dead load of a structure, a factor which can be computed with great accuracy. Live loads have practically no effect on settlement unless they are long continued, but of course they have the same effect on the stresses in the footings as any other load. Accordingly it is important that the unit bearing pressure on the soil under all footings of a building be the same under dead load in order that equal settlement of all parts may occur. If the bearing capacity of the earth under one part of a structure differs from that under the rest careful studies of the relative capacities must be made and bearing pressures used that will result in equal settlement. In some types of buildings where the live loads are certain of realization for long continued periods a portion of the live load is added to the dead load and footings are proportioned for equal unit pressures under the combination. A fact to be kept in mind when deciding upon the proportion of live load to be so used and the portions of the structure where live load concentrations are to be expected, is that a large proportion of the total settlement usually has taken place by the time the building is ready for occupancy. Another factor to be considered here is the reduction to be made in live load on girders and columns. (See Arts. 95 *a* and 111.)

The actual procedure in computing footings is as follows: the

bearing area of the footing under the column with the smallest proportion of dead load (one of the interior columns) is calculated by dividing the total load on the soil (column load, live and dead plus the weight of the footing), by the allowable bearing pressure for the earth. Next the unit pressure under this footing is determined for dead load only (or for dead load plus one-half the live load, the usual proportion chosen when live load is included). Then all other footing areas are determined by dividing their dead load totals by this computed dead load unit bearing.

All footings should settle evenly without tipping and this requires a symmetrical distribution of pressure on the base. The exact distribution of pressure is not known but when the load on a footing is vertical it is assumed as uniform for nearly all calculations. This assumed uniformity of pressure is obtainable by proportioning the bearing area so that its centroid lies on the line of action of the load. Accordingly the footing under a single column is concentric with the column. A footing of this sort is shown on Computation Sheet IF1, page 279.

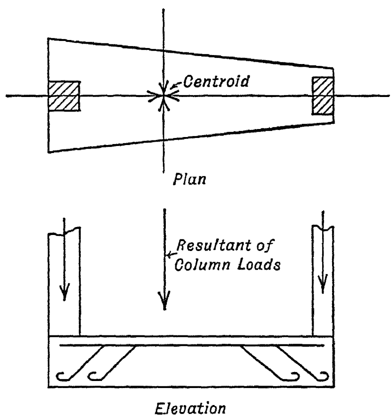


FIG. 89

A combined footing is one that supports two or more columns. The line of action of the resultant of the dead column loads (or dead load plus a fraction of the live) should pass through the centroid of the base of the combined footing. A common type is illustrated in Fig. 89.

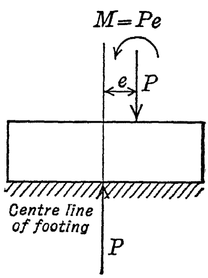


FIG. 90

A wall column is usually subjected to bending as well as to direct compression and so its footing should not be concentric with the column. This moment can be estimated or computed by the methods that have been described and the footing centered so that the uniform base pressure and the axial column load form a couple equal and opposite to the moment. This is illustrated in

Fig. 90. A more common method is to estimate the lines of action of the several loads carried by the lower column above the footing and find the line of action of their resultant, which is the required center line of the footing. This is illustrated by Fig. 91. Beam reactions are usually assumed to act at from one-quarter to one-third the depth of the column from the inside face, or a distance equal to one-half the depth of the beam if that is less. Any error of assumption that places the center of the footing inside the actual resultant load results in a tendency to tip so that the column leans out from the building, causing cracks. So it is desirable that the load resultant (Fig. 91) act a little inside the center of the footing which results in a tipping that tends to force the building together.

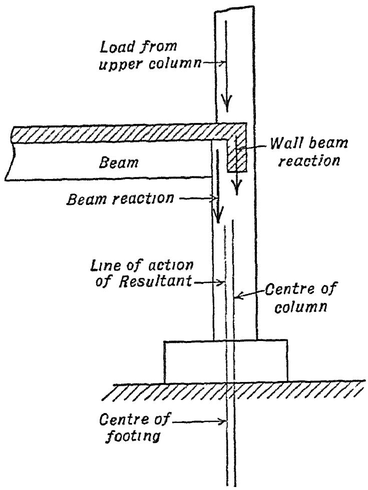


FIG. 91

119. Bearing Capacity of Soils. A table of safe bearing values of soils can be only a very approximate guide as to the proper value to choose in any particular case unless the table has been constructed from the records of experience of the given locality and covers very definitely the conditions at the location. There are many factors that affect bearing value, those of composition, moisture content and degree of confinement laterally. Up to the present we have had no standard of measure by which it was possible to give an accurate description of all the essential properties of soil. The usual descriptions, such as stiff clay, cover a wide range of different materials under one heading. The researches of Dr. Ing. Charles Terzaghi[1] are giving us definite standards and for the first time putting our work with soils on a scientific basis of exact knowledge.

[1] The following publications of Dr. Terzaghi are especially to be noted: "Modern Conceptions concerning Foundation Engineering," Journal of the Boston Society of Civil Engineers, Dec., 1925. "Principles of Soil Mechanics," reprint from Engineering News Record. "Erdbaumechanik," published by Deuticke, Leipzig and Vienna.

In considering any specification for soil pressures it is necessary also to consider the allowable live load reductions. In some building codes extreme conservatism in one of these matters is offset by liberality in the other. In the code of the city of Boston extremely large reductions of the column loads are permitted but the allowable soil values are not unusual. The Boston rules are reprinted below as they give a much better definition than is usual for the various kinds of soil.

ALLOWABLE BEARING ON FOUNDATIONS

	Tons per Sq. Ft.
Solid Ledge Rock. (Naturally formed rock, such as the granites and others of similar hardness and soundness, normally requiring blasting for their removal.)	100
Shale and Hardpan. (Shale: laminated slate or clay rocks removable with more or less difficulty by picking. Hardpan; a thoroughly cemented mixture of sand and pebbles or of sand, pebbles and clay, with or without a mixture of boulders and difficult to remove by picking.)	10
Gravel. (A natural uncemented mixture of coarse or medium grained sand with a substantial amount of pebbles measuring one-fourth of an inch or more in diameter.)	6
Compact Sand and Hard Yellow Clay. (Requiring picking for removal.)	6
Dry or wet Sand of coarse or medium-sized grains. Hard Blue Clay mixed or unmixed with sand. Disintegrated Ledge Rock. (Sand, medium-grain; individual grains readily distinguishable by eye though not of pronounced size. Disintegrated Ledge Rock; residual deposits of decomposed ledge.)	5
Medium Stiff or Plastic Clay, mixed or unmixed with sand. (Stiff and plastic but capable of being spaded.)	4
Fine Grained Dry Sand. (Individual grains distinguished by eye only with difficulty.)	4
Fine Grained Wet Sand, confined	3
Soft Clay, protected against lateral displacement. (Putty-like in consistency and changing shape readily under relatively slight pressure.)	2

For all important structures studies should be made of the foundation material by means of borings and test loads.

To guard against unequal settlement when part of the footings of a structure rest on rock it is common to specify that the soil value for the remaining footings be reduced one-half.

Basement floors often rest directly on the soil. While their load is relatively light there will be large settlement on compressible soils and the resulting failure of the floor may be expensive to repair although not dangerous to the building.

Footings and floors are sometimes subject to uplift due to water pressure. This seldom needs to be considered in footing design

except for hydraulic structures. It is often of importance in designing basement and pit floors. The water pressure is probably not exerted on the entire surface of a concrete floor poured directly on the soil. Mr. J. R. Worcester[1] recommends that such pressure be assumed to act over one-half the area. In designing for water pressure when a non-flexible waterproofing is to be used, it is essential that steel be used to prevent any stress cracks, considering the floor or the pit as a monolith.

120. Footings of Plain Concrete. For large footings reinforcement is usually an economy but plain concrete is used with advantage for the footings of many small structures. The ratio of projection (p) to height of course (h, Fig. 92), is commonly set by rule of thumb, such as $\frac{1}{2}$ for all soil values. This is not logical since the plain concrete section acts like a cantilever beam and the allowable ratio of offset to height is a function of the load, which for the lowest course is the soil pressure. This ratio is ordinarily limited by the tension in the concrete. Taylor, Thompson and Smulski[2] recommend a value of 60 pounds per square inch for this tension which gives ratios of projection to height varying from one-half to one for the usual range of soil pressures.

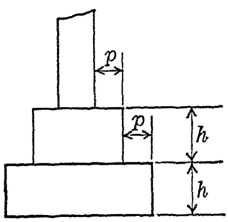


FIG. 92

121. Reinforced Concrete Footings. Experiments at the University of Illinois under the direction of Prof. Arthur N. Talbot[3] in 1908–12 are the basis of modern practice in footing design. These rules are formulated in Chapter XI–H of the Joint Committee Report.

Wall footings are simple cantilever slabs projecting on each side of the wall. Footings for isolated piers are square or rectangular slabs, concentric with the column, made uniform in thickness or with a sloping or stepped top. Such a slab is essentially a system of radial cantilevers projecting from the pier and is designed as a two-way cantilever with reinforcement parallel to the sides. All footings are relatively short heavily loaded beams and therefore shear and bond are of proportionately greater importance than in ordinary slabs.

[1] Journal, Boston Society of Civil Engineers, Vol. I.
[2] "Concrete Plain and Reinforced," Vol. I, page 481, 4th Edition.
[3] Bulletin No. 67, University of Illinois Engineering Experiment Station.

The shear on a vertical surface which is the extension of the face of the column down through the footing, has been much used as a criterion for the total depth required for footings under columns and piers. This shear is called punching shear and is often regarded as pure shear unaccompanied by tension. The allowed value commonly is 6 per cent of the ultimate strength of the concrete. The use of punching shear in design is not mentioned in the 1924 Joint Committee Report and it is quite generally recognized that former practice was unnecessarily conservative in this regard.

The experiments at the University of Illinois proved that the critical section for diagonal tension lies at a distance from the face of the wall or column equal to the depth from the top of the footing to the steel (d). For this reason shear as a measure of diagonal tension is measured on a vertical surface at this section. Since it is difficult and expensive to use diagonal tension reinforcement in footings the diagonal tension shear is kept low.

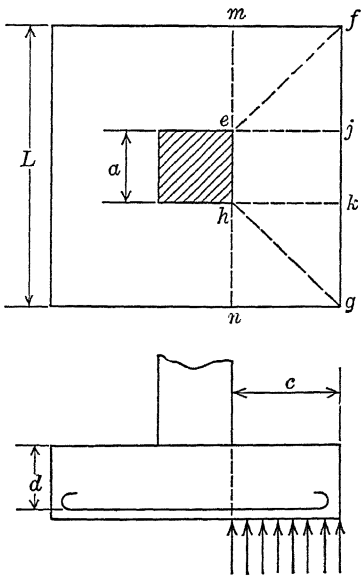


FIG. 93

In wall footings the critical section for bending is at the face of the wall. For column footings the critical section for moment also is at the face of the supported member except in the case when a metal base plate is used, when the theoretical section of maximum moment at the center is that to be used in computation. The bending moment to be resisted by the steel crossing the mn plane in the column footing shown in Fig. 93 is that due to the upward pressure on the trapezoid $efgh$. The assumption of uniform pressure over the whole area results in an unnecessarily large moment and it is proper to assume it non-uniform, taking the center of pressure on the area $ejkh$ a distance $c/2$ from mn and that on the triangles efj and hgk 0.6 c from mn. The moment

thus computed acts on a beam the limit of whose width is the width of the column or pier, a, plus twice the depth, d, of the footing, plus one-half the remaining width of the footing. The steel must be placed in this width with uniform spacing and the concrete stress computed on the exact shape of the concrete section having this width at the face of the pier. If the footing is wider than $a + 2\,d$ it may be desirable to use additional reinforcement outside the effective width. Each step in a stepped footing is a possible critical section.

Footings with sloping tops contain less concrete than the other types but are troublesome and uneconomical on account of the formwork for the top, except for slopes flatter than 2 to 1 which may be poured without forms by using dry mixtures. Builders prefer the stepped top or the single slab. The difficulty with the stepped footing is the tendency to pour the concrete in as many separate operations as there are steps. Unless the pouring proceeds with sufficient continuity so that the footing forms a single homogeneous mass of concrete each step must be designed independently.

Bond stresses usually are very high in footings. When straight rods are used for reinforcement, without hooks at the ends for anchorage, bond usually controls the selection of the steel, the amount required sometimes being twice that for resisting moment. The shear for bond computations is that at the face of the wall or pier. The allowable bond stress for unanchored bars in a spread footing is obviously less than in an ordinary beam since the tension in the bottom tends to pull the concrete away at right angles from the steel. The anchorage required by the Joint Committee with high bond stress is given by providing a 180 degree hook at each end of all bars.

The problem of transferring the load from a highly stressed column with spiral reinforcement to a footing or pier of plain concrete is often difficult. The loaded area can usually be taken as the whole cross-section of the column including fireproofing, and this loaded area is confined laterally by the mass of the pier or footing outside it. The more effective this lateral support the larger can be the local compression as is shown by the formula recommended by the Joint Committee. (See Art. 182, Appendix B.) Some designers make a practice to use spiral reinforcement in the top of piers and footings. The load from the column

reinforcement is transferred to the pier or footing by means of dowels or by use of a metal base plate. An equal number of rods of the same size as the column reinforcement is required for dowels. At the bottom of the column they must carry a stress equal to that in the column reinforcement in the upper part of the column and so must extend above the footing a distance sufficient to develop this stress in bond.

On account of varying soil conditions and the presence of different levels in the basement floor due to special constructions such as pits for elevators or machinery, it is not common for all footings in a building to have their bases at the same elevation. It is not convenient to have a variety of lengths for the basement columns and the necessity for this can be avoided by using plain concrete pedestals on all footings, extending to the underside of the basement floor. These piers have larger cross-sections than the columns they support and so their use results in more economical footings.

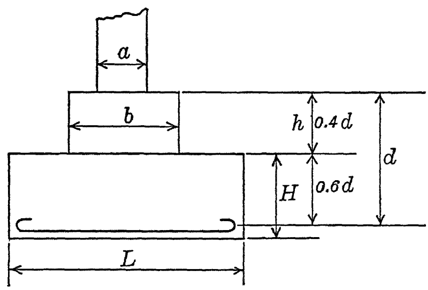


FIG. 94

The following proportions for stepped footings are recommended by Taylor, Thompson and Smulski[1] (Fig. 94):

Ratio Side of Pedestal to Side of Base a/L	Width of Top Block b
0.10	$0.36L$
0.15	$0.38L$
0.20	$0.40L$
0.25	$0.45L$
0.30	$0.50L$
0.35	0 $52L$
0.40	$0.55L$

[1] Concrete Plain and Reinforced, Vol. I, page 490, 4th Edition.

Ratio of depth of top block to total depth (d), 0.4. In the example that follows slightly different proportions are shown which are those used by the Kalman Steel Co., and given in their designers' manual, "Useful Data."

The width of the top block should be at least 4 inches wider all around than the pedestal in order that there may be room for setting the forms. Similarly the pedestal must be larger than the column which it supports.

122. Design of an Interior Column Footing. (Computation Sheets IF1–IF2.) The footing here designed is that required under a typical interior column of the flat slab building under study in these chapters. The data are assembled at the top of the first sheet. The sketch there shown was built up step by step as the computations proceeded.

This is a stepped footing, top block cast integrally with the base, and designed by the older method of punching shear. No pedestal is used. The calculations show that the average thickness for a footing of these proportions is about two-thirds of the total depth, a convenient bit of design data to assist in estimating the weight of footings rapidly. Great precision in the matter of weight is not necessary.

The steel is protected by about $3\frac{1}{2}$ inches of concrete as required by the Joint Committee.

The shear used in calculating the bond stress was the same as the upward load acting on the trapezoid used in determining the bending moment. It is assumed that this is the shear bringing bond stresses to one end of one set of rods.

There are two critical sections for moment in this footing, that at the face of the column and that at the edge of the cap or upper block. The second was first considered as the other was seen by inspection to be less critical. It is thus a matter of indifference in this case whether the cap be cast with the base or not since it satisfies the requirements for a plain concrete footing.

The moment involved in the judgment above noted as obtained by inspection is that computed in the second part of this design where the details for a footing with a sloped top were considered. In this design the slope of the top was limited to the 2–1 already stated to be about the greatest possible for the upper sloping surface without forms. But for that limitation the thickness at the

COMPUTATIONS FOR INTERIOR COLUMN FOOTING

Sheet IF1

Footing: Interior Column.

Loads:		Stresses:	Column:	
Live	166,000	$f_s = 16{,}000\#/\square''$	1-3 mix	6-1″ϕ
Dead	189,700	$f_c = 650\#/\square''$	$n = 10$	24″ diam. core.
	355,700#	$v = 40\#/\square''$ Diagonal Tension		
		$= 120\#/\square''$ Punching Shear		
		$u = 75\#/\square''$ [138]	Soil value $= 8000\#/\square'$	

Column: 28″ Diam.

Depth:

Soil Reaction under column —

$$\text{approx.} = 8000 \times \frac{\pi}{4} \times \overline{2.33}^2$$

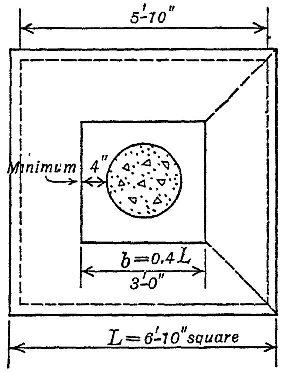
$$= 34{,}300\#$$

Total Punching Shear

$$355{,}700 - 34{,}300 = 321{,}000\# \pm$$

$$H + h = \frac{321{,}000}{120 \times \pi \times 28} + 4'' = 35''$$

$$H = 0.6 \times 35 = 21'' \quad h = 14''$$



Size:

Weight of footing: (approx.)

Footing of same volume and uniform thickness has total thickness

$$= H + \frac{\text{volume of cap}}{L^2}$$

$$= 0.6(H+h) + \frac{(0.4\,L)^2[(0.4)(H+h)]}{L^2}$$

$$= 0.664\,(H+h)$$

Wt. of base

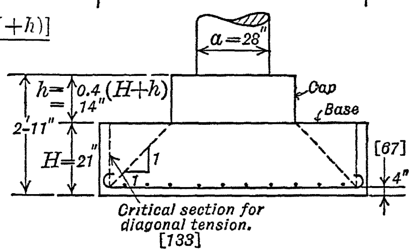
$$= 0.664\,(H+h)\left(\frac{150}{12}\right)$$

$$= 8.3\,(H+h)\#/\square'$$

$$= 8.3 \times 35 = 290\#/\square'$$

Net pressure allowable on footing

$$= 8000 - 300 = 7700\#/\square'$$



$$L^2 = \frac{355{,}700}{7700}$$

$L = 6.8'$ Make $L = 6'\text{-}10''$

$b \geqq 0.4 \times 6.83 \times 12 \geqq 33''$

$\geqq 28 + 2 \times 4 = 36''$

Make $b = 36''$

Diagonal Tension

$$\text{Net pressure} = \frac{355{,}700}{\overline{6.83}^2 = 46.7\square'} = 7600\#/\square'$$

Total shear on critical section

$$= 7600\,(46.7 - \overline{5.83}^2) = 96{,}500\#$$

$$\text{Unit shear } v = \frac{96{,}500}{(4 \times 70)\,(\tfrac{7}{8})\,(17)} = \frac{V}{bjd} = 23\#/\square'' \quad < 40\#/\square'' \qquad \text{O.K.}$$

edge would have been made at least 10 inches, following Art. 175 of the Joint Committee.

It is obvious from the previous computation that the shear stress is within proper limits. Had it been necessary to compute the diagonal tension shear it would have been necessary to calculate the depth of the section at a distance *d* from the face of the equivalent square column assumed to replace the round column in the calculations.

A footing about two-thirds as thick as this would have resulted from neglect of the punching shear, using the same limiting slope, and keeping the shear low enough to avoid using stirrups. For footings of common proportions the concrete stresses are very low but for careful design the exact shape of the section should be used in computing the steel ratio.

123. Footings on Piles. Concrete footings are used with both wood and concrete piles. The design of an interior footing with pile supports proceeds on exactly similar lines to those outlined above. The piles bring concentrated reactions to the base which must be brought into the footing by bearing and punching shear above and around each pile. The moments to be resisted are computed as before except that the forces acting through the piles are often assumed to be definitely located with lever arms measured perpendicularly to the face of the column or pier. As a matter of fact a variation of a foot in the location of individual pile heads is not uncommon in field work. For this reason some engineers design these footings as though the base pressure were uniformly distributed. The steel is placed in two layers and directions as before for a rectangular footing and not radially to each pile. The exact shape of the footing often deviates slightly from the rectangular but this is ignored in design.

124. Combined Footing. (Computation Sheets CF1–CF2.) The first step in designing a combined footing to carry an exterior and an interior column, like those previously designed, is to determine the area required for the base, which must be the sum of the areas required by the individual footings. This was first done without taking account of the weight of the footings which did not give accurate results since the weight of the combined footing turned out to be much greater per unit of bearing area than the interior footing. The first trial width of 3 feet 4 inches was found to be inadequate for the combination of live and dead loads on ac-

COMPUTATIONS FOR INTERIOR COLUMN FOOTING

Sheet IF2

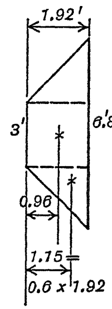
Steel

Moment at face of cap

$(7600 \times 3 \times 1.92)\ (0.96) = 42{,}200'\#$

$(7600 \times \overline{1.92}^2)\ (1.15) = \underline{32{,}200}$

$74{,}400'\#$



$$A_s = \frac{M}{f_s jd} = \frac{74{,}400 \times 12}{16{,}000 \times \frac{7}{8} \times 17} = 3.75\square''$$

$$19\text{-}\tfrac{1}{2}''\phi = 3.72\square''$$

Check of p & f_c:

$$p = \frac{3.75}{82 \times 17} = 0.0027 < 0.0077 \text{ limit for } 16{,}000 - 650$$

Bond: —

Shear at edge of cap $= 7600\ (46.7 - \overline{3.0}^2)\frac{1}{4}$

$V = 71{,}700\#$

$$u = \frac{vb}{\Sigma o} = \frac{V}{jd\Sigma o} = \frac{71{,}700}{\frac{7}{8} \times 17 \times 19 \times \frac{1}{2} \times \pi} = 162\#/\square''$$

$> 0.75 \times 100 = 75\#/\square''$ *allowable* (138)

Moment at face of Column. By inspection — computation unnecessary

All bars must be hooked both ends

Use $19\text{-}\frac{1}{2}''\phi$ *each way — All bars hooked*

Bearing on cap: —

$$f_c = \frac{355{,}700}{\frac{\pi}{4} \times \overline{28}^2 + 14 \times 6 \times 0.785} = 522\#/\square''$$

$= 682\square''$ ← *transformed area*

$$< 0.25 \times 2000\sqrt[3]{\frac{9}{4.3}} = 630\#/\square'' \text{ O.K.}$$

[182]

Design with Sloped Top.

Same as 1st design except sloping top: —

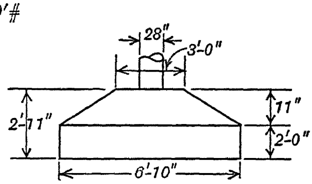
Steel: $M = \frac{w}{2}(a + 1.2\ c)c^2$ [176]

Here $a = \sqrt{\frac{\pi}{4} \times \overline{2.33}^2} = 2.1'$

$= \frac{7700}{2}(2.1 + 1.2 \times 2.4)\overline{2.4}^2$

$c = \frac{6.8 - 2.1}{2} = 2.4$

$= 110{,}000'\#$



$$A_s = \frac{M}{f_s jd} = \frac{110{,}000 \times 12}{16{,}000 \times \frac{7}{8} \times 31''} = 3.04\square''$$

$$16\text{-}\tfrac{1}{2}''\phi = 3.14\square''$$

Bond: $V = \frac{1}{4} \times 7700\ (\overline{6.83}^2 - \frac{\pi}{4} \times \overline{2.33}^2) = 81{,}700\#$

$$u = \frac{vb}{\Sigma o} = \frac{V}{jd\Sigma o} = \frac{81{,}700}{\frac{7}{8} \times 31 \times 16 \times \frac{1}{2} \times \pi} = 120\#/\square''$$

$> 75\#/\square''$

$\therefore$ *Use Hooked Ends*

count of the excessive pressure under one end. The depth was estimated and the weight of the footing found, which made it possible to find the actual width required, 3 feet 8 inches. It was necessary to check the dead unit pressure under the revised base area and compare with that under an independent interior column footing. Had it not happened that the two agreed a revision might have been necessary.

The shear and moment curves were drawn for this foundation beam, something which is often necessary in work of this type. The dotted portions of these curves show the results that are obtained when the width of the columns is neglected.

The design of the stirrups offers no new problem. Several loops would be used with relatively heavy material, such as $\frac{5}{8}$-inch rounds. For the main reinforcement the minimum number of bars was chosen of the largest size, so that when in place in the top of the footing there would be as little interference as possible with the pouring of the concrete.

125. Connected Footings. In place of a combined footing in a situation where a projection beyond the wall is impossible, use is often made of a wall column footing carrying the wall column near its outside edge and connected with the nearest interior column footing by a beam, called a strap, which counterbalances the eccentricity and maintains uniform pressure under the outer footing. A strap of this sort is illustrated in Fig. 95 and the design of another is carried through in outline on Computation Sheet SF1.

In this design the same columns and loads were taken as before, and the common condition assumed of a considerable distance between the bottom of the footings and the basement floor. Caisson piers were used, of plain concrete.

The location of the exterior pier was found by a series of trial computations only the last of which is shown. This pier was enlarged at the bottom to give a rectangular bearing area entirely within the lot. If unequal settlement of the two piers occurs there is possibility of tension developing in the bottom of the strap. To provide for this contingency Taylor, Thompson and Smulski[1] recommend the use of one-third as much steel in the bottom as in the top. The details of the computation follow the same argument as previously and no comment is needed.

[1] "Concrete Plain and Reinforced," Vol. I, 4th Edition, page 537.

COMPUTATION FOR COMBINED FOOTING

Sheet CF1

Base Area

Loads:	*Int'r Col.*	*Wall Col.*	*Total*
Live	*166,000#*	*83,000#*	*249,000*
Dead	*189,700*	*141,200*	*330,900*
Total	*355,700#*	*224,200#*	*579,900*

$f_s = 16{,}000\#/\square''$ $f_c = 650\#/\square''$
$v = 120\#/\square''$ $u = 100\#/\square''$
$n = 15$
Soil Value $= 8000\#/sq.\ ft.$

Int'r Col. Footing: *Bearing under dead column load* $= \dfrac{189{,}700}{6.83^2} = 4050\#/\square'$

Wall Column Footing: *Area required* $= \dfrac{141{,}200}{4050} = 35.0\square'$

Under total load — Bearing $= \dfrac{224{,}200}{35.0} = 6400\#/\square' < 8000\#/\square'$

Combined Footing:

Base area $= 46.7 + 35.0 = 81.7\square'$

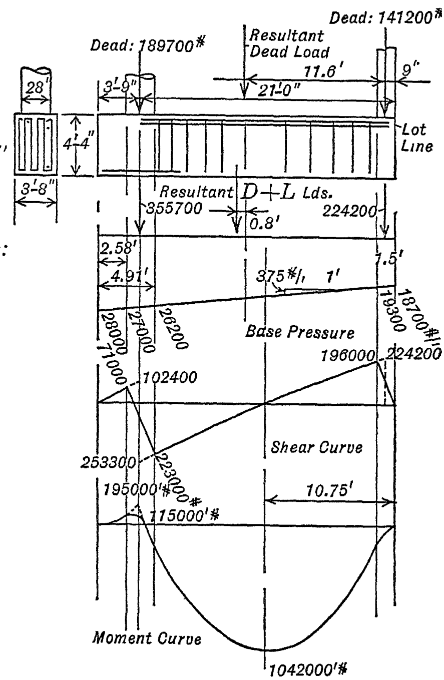
Resultant dead load acts

$$\frac{189{,}700 \times 20.25}{330{,}900} = 11.6'$$

from centre of wall column

$\therefore$ *Length of Footing* $=$
$2\,(11.6 + 0.75) = 24.7' = 24'\text{-}9''$

$\therefore$ *Width* $= \dfrac{81.7}{24.75} = 3.3' = 3'\text{-}4''$



Stresses due to Dead + Live Loads:

Resultant acts

$$\frac{355{,}700 \times 20.25}{579{,}990} = \begin{array}{r} 12.4' \\ \underline{11.6} \end{array}$$

Eccentricity $= 0.8' = e$

Pressure:—

$$f = \frac{P}{h}\left(1 \pm \frac{6\,e}{h}\right) \text{ lbs./ft.}$$
$$= \frac{579{,}900}{24.75}\left(1 \pm \frac{6 \times 0.8}{24.75}\right)$$
$$= 23{,}400\ (1 \pm 0.20)$$
$$= 28{,}000 \text{ and } 18{,}700\#/'$$

Maximum unit pressure

$$= \frac{28{,}000}{3.3} = 8500\#/\square' > 8000$$

Footing should be wider
(See next sheet)

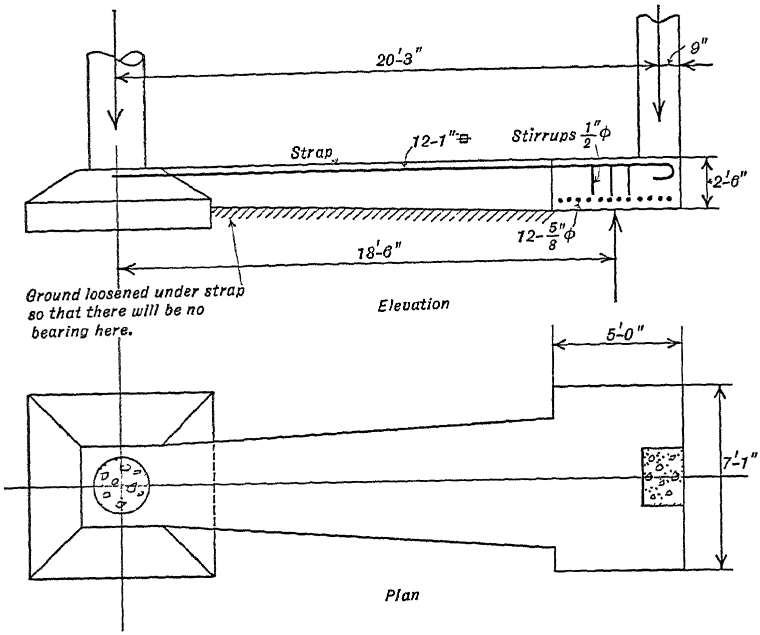


FIG. 95

126. Foundation Walls. When the spacing of the wall columns does not exceed about 18 feet it is economical to support the first story columns and floor beams on a heavy foundation wall at least 12 inches and better 16 inches or more in thickness. The building code of the locality sometimes governs the thickness. Analyzed as a structural member such a wall is a continuous beam with a uniform upward load and a series of concentrated downward loads. When the depth of the wall is greater than about 10 feet the effect of the lateral earth thrust should be considered. A foundation wall is usually of uniform thickness but is sometimes built with enlarged sections, called pilasters, under the column.

In the building designed in this and the previous chapters the wall columns were carried down to either an independent or a combined footing, and a light basement wall constructed between them to hold back the earth. It is not practicable to attempt to pour reinforced walls that are thinner than 12 inches unless the forms are raised gradually to the full height which is not the usual procedure. These walls are designed as slabs spanning horizon-

COMPUTATION FOR COMBINED FOOTING

Sheet CF2

Depth and Width

Assuming 3′-4″ width and using $v = 120\#/\square''$ *with diagonal tension reinforcement:*

$$d = \frac{V}{bjv} = \frac{223{,}000}{40 \times \frac{7}{8} \times 120} = 53''$$

$$\begin{array}{l} 4'' \text{ cover} \\ \hline 57'' \text{ total.} \end{array}$$

$$\begin{array}{lr} \text{Dead weight of footing} = \dfrac{57}{12} \times 150 = & 700\#/\square' \pm \\ \text{Maximum from column loads} & 8500 \\ \hline \text{Maximum total pressure} & 9200 > 8000\#/\square' \text{ allowed} \end{array}$$

$\therefore$ *Make width* $= 28{,}000 \div (8000 - 700) = 3.83'$ *Use 3′-8″*

Make depth: $d = \dfrac{40}{44} \times 53 = 48''$

$$\begin{array}{l} 4'' \text{ cover} \\ \hline 52'' \text{ total @ } 650\#/\square' \end{array}$$

Steel:

Top: $A_s = \dfrac{M}{f_s jd} = \dfrac{1{,}042{,}000 \times 12}{16{,}000 \times \frac{7}{8} \times 48} = 18.65\square''$ $\quad 12\text{-}1\tfrac{1}{4}\square = 18.7\square''$ *in top*

Bottom: $A_s = 18.65 \times \dfrac{115}{1042} = 2.06\square''$ $\quad 2\text{-}1\tfrac{1}{4}\square = 3.1\square''$ *in bottom under interior column.*

Bond: Number of bars required in top at columns:

$$\left(\Sigma o = \frac{vb}{u}\right) \quad N = \frac{120 \times 44}{4 \times 1\frac{1}{4} \times 100} = 11 \text{ bars}$$

Number required in bottom at interior column:

$$N = 11 \times \frac{71{,}000}{223{,}000} = 4 \text{ bars}$$

Check of Base Pressures

$$\begin{array}{lr} \text{Base Pressure under dead load } \dfrac{330{,}900}{3.67 \times 24.75} & = 3650\#/\square' \\ \text{Weight of footing} & 650 \\ \hline \text{Unit pressure — dead load} & = 4300\#/\square' \end{array}$$

For Interior Footing unit pressure — dead load $= 4340 = 4050 + 290$

tally from column to column or vertically from the support offered by the floor slab to a horizontal beam running between columns, or as a two-way slab. Sometimes the lower support is assumed to be furnished by the basement floor but this is not good practice unless special precautions are taken to see that this floor is cast in sufficient season to be available when the supports of the wall are removed and the earth fill is in place. The connection between the wall and the first floor is often broken by windows or vent openings. In many of these cases the wall can be made to span vertically (which is the most economical for basement depths up to about 12 feet), being supported at the top by a horizontal beam made by reinforcing the upper section of the wall with horizontal bars.

Walls of this type may be built to provide their own vertical support with a cantilever wall footing if necessary, or with steel top and bottom to carry the stresses due to beam action in spanning between supporting wall columns.

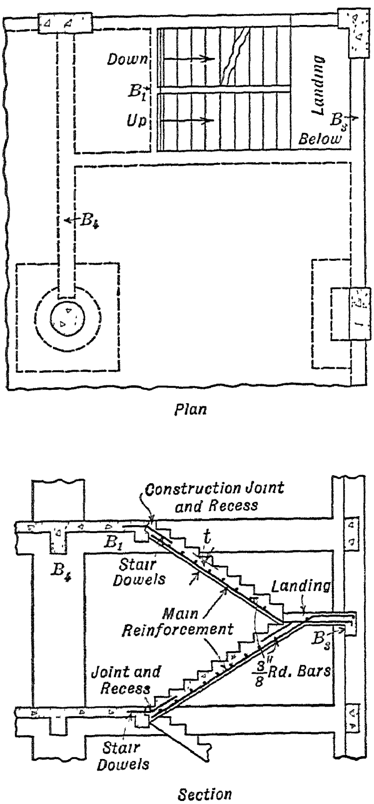
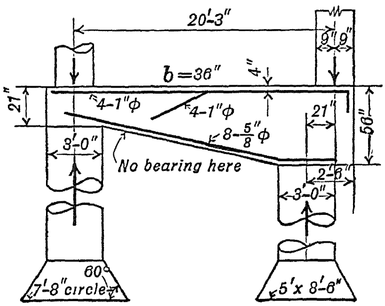


FIG. 96

127. Stairs. No treatment of building design however brief would be complete without some description of stairs. In reinforced concrete construction stairs are inclined slabs with triangular treads formed on the upper surface. In Fig. 96 is shown an arrangement suitable for the flat slab building in these pages. The stairs shown are designed as simply supported slabs with a span equal to the horizontal distance between the floor beam, $B1$, and the landing beam, $B3$. The effective thickness of the in-

COMPUTATION FOR FOOTING WITH STRAP

Sheet SF1



Loads. —

Interior Column:

Dead	*190,000*
Live	*166,000*
Caisson } *Strap* }	*26,000*
	382,000#

Wall Column:

Dead	*141,000*
Live	*83,000*
Caisson } *Strap* }	*27,000*
	251,000#

Uplift: *Dead,* $141{,}000 \times 1.75 \div 18.5 = 13{,}000$

Live, $83{,}000 \times$ " " $= 8{,}000$

$21{,}000\#$

Interior Column:

Total load on foundation: $382 - 21 = 361{,}000\#$

Dead " " " $216 - 13 = 203{,}000\#$

Wall Column Footing:

Total load on foundation: $251 + 21 = 272{,}000\#$

Dead " " " $168 + 13 = 181{,}000\#$

Areas of Footings. —

Interior: $\dfrac{361{,}000}{8000} = 45.1$ *sq. ft.* *7'-8" diam. Bell of Interior Caisson*

$\dfrac{203{,}000}{46} = 4400\#/\text{sq. ft.}$ = *unit pressure under dead load only.*

Exterior:

$\dfrac{181{,}000}{4400} = 41.2$ *sq. ft.* $\dfrac{272{,}000}{42.5} = 6400$ *under total load* < 8000 *O.K.*

5' × 8'-6" caisson base exterior.

Shear: At face of Wall Column: (strap neglected): $v = 120\#/\square''$ *(stirrups not shown)*

$$d = \frac{197{,}000}{36 \times \frac{7}{8} \times 120} = 52'' \quad \textit{Make total depth 56''}$$

Interior: Without stirrups: $v = 40\#/\square''$

$$d = \frac{21{,}000}{36 \times \frac{7}{8} \times 40} = 17'' \quad \textit{Total depth} = 21''$$

Reinforcement

$$M = 21{,}000 \times 17' = 357{,}000'\#$$

$$A_s = \frac{357{,}000 \times 12}{16{,}000 \times \frac{7}{8} \times 52} = 5.90\square'' \; 8-1''\phi$$

$$p = \frac{6.25}{36 \times 52} = 0.0034 < 0.0077$$

$\therefore$ *O.K.*

clined portion of the slab is the distance d shown in Fig. 97. In computing the dead weight per horizontal foot for use in design the inclined part should be considered to have the total thickness of t_1: the treads add 40 to 50 pounds per horizontal square foot.

The dimensions for rise and tread vary in any structure but should be kept as closely the same as possible for all stairs in any single building. The dimensions shown in Fig. 97 are common.

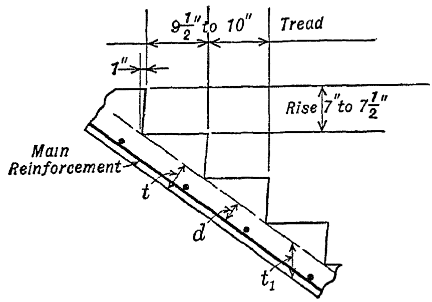


FIG. 97

Stairs are usually poured after the floors are finished. The connection to the supporting floor beams is by means of recessed joints and dowels as shown on Fig. 96. The arrangement of the reinforcement in the stair shown in section should be studied. The main rods do not run from the lower floor beam to the landing beam $B3$ continuously in the bottom of the slab. The bars from the inclined slab extend into the top of the landing slab and run over to the landing beam where they should be hooked. The stiffness of this beam brings negative moment into the landing. The bars in the bottom of the landing slab opposite this flight extend in their turn into the top of the inclined slab and then bend down for a short distance. Were bars extended continuously in the bottom of this slab around this reëntrant angle they would exert a strong bursting pressure on the concrete as they tended to straighten under stress. For temperature reinforcement it is customary to place a single $\frac{3}{8}$-inch round bar under each tread.

CHAPTER XVI

ARCHES

128. The first arches were curved structures with converging reactions, made of wedge-shaped stone blocks, so proportioned that the line of action of the resultant normal stress at any section was wholly compression. The modern arch of plain concrete likewise must be designed so that the line of resistance lies within the middle third. Reinforced with steel somewhat smaller sections may be used since such an arch can carry a large bending moment with tension in one face.

The most common type of reinforcement is a series of longitudinal bars in top and bottom of the section, following the curve of the arch ring. Large concrete arches often employ a structural steel arch as reinforcement. The great advantage of this is that it is possible to design the steel arch to carry the weight of the forms and the wet concrete of the rib, thus avoiding the use of false work and making more economical use of the steel. Arches thus reinforced are often made with hinges at the supports and sometimes with one also at the crown. The more common type of concrete arch is built without hinges and the discussion of this chapter is limited to that variety.

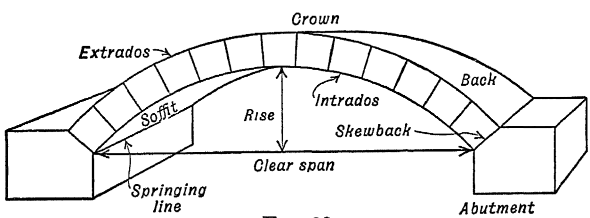


FIG. 98

Hinges serve to fix the point of application of the force acting through them and to eliminate bending. The three-hinged arch is statically determinate and is not subject to stress due to temperature changes and to settlement of the abutment. The hingeless type, like all indeterminate structures, is acted upon by both these influences.

The technical names of the principal parts of an arch are given in Fig. 98. An arch may consist of a single ring called the barrel

or of two or more parallel ribs. The haunch is that portion of the rib or barrel midway between crown and springing. The spandrel is the space between the upper surface of the arch, the back, and the roadway. Barrel arches are of two sorts: the filled spandrel with the roadway built on earth filled in above the arch which is built with side or spandrel walls; the open spandrel with columns or cross walls built on the back of the arch to carry the beams and slab of the floor system.

129. Arch Analysis. A hingeless arch is indeterminate to the third degree, there being six unknown elements of reaction as shown by Fig. 99. In order to analyze the stresses it is necessary to find three equations in addition to the three of equilibrium for a non-concurrent co-planar force system. This may be done in various

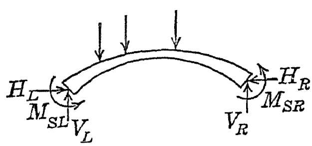


FIG. 99

ways by consideration of the relations between the elastic deformations of the arch and the internal and external stresses. The most convenient and simple method of the many available is probably that of Least Work presented by Professor C. M. Spofford in his "Theory of Structures." There are two great advantages to this method: first, its ease and accuracy of application, and second, the fact that it is the application of a simple and familiar theorem and so involves the minimum of new detail for the learner.

In order to apply this or any other method of analysis based on the elastic properties of the arch it is first necessary to assume a section of known dimensions. The labor of an exact analysis is so great that it is desirable to have available some simple method for arriving at a trial section that will require little if any change upon closer study. Mr. Victor S. Cochrane[1] has developed a simple and speedy method of applying the elastic theory to

[1] "Design of Symmetrical Hingeless Arches," Victor S. Cochrane, in Proceedings of the Engineers' Society of Western Pennsylvania, Nov., 1916. Convenient curves for the solution of Mr. Cochrane's equations are given in "Concrete Engineers' Handbook," Hool & Johnson.

symmetrical hingeless arches which is somewhat approximate but sufficiently accurate for the final design of structures of moderate span where great refinement is not attempted. Mr. Charles S. Whitney[1] has prepared a similar adaptation of the elastic theory, equally simple and rapid of use, which even exceeds the longer methods in accuracy when applied to arches proportioned in accordance with his fundamental equations. Mr. Whitney's method is presented in part only and very briefly in a later article.

Stone arches are usually analyzed by the line of thrust or static method, of which there are several variations.[2] The most favored of these assumes that for any loading the crown thrust is the minimum consistent with equilibrium. For symmetrical loading on arches with a central angle[3] of not more than 90 to 120 degrees this criterion would indicate a line of resistance passing through the upper middle third point at the crown and the lower middle third point at the springing. If this line of resistance lies within the middle third at all sections the arch is considered to be satisfactory for this loading. The criterion for unsymmetrical loading seems to amount to this: if a line of resistance can be drawn within the middle third at all points the arch is satisfactory.

The static method is used by many engineers for the preliminary and even for the final design of monolithic arches of reinforced concrete. Those who are impressed with the uncertainties of the elastic theory due to doubtful preliminary assumptions consider this practice satisfactory for structures of moderate span. The two methods give results in reasonably close agreement. The static method takes no account of temperature stresses and abutment movements.

130. Proportions. The axis (the center line of the arch ring) should conform very closely to the dead load line of resistance,

[1] "Design of Symmetrical Concrete Arches," Charles S. Whitney, Transactions, Am. Soc. C. E., Vol. LXXXVIII, page 931, 1925. Complete tables and diagrams for this method are given in "Concrete Designers' Manual," 2nd Edition, Hool & Whitney. A valuable paper by Mr. Whitney, on the "Analysis of Continuous Concrete Arch Systems," was printed in the Proceedings, Am. Soc. C. E., May, 1926.

[2] The principal methods are discussed simply and clearly in Baker's "Treatise on Masonry Construction."

[3] For arches with a central angle equal to or greater than these limits a plane of maximum rupture develops at about this location in the ring which should be treated as the actual springing line section.

thus eliminating so far as possible bending under the permanent and major part of the load and reducing stresses to the least possible. The shape of this curve would be parabolic were the dead load uniformly distributed. Since the intensity of this load increases towards the ends of the span, the theoretical curve lies above a parabola and in preliminary computations of weight for arches of moderate ratio of rise to span may be assumed as the segment of a circle through the crown and the two springings. With the dead weight of the structure known with reasonable accuracy, a satisfactory curve for the axis may be obtained by passing an equilibrium polygon through the crown and springing points of the axis. This assumes no bending moment at these two sections, which is not far from the truth if the arch is constructed in such a manner that the shrinkage of the concrete on setting has no effect. Shrinkage is an uncertain factor which is not computed except in the case of long span structures where construction is carried on in such a manner as to eliminate this element so far as possible.

There is no thoroughly satisfactory way of estimating in advance of computation the proper thickness of the arch ring at the crown. Comparison with existing arches of admittedly good design is an excellent course.[1] Several formulas have been proposed but none is known to the writers which includes all the factors. The following empirical expression devised by Mr. F. F. Weld[2] gives very conservative results which may be useful in preliminary weight computations.

$$d_c = \sqrt{L} + \frac{L}{10} + \frac{w_L}{200} + \frac{w_c}{400}$$

where d_c = crown thickness in inches;
L = clear span in feet;
w_L = live load in pounds per square foot;
w_c = dead load at crown in pounds per square foot.

The thickness of the arch at the springing is usually about twice or more that at the crown, with a range from about 1.5 to 3.

[1] The greatest compendium in this field is "Grandes Voûtes," in six large volumes by Paul Séjourné, Professeur à l'École Nationale des Ponts et Chaussées, Paris, and Ingenieur en Chef des Ponts et Chaussées. Professor Séjourné gives pictures and the details of design and construction of most of the masonry and concrete arches of 40 metre span and more, built in all countries up to 1916.

[2] "Engineering Record," Nov. 4, 1905.

131. Loads. The live loads used in the design of an arch rib or barrel are the same as those for any other highway or railway bridge. Many arches are designed for a uniform live load on the deck, of sufficient intensity per square foot to be equivalent to the maximum concentrations expected. For a railway arch this is the distributed weight of locomotives and train; for highway arches the distributed weight of trolley cars and from 50 to 150 pounds per square foot on the roadway, 40 to 100 pounds per square foot on the sidewalk, — figures that vary with the type of traffic and the length of span.

Impact allowances vary from about 15 to 30 per cent for open spandrel arches; for filled spandrel structures a less allowance is made or the item is omitted entirely.

The dead loads on an arch may be computed from these data: concrete at 150 pounds per cubic foot; earth at 100 or 120 pounds per cubic foot, the latter figure being used when there is possibility that the fill may be saturated. It is customary to ignore the horizontal component of earth thrust on the back of the arch ring.

132. Temperature Stress. The resistance to movement offered by the fixed connection to the abutment results in temperature stresses in hingeless arches. In the northern part of the United States it is customary to proportion arches so that the stresses set up by a range of 80° F. are within proper bounds, the temperature drop being commonly taken as larger than the rise.

Expansion joints must be provided in the side walls of filled-spandrel arches and in the floor system of open-spandrel structures.

Example 51. The preliminary proportioning of an arch is illustrated in a general way by Fig. 100, which shows the half elevation of a filled spandrel structure. The material above the arch ring weighs less per cubic foot than the masonry and so for convenience of graphic solution the dash line *ab* is drawn showing the height at which this material would stand were it compressed to the same density as the masonry. The total dead load is then divided into any desired number of parts as shown by the vertical dotted lines, the number depending on the span, 8 to 10 being common. For simplicity 3 only were used in the figure. It is equally correct to take the left limit of load through *c* instead of as shown. The line of action of each load division is found graphically and the magnitude computed. An equilibrium polygon is then passed through crown and springing by the familiar method shown. The crown thrust is horizontal since the arch and loading are symmetrical.

It should be noted that the vertical divisions shown are independent of any divisions made of the arch ring for purposes of analysis.

By use of the data in Art. 133 the axis may be plotted directly without the drawing of an equilibrium polygon.

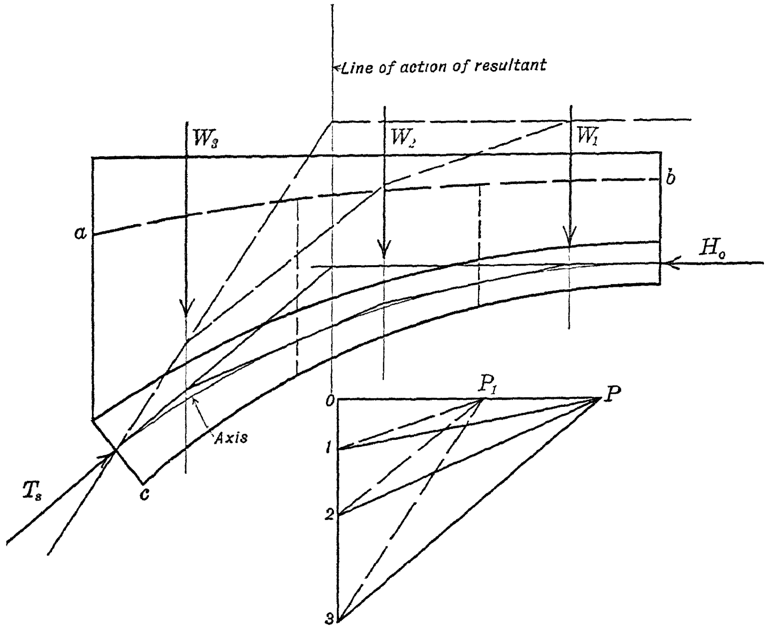


FIG. 100

133. Whitney's Method.[1] In order to obtain formulas by actual integration mathematical expressions for the arch axis and the variation of rib thickness are needed. Using those of Strassner[2] in the general elastic equations for symmetrical hingeless arches Mr. Whitney derived definite formulas for reactions. From these he constructed influence lines, tables and diagrams which eliminate all involved calculation. With these helps it is a quick and simple matter to study the effect of variations in crown and springing line thickness and determine the most economical and satisfactory section.

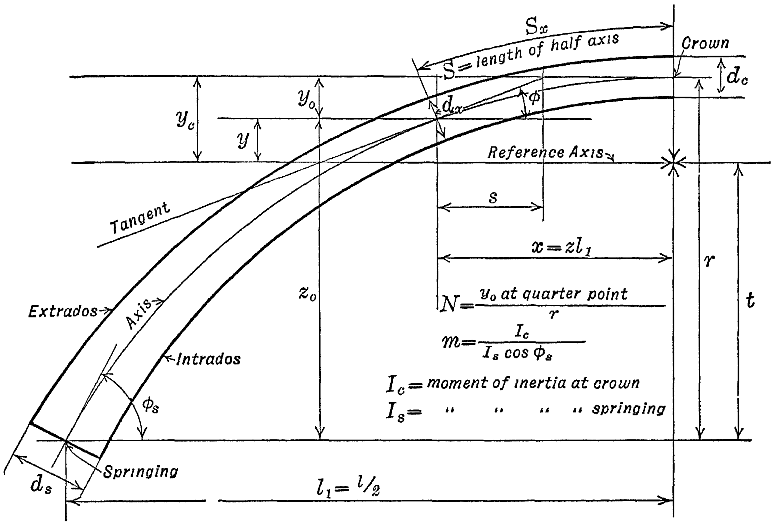
The best axis for any arch within their range may be determined by the values given in Tables 2–3 (at end of this article) which give the coördinates of points on the curve and the intercepts neces-

[1] Transactions Am. Soc. C. E., 1925.

[2] "Neuere Methoden zur Statik der Rahmentragwerke und der elastischen Bogentrager."

sary to establish the slope of the axis at each tabulated point. In addition to the information regarding notation given in Fig. 101 the following are used:

w_s = dead load per linear foot at springing;
w_c = dead load per linear foot at crown.



Notation for Arch Analysis
FIG. 101

The formula which is the basis of these tables was derived for an arch with distributed loads. To use these data for one with an open spandrel w_c and w_s should be computed as though the actual loads supported by the arch, including the spandrel columns, were carried to the arch by a spandrel wall or filling.

The variation of rib thickness is given by the following equation (see Fig. 101):

$$d_x = d_c c \sqrt[6]{1 + \tan^2 \phi} \tag{47}$$

where

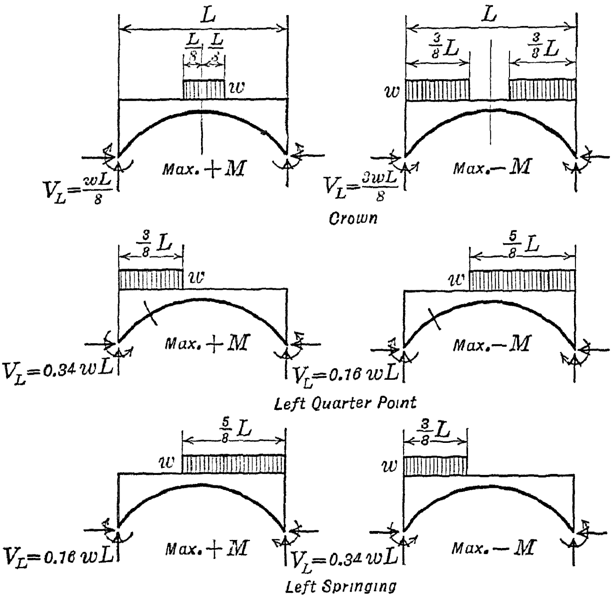
$$c = \frac{1}{\sqrt[3]{1 - (1 - m)z}} \cdot \tag{47a}$$

Tables 4–5 are given to facilitate the solution of these expressions.

Two factors are sufficient to determine definitely any arch whose proportions conform to these data: N, the ratio y_0/r at the

quarter-point, and m, the ratio $I_c/I_s \cos \phi_s$, factors which designate respectively the form of the arch axis and the form of the rib.

Space forbids the reproduction of the influence lines which are presented in Mr. Whitney's paper, which show clearly the position of the loading for maximum stress at the critical sections of an arch, crown, springing and quarter-point. With the aid of these influence lines Figs. 104 to 109 (at end of this article) were pre-



Position for Maximum Stress.
Live Load = w lbs. per ft. on rib.

Fig. 102

pared, which give coefficients for obtaining the maximum values of live load moment at the crown, quarter-point and springing, with the corresponding horizontal component of crown thrust. In order to obtain the arch stresses it is necessary in addition to know the vertical component of reactions for each loading. In the absence of the influence lines approximate values of these may be taken from Fig. 102.

Temperature stress may be computed by aid of Fig. 110 which gives the value of the horizontal reaction induced by temperature variation. A fall in temperature causes the arch to contract and

tend to draw away from the abutments, setting up the horizontal pull and negative springing line moment shown in Fig. 103*a*. The forces acting on the half-arch are shown in Fig. 103*b* with the pull and the positive moment at the crown replaced by a single force acting a distance y_c below the crown. Fig. 110 gives simply the value of H_T and Table 6, which gives the values of y_c, is printed here.

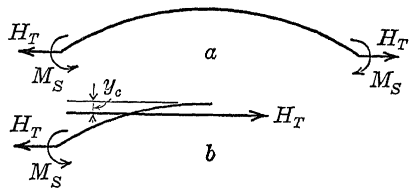


FIG. 103

Any load, live or dead, causes compression in an arch rib and consequently a general shortening of the fibers. This shortening sets up a stress of exactly the same sort as a fall of temperature as shown in Fig. 103 and must be combined with the live and dead load stresses to give the true stress. This effect is called rib-shortening. The amount of horizontal pull on the abutments thus induced may be expressed as

$$H_{RS} = \text{approximately } -Hu' \tag{48}$$

where H is the thrust produced by the loading, live or dead or the two together, and

$$u' = \frac{I_c}{A_c C_m' C r^2}\,. \tag{48a}$$

Here A_c is the cross-sectional area at the crown and C and C_m' are coefficients given by Table 7 and Fig. 111 respectively.

Mr. Whitney makes the following statement which should be carefully noted by all making use of this material in design. "The data in the paper can be used by one who is not an expert in arch analysis, provided the designing is done under the direction of an engineer who is thoroughly familiar with both the practical and theoretical aspects of arch design. The design of arches should not be entrusted to a novice. The writer's paper is limited to the mathematical considerations of design and does not attempt to treat the equally important practical considerations. No tables or diagrams should be used without a thorough knowledge of their basis and limitations." It is hoped that the reproduction of this portion of Mr. Whitney's paper will lead many to make a thorough study of this very important contribution to the literature of arches.

TABLE 2

ARCH AXIS COÖRDINATES

$\frac{w_s}{w_c}$	N	Values of $\frac{y_0}{r}$										
		Point 0 (Springing line)	Point 1	Point 2	Point 3	Point 4	Point 5 (Quarter-point)	Point 6	Point 7	Point 8	Point 9	Point 10 (Crown)
		$z = 1.0$	$z = 0.9$	$z = 0.8$	$z = 0.7$	$z = 0.6$	$z = 0.5$	$z = 0.4$	$z = 0.3$	$z = 0.2$	$z = 0.1$	$z = 0.0$
1.000	0.250	1.000	0.8100	0.6400	0.4900	0.3600	0.2500	0.1600	0.0900	0.0400	0.0100	0.0000
1.167	0.245	1.000	0.8060	0.6339	0.4834	0.3539	0.2450	0.1563	0.0878	0.0390	0.0098	0.0000
1.347	0.240	1.000	0.8019	0.6277	0.4769	0.3178	0.2400	0.1527	0.0856	0.0380	0.0095	0.0000
1.543	0.235	1.000	0.7977	0.6214	0.4701	0.3416	0.2350	0.1493	0.0835	0.0370	0.0092	0.0000
1.756	0.230	1.000	0.7934	0.6151	0.4632	0.3353	0.2300	0.1458	0.0814	0.0360	0.0090	0.0000
1.987	0.225	1.000	0.7890	0.6087	0.4563	0.3293	0.2250	0.1424	0.0792	0.0350	0.0087	0.0000
2.240	0.220	1.000	0.7847	0.6022	0.4494	0.3229	0.2200	0.1386	0.0771	0.0340	0.0085	0.0000
2.514	0.215	1.000	0.7801	0.5957	0.4425	0.3167	0.2150	0.1351	0.0749	0.0330	0.0082	0.0000
2.814	0.210	1.000	0.7755	0.5891	0.4355	0.3104	0.2100	0.1315	0.0728	0.0320	0.0080	0.0000
3.141	0.205	1.000	0.7709	0.5824	0.4285	0.3041	0.2050	0.1281	0.0707	0.0310	0.0077	0.0000
3.500	0.200	1.000	0.7662	0.5757	0.4215	0.2978	0.2000	0.1245	0.0686	0.0300	0.0074	0.0000
3.893	0.195	1.000	0.7615	0.5689	0.4145	0.2914	0.1950	0.1209	0.0665	0.0290	0.0072	0.0000
4.324	0.190	1.000	0.7567	0.5621	0.4073	0.2851	0.1900	0.1176	0.0644	0.0281	0.0070	0.0000
4.801	0.185	1.000	0.7518	0.5551	0.4000	0.2787	0.1850	0.1140	0.0623	0.0271	0.0067	0.0000
5.321	0.180	1.000	0.7469	0.5481	0.3927	0.2723	0.1800	0.1106	0.0602	0.0262	0.0065	0.0000
5.898	0.175	1.000	0.7420	0.5410	0.3854	0.2659	0.1750	0.1072	0.0582	0.0252	0.0062	0.0000
6.536	0.170	1.000	0.7367	0.5337	0.3781	0.2595	0.1700	0.1037	0.0562	0.0243	0.0059	0.0000
7.244	0.165	1.000	0.7313	0.5264	0.3707	0.2531	0.1650	0.1002	0.0541	0.0233	0.0057	0.0000
8.031	0.160	1.000	0.7259	0.5190	0.3632	0.2466	0.1600	0.0968	0.0521	0.0224	0.0055	0.0000
8.906	0.155	1.000	0.7205	0.5116	0.3557	0.2399	0.1550	0.0934	0.0501	0.0215	0.0053	0.0000
9.889	0.150	1.000	0.7151	0.5040	0.3480	0.2332	0.1500	0.0901	0.0483	0.0206	0.0050	0.0000

TABLE 3

INTERCEPTS TO DETERMINE POSITION OF TANGENTS TO ARCH AXIS

$\frac{w_s}{w_c}$	N	Values of $\frac{s}{l_1}$										
		Point 0 (Springing line)	Point 1	Point 2	Point 3	Point 4	Point 5 (Quarter-point)	Point 6	Point 7	Point 8	Point 9	Point 10 (Crown)
		$z = 1.0$	$z = 0.9$	$z = 0.8$	$z = 0\ 7$	$z = 0.6$	$z = 0.5$	$z = 0.4$	$z = 0.3$	$z = 0.2$	$z = 0.1$	$z = 0.0$
1 000	0 25	0 5000	0 4500	0 4000	0 3500	0 3000	0 2500	0 2000	0 1500	0 1000	0 0500	∝
1 347	0 24	0 4743	0 4311	0 3865	0 3410	0 2943	0 2466	0 1982	0 1492	0 0998	0 0500	∝
1.756	0 23	0 4503	0 4129	0 3734	0 3319	0.2884	0.2432	0 1965	0 1485	0 0996	0 0500	∝
2 240	0 22	0 4279	0.3957	0 3607	0 3229	0 2825	0.2397	0.1946	0.1478	0 0993	0 0499	∝
2 814	0 21	0 4070	0 3792	0 3482	0 3140	0 2765	0 2360	0 1926	0.1469	0 0991	0 0499	∝
3 500	0 20	0 3872	0 3634	0 3360	0 3051	0 2704	0 2323	0 1905	0 1459	0 0988	0 0498	∝
4 324	0 19	0 3686	0 3482	0 3241	0 2962	0 2643	0 2285	0 1884	0 1449	0 0985	0 0498	∝
5.321	0.18	0 3510	0.3335	0 3125	0 2875	0.2583	0 2246	0 1862	0 1439	0 0983	0 0497	∝
6.536	0 17	0 3342	0 3194	0 3011	0 2787	0 2520	0 2206	0 1840	0 1429	0 0980	0 0497	∝
8 031	0 16	0 3182	0 3058	0 2899	0 2700	0 2457	0.2164	0 1817	0 1419	0 0976	0 0496	∝
9 889	0.15	0 3030	0 2926	0.2788	0 2613	0 2393	0 2121	0 1792	0 1407	0 0972	0 0496	∝

TABLE 4

DETERMINATION OF RIB THICKNESS — VALUES OF $\frac{l^2}{r^2}$ TAN² ϕ

$\frac{w_s}{w_c}$	N	Values of $\frac{l^2}{r^2} \tan^2 \phi$										
		Point 0 (Springing line)	Point 1	Point 2	Point 3	Point 4	Point 5 (Quarter-point)	Point 6	Point 7	Point 8	Point 9	Point 10 (Crown)
		$z = 1.0$	$z = 0.9$	$z = 0.8$	$z = 0.7$	$z = 0.6$	$z = 0.5$	$z = 0.4$	$z = 0.3$	$z = 0.2$	$z = 0.1$	$z = 0.0$
1 000	0 25	16 000	12 960	10 240	7 840	5 760	4 000	2 560	1 440	0 640	0 160	0
1 347	0 24	17 780	13 846	10 552	7 821	5 583	3 788	2 377	1 316	0 580	0 145	0
1 756	0 23	19 722	14 769	10 852	7 791	5 403	3 578	2 200	1 199	0 523	0 130	0
2 240	0 22	21 841	15 730	11.149	7.748	5 220	3 371	2 029	1 088	0 468	0 116	0
2 814	0 21	24 150	16.731	11 443	7 694	5 034	3 166	1 864	0 983	0 417	0 102	0
3 500	0 20	26 676	17 780	11 734	7 629	4 844	2 964	1 706	0 883	0 369	0 089	0
4 324	0 19	29 441	18 885	12 020	7 552	4 650	2 765	1 554	0 789	0 325	0 076	0
5 321	0 18	32 476	20 047	12 297	7 463	4 450	2 569	1 408	0 701	0 284	0 065	0
6 536	0 17	35 812	21 269	12 564	7 360	4 243	2 376	1 268	0 619	0 246	0 056	0
8 031	0 16	39 494	22 553	12 820	7 238	4 029	2 187	1.135	0 541	0 211	0 048	0
9 889	0 15	43 572	23 899	13 063	7 095	3 809	2 001	1 008	0 467	0 180	0 011	0

TABLE 5

DETERMINATION OF RIB THICKNESS — VALUES OF c

VALUE OF $c = \dfrac{1}{\sqrt[3]{1-(1-m)z}}$

m	Point 0 (Springing line)	Point 1	Point 2	Point 3	Point 4	Point 5 (Quarter-point)	Point 6	Point 7	Point 8	Point 9	Point 10 (Crown)
	$z = 1.0$	$z = 0.9$	$z = 0.8$	$z = 0.7$	$z = 0.6$	$z = 0.5$	$z = 0.4$	$z = 0.3$	$z = 0.2$	$z = 0.1$	$z = 0.0$
1.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
0.8	1.077	1.068	1.060	1.052	1.044	1.036	1.028	1.021	1.014	1.007	1.000
0.6	1.186	1.160	1.137	1.116	1.096	1.077	1.060	1.044	1.028	1.014	1.000
0.5	1.260	1.221	1.186	1.154	1.126	1.101	1.077	1.056	1.036	1.017	1.000
0.4	1.357	1.295	1.244	1.199	1.160	1.126	1.096	1.068	1.044	1.021	1.000
0.3	1.494	1.393	1.315	1.252	1.199	1.154	1.116	1.082	1.052	1.024	1.000
0.25	1.587	1.454	1.357	1.282	1.221	1.170	1.126	1.089	1.056	1.026	1.000
0.20	1.710	1.529	1.405	1.315	1.244	1.186	1.137	1.096	1.060	1.028	1.000
0.15	1.882	1.621	1.462	1.352	1.268	1.203	1.149	1.103	1.064	1.030	1.000

TABLE 6
VALUE OF y_c

N	$\frac{w_s}{w_c}$	Value of $\frac{y_c}{r}$					
		$m = 0.15$	$m = 0.20$	$m = 0$ 25	$m = 0$ 30	$m = 0.40$	$m = 0.50$
0.25	1 000	0 2101	0 2222	0 2333	0 2136	0 2619	0 2778
0 24	1 347	0 2044	0 2163	0 2273	0 2374	0 2556	0 2713
0 23	1 756	0 1985	0 2103	0 2212	0 2312	0 2491	0 2647
0 22	2 240	0 1926	0 2043	0 2150	0 2250	0 2427	0 2580
0.21	2 814	0 1867	0 1983	0 2089	0 2187	0 2362	0 2513
0 20	3 500	0 1808	0 1922	0 2027	0 2124	0 2296	0 2446
0.19	4 324	0 1748	0 1861	0 1964	0 2060	0 2230	0 2378
0 18	5 321	0 1688	0.1799	0 1901	0 1996	0 2164	0 2309
0 17	6 536	0 1628	0 1738	0 1838	0 1931	0 2097	0 2240
0 16	8 031	0 1567	0 1675	0 1774	0 1865	0 2029	0 2170
0 15	9 889	0 1506	0 1612	0 1709	0 1799	0 1960	0 2099

TABLE 7
RIB SHORTENING

N	$\frac{w_s}{w_c}$	Value of C ($E = 2{,}000{,}000$)					
		$m = 0.15$	$m = 0.20$	$m = 0.25$	$m = 0.30$	$m = 0.40$	$m = 0.50$
0 25	1 000	0 0329	0 0370	0 0410	0 0448	0 0520	0 0588
0 24	1 347	0 0320	0 0361	0 0400	0 0437	0 0509	0 0577
0 23	1 756	0 0312	0 0352	0 0390	0 0428	0 0498	0 0566
0 22	2 240	0 0302	0.0342	0.0380	0 0417	0 0486	0 0553
0 21	2 814	0 0294	0 0332	0 0370	0 0407	0 0475	0.0541
0 20	3 500	0 0285	0 0323	0 0361	0 0396	0 0466	0 0530
0 19	4 324	0 0276	0 0314	0 0351	0 0386	0 0455	0 0519
0 18	5 321	0 0267	0 0305	0.0341	0 0376	0 0443	0 0507
0 17	6 536	0 0258	0 0295	0 0331	0 0366	0 0432	0 0496
0 16	8 031	0 0250	0 0286	0 0322	0 0356	0 0422	0 0484
0.15	9 889	0 0241	0 0277	0 0312	0 0346	0 0411	0 0472

C varies as E

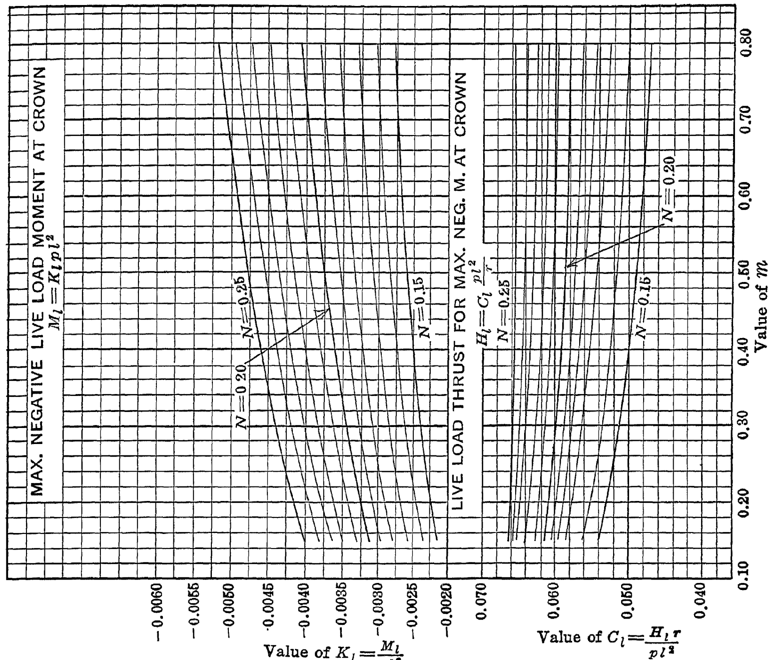


FIG. 105

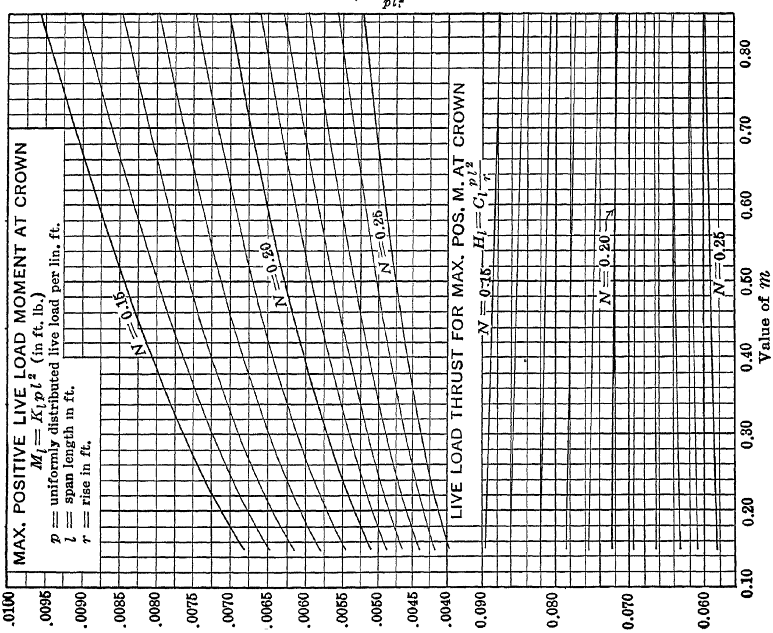


FIG. 104

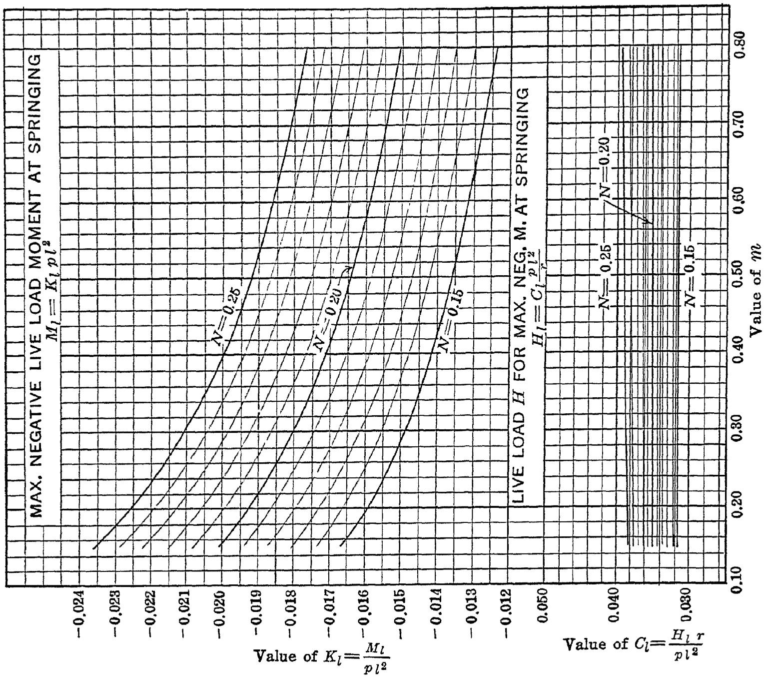


FIG. 107

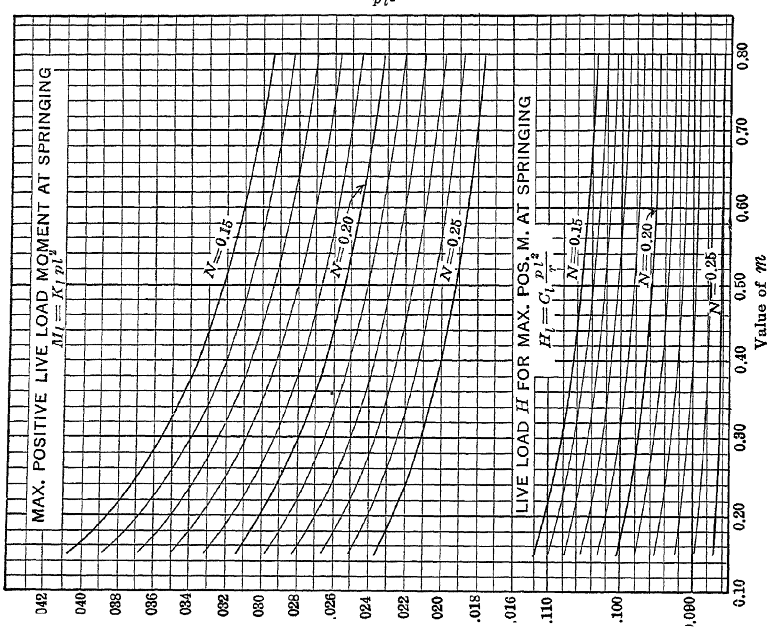


FIG. 106

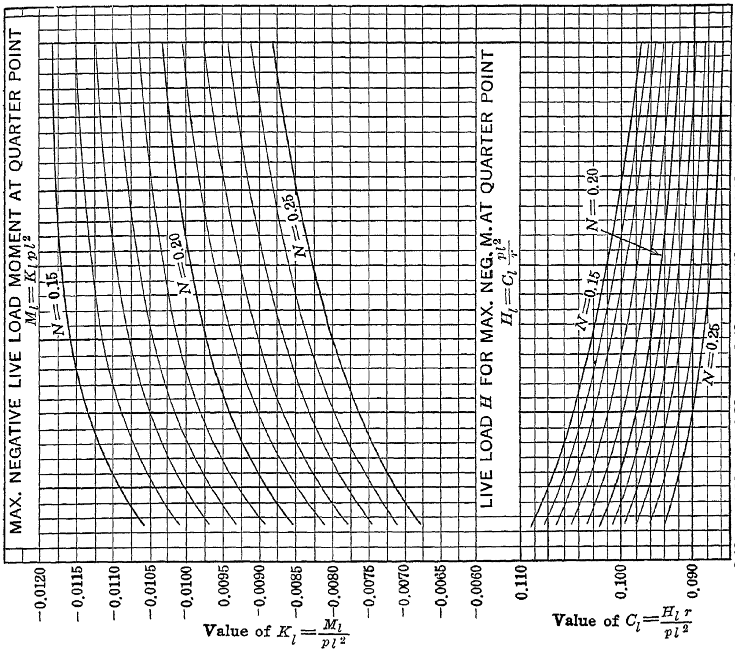


FIG. 109

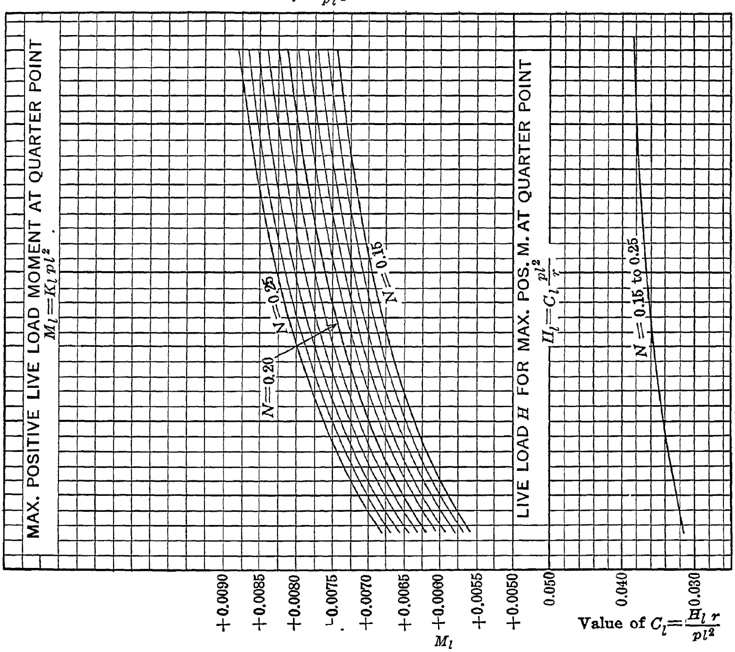
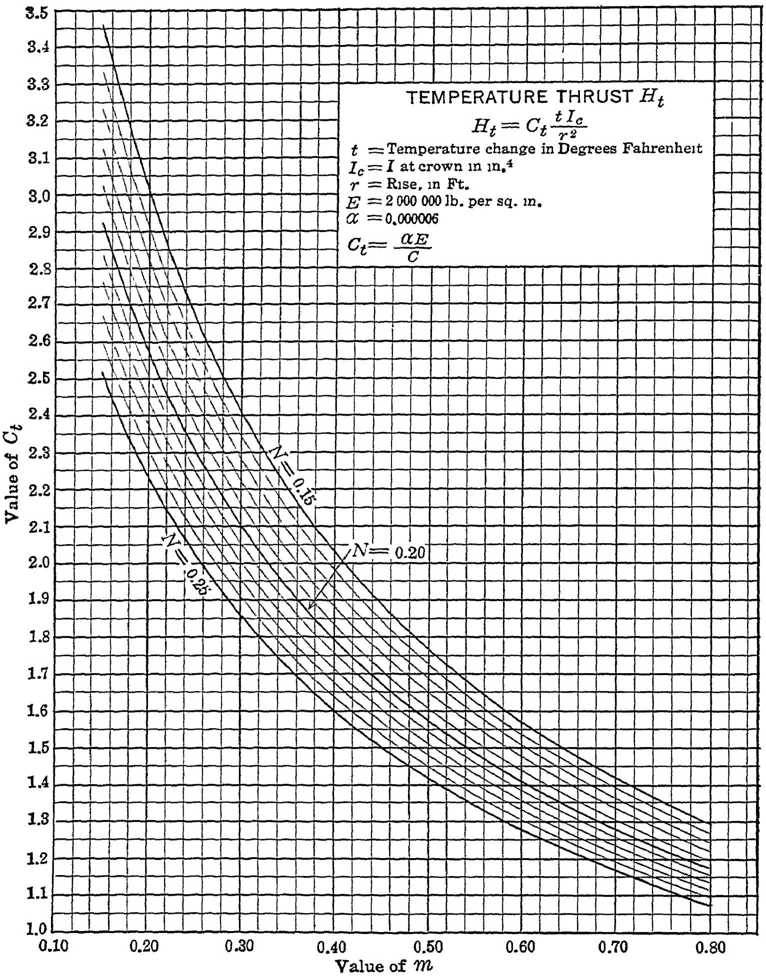


FIG. 108



TEMPERATURE THRUST H_t
$H_t = C_t \frac{t\,I_c}{r^2}$
t = Temperature change in Degrees Fahrenheit
$I_c = I$ at crown in in.^{4}
r = Rise, in Ft.
E = 2 000 000 lb. per sq. in.
α = 0.000006
$C_t = \frac{\alpha E}{C}$
N = 0.15
N = 0.20
N = 0.25
Value of C_t
3.5
3.4
3.3
3.2
3.1
3.0
2.9
2.8
2.7
2.6
2.5
2.4
2.3
2.2
2.1
2.0
1.9
1.8
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
Value of m

FIG. 110

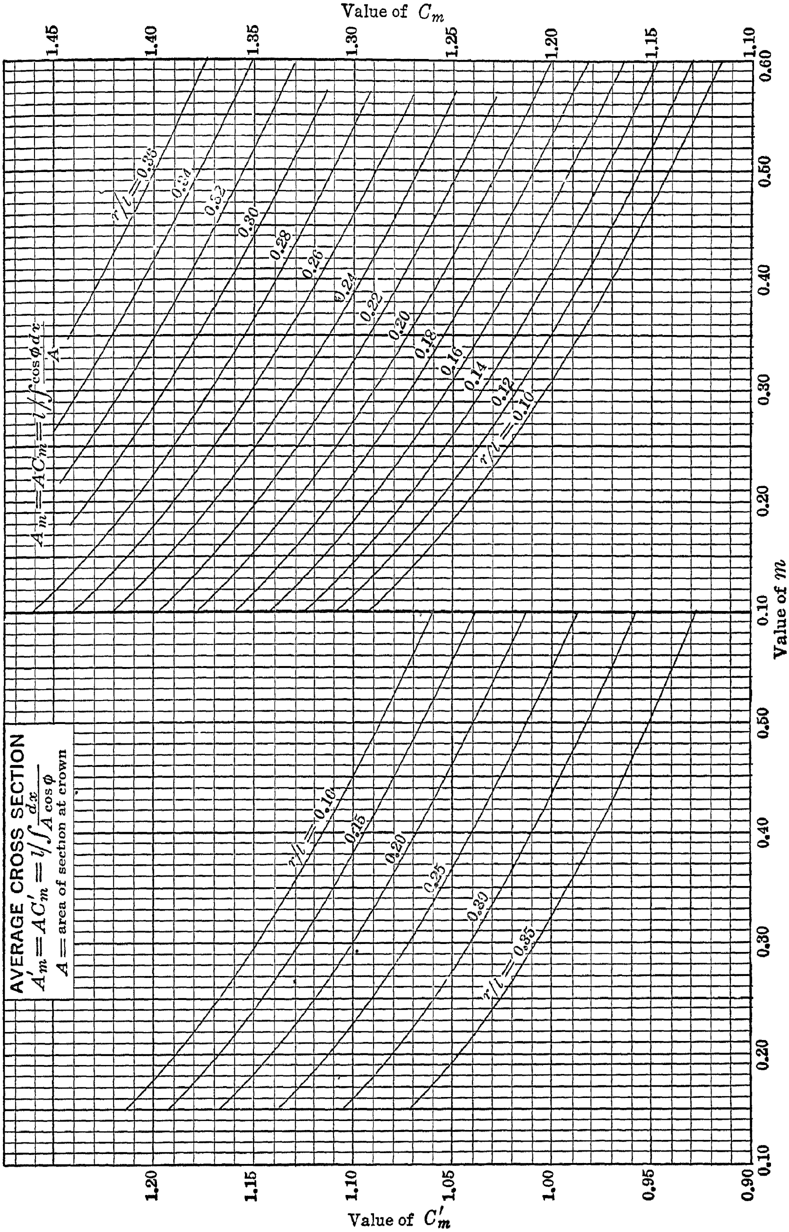

Value of C_m
1.45
1.40
1.35
1.30
1.25
1.20
1.15
1.10
$A_m = AC_m = l/\int \frac{\cos\phi\, dx}{A}$
$r/l = 0.36$
0.34
0.32
0.30
0.28
0.26
0.24
0.22
0.20
0.18
0.16
0.14
0.12
$r/l = 0.10$
AVERAGE CROSS SECTION
$A'_m = AC'_m = l/\int \frac{dx}{A\cos\phi}$
A = area of section at crown
$r/l = 0.10$
0.15
0.20
0.25
0.30
$r/l = 0.35$
1.20
1.15
1.10
1.05
1.00
0.95
0.90
Value of C'_m
0.60
0.50
0.40
0.30
0.20
0.10
0.50
0.40
0.30
0.20
0.10
Value of m

FIG. 111

134. Method of Least Work.[1] All analyses of the hingeless arch based on its elastic action assume that the abutments are rigid and immovable so that the span remains unchanged and there is no rotation of the tangent to the axis at the springing. This is probably never exactly true and in consequence another possible source of error is introduced at the start into any such analysis in addition to those common to all reinforced concrete design: lack of uniformity of material; uncertain value of the modulus of elasticity; uncertain action of concrete in tension; the effect of shrinkage and plastic flow. Consequently an exact theory for the arch, while interesting mathematically, is of doubtful worth actually. By making certain reasonable assumptions the application of the Theorem of Least Work to arch analysis is greatly simplified. The equations for reactions thus derived give results that are closely the same as those obtained by the better known method based on the principles relating to curved bars.[2] The simplifying assumptions are these:

(*a*) The distribution of stress over the cross-section of a curved bar is the same as though the bar were straight. This assumes that the familiar expression, $f = \frac{P}{A} \pm \frac{My}{I}$, applies instead of the exact formula which involves the radius of curvature.

(*b*) The total direct or axial thrust at all sections of the arch is the same and equal to the horizontal component of crown thrust.

(*c*) The work due to shear may be neglected as is done universally in the study of indeterminate structures.

The justification of these assumptions may be briefly stated. The distribution of stress over the cross-section of a curved bar whose depth is 0.06, or less, of the radius of curvature is affected to a negligible degree by the curvature. The work due to direct thrust is so small an element in indeterminate structures that it is often neglected. So an approximation in the magnitude of the direct thrust is entirely proper. The magnitude of the error was found to be a fraction of one per cent in a 100-foot arch with a rise of 20 feet.

[1] This treatment is that given by Professor C. M. Spofford in his "Theory of Structures." It is based on the method given by Müller-Breslau in Zeitschrift des Architekten und Ingenieur Vereins, Hannover, 1884.

[2] A complete statement of the ordinary theory is given in Appendix E.

In addition to the notation already made use of (Fig. 101) that of Fig. 112 and also the following will be used:

XX and YY are two reference axes, the first located so that $\int \frac{y\,dS}{EI} = 0$ and the second being an axis of symmetry.

m_L = moment (foot-pounds) at any point (x, y) on the left half axis of the loads between that point and the crown, the left half of the arch being considered as a cantilever beam fixed at the abutment with the right half replaced by its equivalent M_0, H_0 and V_0.

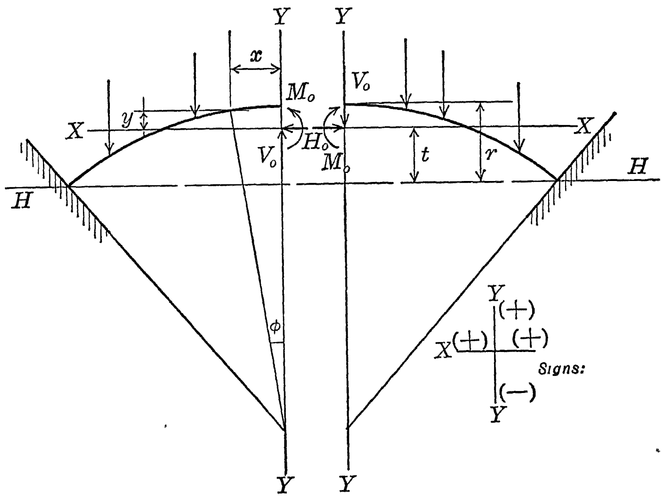


FIG. 112

m_R = same for right half of arch.

M_c = actual bending moment at crown (foot-pounds) = $M_0 - H_0(r - t)$.

M_L = actual bending moment at any point on left half of arch axis.

M_R = actual bending moment at any point on right half of arch axis.

W = total work in entire arch.

I = moment of inertia (ft.[4]) at any section normal to axis.

A = area (square feet) at any section normal to axis.

E = modulus of elasticity (pounds per square foot).

M_0, H_0 and V_0 are all taken as positive when acting in the direction shown in Fig. 112.

In Fig. 112 is shown a loaded arch cut into halves by a section at the crown, with the forces exerted by each portion of the arch upon the other represented by M_0, H_0 and V_0, the moment, horizontal component of thrust and the shear acting at the level of the reference axis XX.[1] With these three quantities known for any

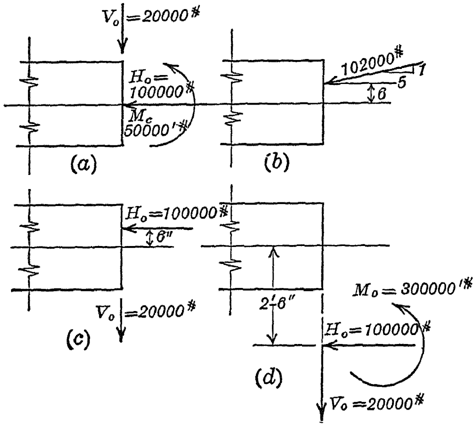


FIG. 113

loading the principles of statics are sufficient for the determination of the moment, thrust and shear at any other section of the arch. By the theorem of least work it may be stated that M_0, H_0 and V_0 always have the least value consistent with equilibrium. They may be determined, therefore, by writing an expression for the total work done in the arch in terms of the loads and these three unknowns, differentiating the expression with regard to each unknown in turn, placing each partial differential equal to zero and solving for M_0, H_0 and V_0.

The total work done is (see Art. 88):

$$W = \int_0^{\frac{S}{2}} \frac{M_L^2\,dS}{2\,EI} + \int_0^{\frac{S}{2}} \frac{M_R^2\,dS}{2\,EI} + 2\,H_0^2 \int_0^{\frac{S}{2}} \frac{dS}{2\,AE}$$

where
$$M_L = M_0 - m_L - H_0 y + V_0 x$$
$$M_R = M_0 - m_R - H_0 y - V_0 x.$$

[1] In Fig. 113 are shown four different ways of representing the action of the right portion of a given arch upon that to the left.

The partial derivatives are:

$$\frac{\partial W}{\partial M_0} = \int_0^{\frac{S}{2}} (M_0 - m_L - H_0 y + V_0 x)\frac{dS}{EI} + \int_0^{\frac{S}{2}} (M_0 - m_R - H_0 y - V_0 x)\frac{dS}{EI}$$

$$\frac{\partial W}{\partial H_0} = \int_0^{\frac{S}{2}} (M_0 - m_L - H_0 y + V_0 x)(-y)\frac{dS}{EI} + \int_0^{\frac{S}{2}} (M_0 - m_R - H_0 y - V_0 x)(-y)\frac{dS}{EI} + 2\,H_0\int_0^{\frac{S}{2}} \frac{dS}{AE}$$

$$\frac{\partial W}{\partial V_0} = \int_0^{\frac{S}{2}} (M_0 - m_L - H_0 y + V_0 x)(x)\frac{dS}{EI} + \int_0^{\frac{S}{2}} (M_0 - m_R - H_0 y - V_0 x)(-x)\frac{dS}{EI}.$$

The XX axis is located so that $\int \frac{y\,dS}{EI} = 0$ and all terms containing this integral multiplied by a constant equal zero. Combining terms and placing each partial differential equal to zero gives

$$\frac{\partial W}{\partial M_0} = 2\,M_0\int_0^{\frac{S}{2}} \frac{dS}{EI} - \int_0^{\frac{S}{2}} \frac{(m_L + m_R)dS}{EI} = 0$$

$$\frac{\partial W}{\partial H_0} = \int_0^{\frac{S}{2}} \frac{(m_L + m_R)y\,dS}{EI} + 2\,H_0\int_0^{\frac{S}{2}} \frac{y^2\,dS}{EI} + 2\,H_0\int_0^{\frac{S}{2}} \frac{dS}{AE} = 0$$

$$\frac{\partial W}{\partial V_0} = \int_0^{\frac{S}{2}} \frac{(m_R - m_L)x\,dS}{EI} + 2\,V_0\int_0^{\frac{S}{2}} \frac{x^2\,dS}{EI} = 0$$

Solving

$$M_0 = \frac{\displaystyle\int_0^{\frac{S}{2}} \frac{(m_L + m_R)\,dS}{I}}{\displaystyle 2\int_0^{\frac{S}{2}} \frac{dS}{I}} \tag{49}$$

$$H_0 = \frac{\displaystyle -\int_0^{\frac{S}{2}} \frac{(m_L + m_R)y\,dS}{I}}{\displaystyle 2\int_0^{\frac{S}{2}} \frac{y^2\,dS}{I} + 2\int_0^{\frac{S}{2}} \frac{dS}{A}} \tag{50}$$

$$V_0 = \frac{\displaystyle\int_0^{\frac{S}{2}} \frac{(m_L - m_R)x\,dS}{I}}{\displaystyle 2\int_0^{\frac{S}{2}} \frac{x^2\,dS}{I}} \tag{51}$$

The XX axis is properly located if

$$t\int\frac{dS}{I}=\int\frac{z_0\,dS}{I}$$

$$t=\frac{\displaystyle\int\frac{z_0\,dS}{I}}{\displaystyle\int\frac{dS}{I}} \tag{52}$$

These expressions are difficult to integrate and working formulas are obtained by substituting for the infinitesimal dS lengths of finite value, ΔS:

$$M_0=\frac{\Sigma\dfrac{(m_L+m_R)}{I}}{2\,\Sigma\dfrac{1}{I}} \tag{49a}$$

$$H_0=\frac{-\Sigma\dfrac{(m_L+m_R)y}{I}}{2\,\Sigma\dfrac{y^2}{I}+2\,\Sigma\dfrac{1}{A}} \tag{50a}$$

$$V_0=\frac{\Sigma\dfrac{(m_L-m_R)x}{I}}{2\,\Sigma\dfrac{x^2}{I}} \tag{51a}$$

$$t=\frac{\Sigma\dfrac{z_0}{I}}{\Sigma\dfrac{1}{I}} \tag{52a}$$

In all cases the summation is over the half arch.

Temperature Stresses. The total work done in an arch by the forces set up by a change of temperature is

$$W=2\int_0^{\frac{S}{2}}\frac{M^2\,dS}{2\,EI}+2\int_0^{\frac{S}{2}}\frac{T^2\,dS}{2\,AE}-2\int_0^{\frac{S}{2}}T\alpha t\,dS$$

where M and T are the moment and thrust due to temperature only; α is the coefficient of expansion and t is the degrees of temperature change. The two terms giving the work done by the thrust may be understood by reference to Fig. 114, where is shown a short length, dS, of an arch. It is assumed that a thrust, T, due

to temperature rise is in action. A further rise of $dt°$ would cause a lengthening of $\alpha\, dt \cdot dS$ were there no restraint, with a negative increment of work equal to $-\int\int T\alpha\, dt\, dS$. The restraint causes the thrust to increase to $T+dT$ and there is a shortening of $\frac{dT \cdot dS}{AE}$. The work done during this movement is

$$\int\int (T + dT/2)\,\frac{dT \cdot dS}{AE}$$
$$= \int\int (T \cdot dT + dT^2/2)\,\frac{dS}{AE},$$

the term in dT^2 being extremely small and negligible. The expression for total work is $W = -\int T\alpha t\ dS + \int \frac{T^2\, dS}{2\,AE}$ as given. In the above expression for work

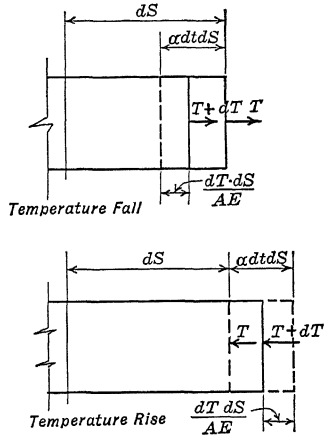


FIG. 114

$$M = M_0 - H_0 y \qquad \text{and} \qquad T = H_0 \cos\phi.$$

Substituting these values and differentiating the work with regard to the two variables in turn gives

$$\frac{\partial W}{\partial M_0} = 2\int_0^{\frac{S}{2}} \frac{M_0\, dS}{EI} - 2\,H_0 \int_0^{\frac{S}{2}} \frac{y\, dS}{EI} = 0$$

$$\frac{\partial W}{\partial H_0} = -2\int_0^{\frac{S}{2}} \frac{(M_0 - H_0 y)y\, dS}{EI} + 2\int_0^{\frac{S}{2}} \frac{H_0 \cos^2\phi\, dS}{AE}$$
$$- 2\int_0^{\frac{S}{2}} \alpha t \cos\phi\, dS = 0.$$

Since $dS = \frac{dx}{\cos\phi}$, $2\int_0^{\frac{S}{2}} \alpha t \cos\phi\, dS = \alpha t l.$

Solving the first equation gives $M_0 = 0$ since $\int \frac{y\, dS}{EI} = 0.$

The solution of the second is:

$$H_0 = \pm \frac{\alpha t l E}{2\int_0^{\frac{S}{2}} \frac{y^2\, dS}{I} + 2\int_0^{\frac{S}{2}} \frac{\cos^2\phi\, dS}{A}}. \tag{53}$$

For a rise of temperature the sign will be $+$, indicating compression.

A convenient working expression for H_0 may be obtained by substituting unity for $\cos^2 \phi$ and by assuming that l, the span, equals the arch axis in length, equals $2\,n\Delta S$, where n is the number of divisions of the half axis.

$$H_0 = \frac{\alpha t n E}{\Sigma \frac{y^2}{I} + \Sigma \frac{1}{A}}. \tag{53a}$$

This formula may be easily corrected for the second approximation by aid of the following table given by Cochrane in the paper referred to previously:

Lengths of the half-arch axis (S) in terms of the span length:

	Rise Ratio $= r/l$				
	0.10	0.15	0.20	0.25	0.30
Open-Spandrel Arch	$0.513\,l$	$0.529\,l$	$0.551\,l$	$0.577\,l$	$0.607\,l$
Filled-Spandrel Arch	$0.515\,l$	$0.534\,l$	$0.559\,l$		

The above table gives accurate values of course only for arches whose axes conform to the equations used in preparing it.

Since the thrust was considered in writing the expressions for work, rib-shortening has been taken account of in these formulas. The effect of thrust appears in the terms in $\Sigma\,1/A$ in the denominators of the expressions for H_0 (Equations 50a, 53a). By omitting these terms the horizontal component of crown thrust is obtained with rib-shortening not taken account of. The difference between this larger value and that obtained with the term included is the crown thrust due to rib-shortening.

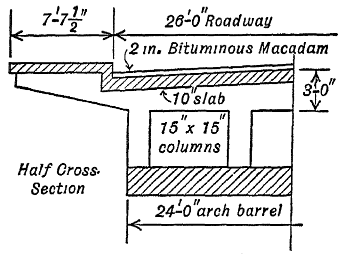
135. Example of Arch Design. (Computation Sheets A1–A11.) On Computation Sheet A1 are given the design data for an open spandrel hingeless arch. The situation at the crossing fixed the clear span and rise at approximately the values shown, which were those finally used. The preliminary sketches and trials to determine these dimensions are not shown.

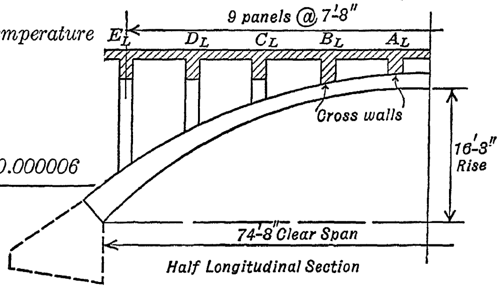
135a. Preliminary Analysis. (Computation Sheet A1–A5.) By use of a formula for crown thickness and by trial computations of the same sort as those carried through on these sheets but made with less precision, it was judged that the trial dimensions shown offered a reasonable solution worthy of more careful study. The area of the reinforcement at the crown was taken at the value

COMPUTATIONS FOR HINGELESS ARCH

Sheet A1

Data. *Live Load:*
On Floor & Sidewalk 90 lbs./sq. ft.
Impact: 25%
Specifications:
1924 Joint Committee Report except as noted.
Concrete: 28 day strength 2000#/□″
Stresses: $n = 12$
Dead + Live Loads
$f_c = 650\#/□''$
$f_s = 16{,}000\#/□''$
Allow 25% increase with temperature stresses.
Temperature Change:
Fall: 60° F.
Rise: 20° F.
Coefficient of Expansion = 0.000006





Loads on Arch Barrel.
Dead: 10″ Slab 125

$$\text{Stem: } \frac{350\#/'}{7'.67} = 46$$

$$\overline{171} \times 41.25 = 7050$$

2″ Bituminous Macadam @ 120#/cu. ft.

$$20 \times 26 \qquad 520$$

$$w_d = \overline{7570} \div 24 = 315\#/'$$

Live + Impact:

$$\text{Uniform: } w_L = \frac{90 \times 41 \times 1.25}{24} = 192\#/'$$

Preliminary Analysis. Whitney's Method. See Art. 133

Trial Dimensions: Crown Thickness:	15″	$= d_c$
Springing Line Thickness:	28″	$= d_s$
Span of Arch Axis:	76.5′	$= l$
Rise of Arch Axis:	16.1′	$= r$
Reinforcement:	1% at crown	

$$A_s = 0.01 \times 15 \times 12 = 1.8□''/'$$

Use $\frac{3}{4}''$ ϕ *6″ o.c. in each face.*

Data required for analysis: Moment of Inertia:

$$I_c = \frac{12 \times 15^3}{12} + 11 \times 0.88 \times 2 \times \overline{5.5}^2 = 3960''^4$$

$$I_s = \frac{12 \times 28^3}{12} + 11 \times 0.88 \times 2 \times \overline{12}^2 = 24{,}700''^4$$

Crown Section Area: $A_c = 12 \times 15 + 11 \times 0.44 \times 4 = 199□''$

usually chosen, about 1 per cent of the cross-sectional area at the crown. The same rods were used throughout the length of the barrel and so the steel ratio at the springing is considerably less than at the crown. The necessary data were computed for the application of Whitney's method. From the first draft of Fig. 115 an estimate was made of ϕ_s and y_0/r for computing values of m and N. Inspection of Figs. 105 and 106 and consideration of the differences between the temperature rise and fall made it evident that it was not necessary to compute the maximum negative moment at the crown nor the maximum positive moment at the springing. With the aid of the references on the computation sheets all the details of this analysis should be plain.

The stresses found were considered satisfactory, the slight excess of compressive stress at the springing with temperature effect included not being sufficient to make a change necessary. The crown stresses are very low and a thinner section is possible. However 15 inches was judged as thin as it was desirable to use for good architectural proportions.

It should be remembered that this analysis is approximate only when the arch section differs from the theoretical basis of the method.

135b. Arch Axis. In the first draft of Fig. 115 no attempt was made to do more than approximate the shape of the axis and ring. This was sufficient for calculating the dead loads fairly closely. A careful determination of the curve of the axis followed the preliminary analysis. Instead of laying out the dead load equilibrium polygon on paper the elevation of this polygon at each panel point was computed, thus saving time, the more since the moments then computed were required for later steps in the analysis.

The moment about the springing of all of the dead loads on one half of the arch divided by the rise of the axis gave the crown thrust. Since there is no moment at any point on the equilibrium polygon, the moment about any panel point of the dead loads between it and the crown, divided by the crown thrust gives the distance to the polygon from the crown.

135c. Arch Ring. The following table (No. 8) is given by Cochrane for laying out the arch ring. It is reproduced here for comparison with Whitney's section and also because of its ease of application. The two thicknesses computed by Whitney's method are somewhat thicker than those recommended by Coch-

COMPUTATIONS FOR HINGELESS ARCH

Sheet A2

Preliminary Analysis, Continued.

Constants:

$$m = \frac{I_c}{I_s \cos \phi_s} = \frac{3960}{24{,}700 \times 0.70} = 0.23$$

$$N = \frac{y_0(Q.Pt.)}{r} = \frac{3\ 55}{16.1} = 0.22$$

See Fig. 101

for Live Loads:

$$pl = 192 \times 76.5 = 14{,}700\#$$

$$pl^2 = 192 \times \overline{76.5}^2 = 1{,}120{,}000'\#$$

$$pl^2/r = 69{,}800\#$$

Temperature Stresses: (*Whitney*)

For 60° Fall: $n = 12$ $E = 2{,}500{,}000\#/\square''$

$$H_T = \left(-2.3\,\frac{60 \times 3960}{\overline{16.1}^2}\right)\frac{15}{12} = -2640\#$$

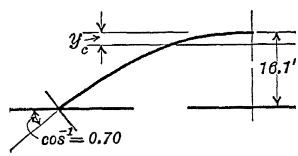
From Fig. 110 based on n = 15

$$M_c = 2640 \times 3.38 = +8920'\#$$

$$M_s = 2640\,(16.1 - 3.4) = -33{,}500'\#$$

$$T_s = 2640 \times 0.70 = -1850\#$$

For 20° Rise divide above values by (−3)



$y_c = 0.21 \times 16.1 = 3.38'$
From Table 6

Maximum Positive Moment at Crown. (*Whitney*)

$pl^2 = 1{,}120{,}000'\#$ $pl^2/r = 69{,}800\#$ $m = 0.23$ $N = 0.22$

	Thrust = H_0		*Moment* = M_C		*Rib Shortening*
Dead		*25,000			$u' = \dfrac{3960 \times 2/2.5}{199 \times 0.037 \times 1.1 \times \overline{16\ 1}^2 \times 144}$
Live	$0.0665\,\dfrac{pl^2}{r}$	† 4,600	$0.0049\ pl^2$	† +5,500	(*Equation 48a*)[1]
R.S.		− 300	$296 \times 3.38 =$	1,000	$= 0.010$
		29,300		+6,500	$H_{RS} = 0.010 \times 29{,}600 = -296\#$
D+L		29,600		+5,500	
Temp.		−2,600		8,900	[1]$C = 0.037 \times 2.5 \div 2$
R.S.		− 300		1,000	
		26,700		15,400	

* *From trial computations like that following for dead load equilibrium polygon.*
† *From Fig. 104.*

rane and approximate those finally chosen. These two series of trial depths were plotted on Fig. 115 and smooth curves drawn for axis, intrados and extrados. In a long span arch it would be desirable to be much more precise in the lay-out, making the axis fit the dead load equilibrium polygon more exactly and making the thickness agree with the section recommended by Mr. Whitney.

TABLE S

THICKNESSES OF TYPICAL ARCHES[1]

Value of $\frac{S_x}{S}$	Values of $\frac{d_x}{d_c}$							
	d_s/d_c							
	1 5	1 75	2	2 25	2 5	2 75	3	3 25
0	1 000	1 000	1.000	1.000	1 000	1 000	1 000	1 000
0.05	1 007	1.006	1.005	1 004	1 003	1 002	1 001	1 000
0 15	1 021	1.018	1.015	1 012	1 009	1.006	1 003	1 000
0.25	1.035	1.030	1.025	1 020	1 015	1.010	1 005	1 000
0 35	1 049	1.042	1.035	1.028	1 023	1.021	1 023	1 030
0 45	1.063	1.054	1.048	1.048	1.057	1.070	1.083	1.101
0.55	1.077	1.072	1 085	1.105	1.133	1.165	1.193	1.231
0.65	1.095	1.125	1.168	1.215	1.269	1.328	1.385	1.455
0.75	1.145	1.223	1 311	1 403	1.508	1.625	1.737	1 865
0 85	1.245	1.393	1 547	1 700	1.862	2.025	2.185	2.355
0 95	1.406	1.621	1 837	2 055	2.277	2.495	2 709	2.932
1.00	1.500	1.750	2 000	2.250	2.500	2.750	3 000	3.250

[1] Reprinted from a paper by Victor H. Cochrane, "The Design of Symmetrical Hingeless Concrete Arches," Proceedings of The Engineers' Society of Western Pennsylvania, November, 1916.

Of the three curves drawn, the extrados and intrados must be chosen and designated so that construction may follow the desired lines. Usually these curves are made up of one or more circular segments. In this case a single segment fitted the requirements and the result is a segmental or single-centered arch. Had an arc of different radius been used towards the springing it would have been called a three-centered arch. A smooth curve suffices for the axis and it is not necessary to determine its curvature although it is often done as here.

135d. Analysis by Least Work. Arch Constants. (Computation Sheets A6.) From Fig. 115 the values of z_0, d_x and x were

COMPUTATIONS FOR HINGELESS ARCH

Sheet A3

Preliminary Analysis, Continued.

Approximate stress analysis — using Plate XV assuming $\frac{d'}{h} = \frac{1}{10}$

Dead + Live + Arch Shortening: $p = 0.005$ *each face.*

$$\text{Eccentricity:} \qquad e = \frac{6500 \times 12}{29{,}300} = 2.7''$$

$$e/h = \frac{2.7}{15} = 0.18$$

$$\text{Max. } f_c = \frac{6500 \times 12}{12 \times \overline{15}^2 \times 0.105} = 280\#/\square'' < 650$$

Dead + Live + Temperature + Arch Shortening.

$$e = \frac{15{,}400 \times 12}{26{,}700} = 6.9'' \quad e/h = 0.46$$

$$f_c = \frac{15{,}400 \times 12}{12 \times \overline{15}^2 \times 0.122} = 560\#/\square'' < 810$$

Maximum Negative Moment at Springing. (Whitney)

	T_s		M_s		Rib Shortening
Dead	See below	+34,000			$H_{RS} = -Hu' = 0.010(25{,}000 + 2500) = -275\#$
Live	See below	5,200	$0.0202\ pl^2 =$	† −22,700	
R S.	0.104×1850	− 200	0.104×33,500=	− 3,500	
		+39,000		−26,200	$\frac{275}{2640} = 0.104$
Temp		− 1,900		−33,500	
		+37,100		−59,700	

† From Fig. 107.

Data for Max. (−M) at springing.

Dead Load Thrust at Springing:

$T = H \cos \phi_s + V \sin \phi_s$

$= 34{,}000\#$

Live Load Thrust:

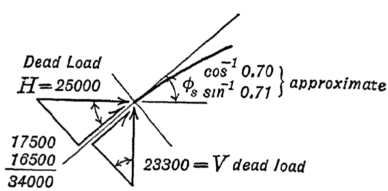
$H = 0.036 \frac{pl^2}{r} = 2500\#$

from Fig. 107

$V_L = 0.34 \times 192 \times 76.5 = 5000\#$

from Fig. 102

$T = 5200\#$



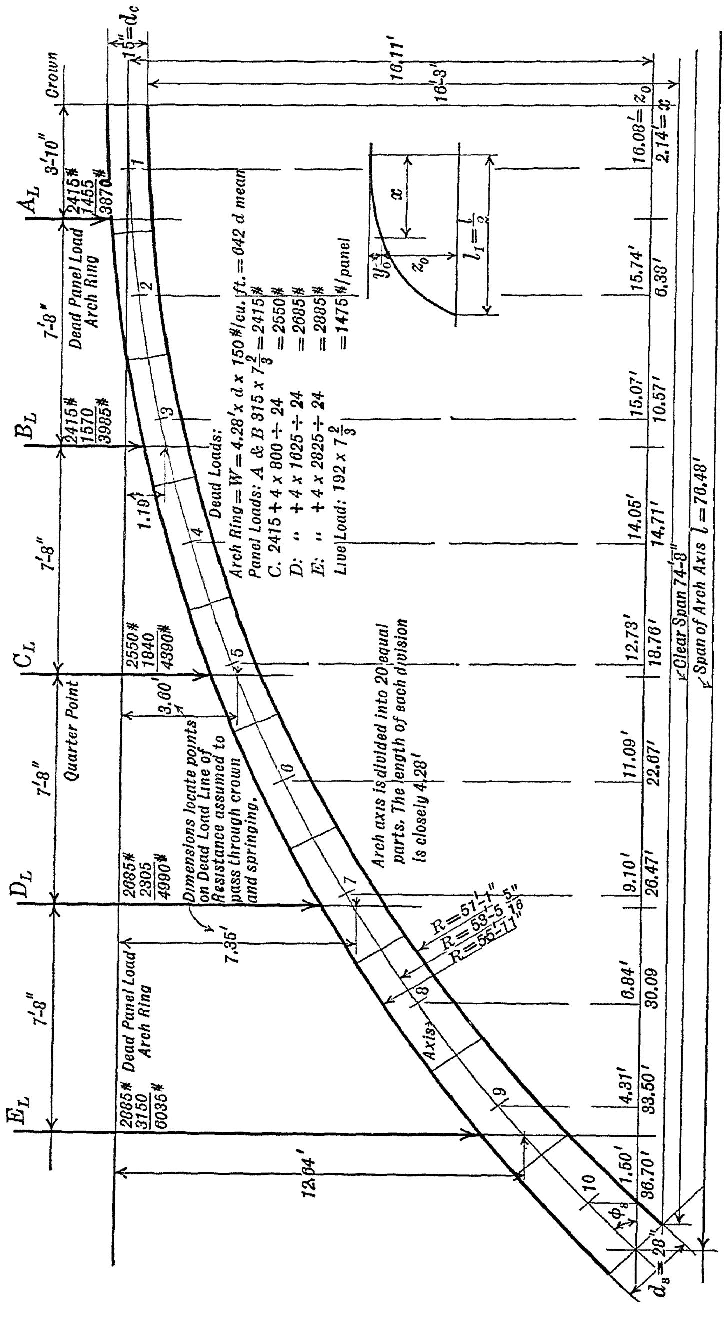

Crown
Quarter Point
Dead Panel Load
Arch Ring
Dead Loads:
Arch Ring = W = 4.28' x d x 150 #/cu. ft. = 642 d mean
Panel Loads: A & B 315 x 7 2/3 = 2415 #
C. 2415 + 4 x 800 ÷ 24 = 2550 #
D: " + 4 x 1625 ÷ 24 = 2685 #
E: " + 4 x 2825 ÷ 24 = 2885 #
Live Load: 192 x 7 2/3 = 1475 #/panel
Dimensions locate points on Dead Load Line of Resistance assumed to pass through crown and springing.
Arch axis is divided into 20 equal parts. The length of each division is closely 4.28'
Clear Span 74'-8"
Span of Arch Axis l = 76.48'
Axis

FIG. 115

COMPUTATIONS FOR HINGELESS ARCH

Sheet A4

Preliminary Analysis, Continued.

Stresses at Springing: — Plate XV

Dead + Live + Arch Shortening: —

$$e = \frac{26{,}200 \times 12}{39{,}000} = 8.1'' \qquad \frac{e}{h} = \frac{8.1}{28} = 0.29$$

$$p = \frac{2 \times 0.44}{12 \times 28} = 0.0026 \text{ in each face}$$

$$f_c = \frac{26{,}200 \times 12}{12 \times \overline{28}^2 \times 0.107} = 310\#/\square'' < 650 \qquad \text{O.K.}$$

$$f_s = 3000\#/\square'' \text{ about.}$$

Dead + Live + Temperature + Arch Shortening.

$$e = \frac{59{,}700 \times 12}{37{,}100} = 19.3'' \qquad \frac{e}{h} = \frac{19.3}{28} = 0.69$$

$$f_c = \frac{59{,}700 \times 12}{12 \times \overline{28}^2 \times 0.094} = 815\#/\square'' > 810 \qquad \text{O.K.}$$

Dead Load Equilibrium Polygon

Load		*Arm*	*Moment*	*Section*	y_0
			0	*A*	0
A	3870	7.67	29700	*B*	1.19
B	3985				
	7855	7.67	60200		
			89900	*C*	3.60
C	4390				
	12245	7.67	94000		
			183900	*D*	7.35
D	4990				
	17235	7.67	132000		
			315900	*E*	12.64
E	6035				
	23270	3.74	87000		
		16.11	)402900	*Springing*	16.11
		H =	25000		

scaled and recorded as shown in the tables on Computation Sheet A6. In doing this work it is best to plan to do the final work on the lay-out and the scaling all on the same day. An overnight change of humidity may easily cause sufficient expansion or contraction of the paper to be very troublesome. A desirable scale is about 2 to 3 feet to the inch.

135e. Dead Stresses. (Computation Sheet A7.) No explanation of these figures should be necessary except to note that the computations of m_L are omitted.

135f. Influence Table. (Computation Sheets A8–A9.) It was assumed that the critical sections were those at crown and springing which Cochrane found to be true for arches conforming at all closely to his assumed proportions.

The only comment necessary for the explanation of this table is to call attention to the figure and formulas following it which explain the headings in the left-hand column. The moment and thrust at the right springing with loads on the left equal those at the left springing with loads at the corresponding points on the right. Therefore it was necessary to show only one-half the load points in the table. All the figures required by the computation are shown.

135g. Table of Maximum Stress. (Computation Sheet A10.) The influence table showed the panel points to load for maximum positive and negative moments at crown and springing and the values for a unit load. These coefficients multiplied by the live panel load gave the maximum stresses.

135h. Temperature Stress. (Computation Sheet A10.) Considerable doubt exists as to the accuracy of computations for temperature stress on account of the uncertain values of the coefficient of expansion and the modulus of elasticity. The values used here are representative.

135i. Summary of Unit Stresses. (Computation Sheet A11.) The values here found are well within the limits set and more precise computation is unnecessary. The steel stresses were approximated by aid of the dotted curves for f_s/f_c on Plate XV.

A comparison of the results with those of the preliminary analysis shows that the differences are small except as regards dead load moment. The true dead load equilibrium polygon does not pass through the crown and springing. It should be realized that on account of the shortening of the arch fibers under load

COMPUTATIONS FOR HINGELESS ARCH

Sheet A5

Preliminary Analysis, Continued.

Arch Thickness

$d_s/d_c = 1.87$ *After Cochrane* *See Table 8.*

S_x/S	d_x/d_c	d_x (ft.)	S_x/S	d_x/d_c	d_x
0	1 000	1.250	0 55	1.079	1 348
0 05	1.0055	1 257	0.65	1.147	1.432
0.15	1.0165	1.270	0 75	1.267	1.580
0.25	1.0275	1.285	0 85	1.470	1.838
0.35	1 0385	1.298	0.95	1.729	2.160
0.45	1 051	1.314	1 00	1.870	2.333

The section above is somewhat thinner than that used.

Length of half-axis = 0.557 × 76.48 = 42.65 *ft.* (*Table on page 314.*)

After Whitney

Point (Fig. 115)	S_x/S	$z = x/38.24$	$\sqrt[6]{1 + \tan^2 \phi}$	c	d_c	d_x
4	0 35	0 385	1.014	1.124	15	17.1
8	0 75	0.786	1.066	1.365	15	21.9

Comparison:

	Whitney	Cochrane	Used
4	17.1 in.	15.6 in.	16.7 in.
8	21.9	19.0	22.3

there will always be moment under dead load at the crown and springing of a hingeless arch as usually constructed.

136. Long Span Arches. Besides temperature stress and the secondary effect of rib-shortening already referred to, careful consideration must also be given, especially in long-span construction, to the settlement of abutments and to the shrinkage of the concrete on setting and hardening. All these effects may be cumulative and tend to produce the same kind of stress in the rib, heavy positive moment at the crown and negative moment at the springing. So they all may be summed up in the term arch-shortening. The stresses due to this bending are a very large proportion of the maximum stress and it becomes a matter of considerable economic and practical importance to eliminate them as far as possible.

The first method devised for this purpose was the use of temporary hinges at crown and springings, which are closed with concrete after all or most of the dead load is in position. The arch acts as a three-hinged arch for dead load and is statically determinate, not affected by arch-shortening. Since the major part of the settlement of abutments usually takes place during construction this element is thus largely eliminated. The hinges are placed so as to neutralize most effectively the arch-shortening moments, below the axis at the crown and above it at the springing.

As devised by Considère[1] such temporary hinges consist of short lengths of reduced cross-section, heavily reinforced with longitudinal steel and spirals, designed for such heavy stress that the concrete is ductile and offers little or no resistance to bending. As shown in Fig. 116, a sufficient length of the main reinforcement is exposed so that it yields easily to the small movement that accompanies the adjustment.

The more recent method, best known through the work of Freyssinet[2] in France, consists in inserting hydraulic jacks between

[1] "Reinforced Concrete Bridges," by W. L. Scott, Crosby, Lockwood & Son, London. "Two Reinforced Concrete Bridges in France," by W. L. Scott, in Engineering News-Record, Dec. 6, 1923. It is stated by Mr. J. F. Brett (letter in Engineering News-Record, Nov. 5, 1925) that the saving from the use of temporary hinges amounts to from 20 to 25 per cent of the cost of the structure.

[2] "Long Span Concrete Arch Design in France," by Charles S. Whitney in Engineering News-Record, Sept. 18, 1924. "Le Pont de Villeneuve-sur-Lot," E. Freyssinet, Le Génie Civil, July 30, Aug. 6–13, 1921.

COMPUTATIONS FOR HINGELESS ARCH

Sheet A6

Analysis by Method of Least Work; Reference Axis:

Sect.	z_0	d_x	$d_x^3/12$	$0.0336(d_x - 0\ 33)^2$	I	z_0/I	$1/I$
1	16 08	1 25	0 163	0.0284	0 1914	84 1	5.22
2	15 74	1 27	0 171	0.0297	0 2007	78 4	4.98
3	15 07	1.31	0 187	0.0323	0 2193	68 7	4.56
4	14 05	1 39	0 224	0 0378	0.2618	53 7	3.82
5	12.73	1 48	0.270	0.0444	0.3144	40 5	3.18
6	11 09	1 58	0.329	0 0525	0.3815	29 1	2.62
7	9 10	1.70	0 410	0 0631	0.4731	19 2	2 11
8	6 84	1 86	0 536	0 0787	0.6147	11.1	1 63
9	4 31	2 03	0 697	0 0971	0.7941	5.4	1 26
10	1 50	2.20	0 888	0 1175	1.0055	1 5	0.99
						391 7	30 37

$$A = d_x + \frac{11 \times 2 \times 0.88}{144}$$
$$= d_x + 0.1345 \text{ sq. ft.}$$

$$I = \frac{d_x^3}{12} + 0.1345\left(\frac{d_x}{2} - \frac{1}{6}\right)^2$$
$$= \frac{d_x^3}{12} + 0.0336\,(d_x - 0.33)^2 \text{ ft.}^4$$

$$t = \frac{\Sigma \frac{z_0}{I}}{\Sigma \frac{1}{I}} = \frac{391.7}{30.37} = 12.90 \qquad y_c = r - t = 16.11 - 12.90$$
$$= 3.21 \text{ ft.}$$

Arch Constants:

Sect.	x	$y = z_0 - t$	x^2/I	y^2/I	$1/A$
1	2.14	3 18	23.9	52 8	0 722
2	6.38	2.84	203.0	40 2	0.712
3	10 57	2.17	510	21 5	0 692
4	14.71	1.15	827	5.1	0.657
5	18.76	−0.17	1120	0.1	0 620
6	22.67	−1 81	1347	8.6	0 583
7	26.47	−3.80	1480	30.5	0.545
8	30.09	−6 06	1474	59.7	0.502
9	33 50	−8.59	1414	92 9	0.462
10	36 70	−11.40	1340	129 5	0.428
	38.24		9739	440.9	5 923

$$M_0 = \frac{\Sigma \frac{m_L + m_R}{I}}{2\Sigma \frac{1}{I}} = 60.74 \qquad H_0 = \frac{-\Sigma \frac{(m_L + m_R)y}{I}}{2\Sigma \frac{y^2}{I} + 2\Sigma \frac{1}{A}} = 893.6$$

$$V_0 = \frac{\Sigma \frac{(m_L - m_R)x}{I}}{2\Sigma \frac{x^2}{I}} = 19{,}478$$

Temperature Stress:

$$H_0 = \frac{\alpha t n E}{\Sigma \frac{y^2}{I} + \Sigma \frac{1}{A}} = 446.8$$

the two sections of the arch at the crown and exerting a thrust definitely known in magnitude and point of application. Precast concrete slabs are then inserted to carry the weight of the structure, made of such thickness that the previously determined

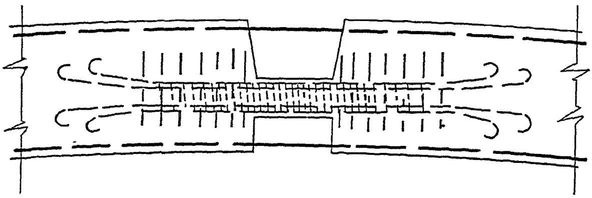


FIG. 116

crown thrust is brought into action. This process results in lifting the arch away from the centering which can accordingly be constructed in a simpler manner than usual, without devices for striking. By making the closure at the desired temperature definite knowledge is had of the temperature which causes no stress, thus removing another element of uncertainty in design.

COMPUTATIONS FOR HINGELESS ARCH

Sheet A7

Least Work. Dead Stresses.

Sect.	$m_L = m_R$	$\dfrac{m_L + m_R}{I}$	$\dfrac{(m_L + m_R)y}{I}$
1	0		
2	9.9	99	+ 295
3	26.1	238	516
4	54.9	419	(1294) 483
5	86.7	552	− 94
6	132.8	696	1260
7	179.3	758	2875
8	239.9	782	4738
9	298.7	753	6470
10	367.2	731	8330
		5028	−22,473

$$M_0 = \frac{5028}{60.74} = 82.8 \text{ kip ft.}$$

$$H_0 = \frac{22{,}473}{893.6} = 25.1 \text{ kips}$$

$$M_c = 82.8 - 25.1 \times 3.21 = 2200'\#$$

$$e = \frac{2200 \times 12}{25{,}100} = 1.1'' \text{ above axis}$$

$$= 0.09' \text{ above axis}$$

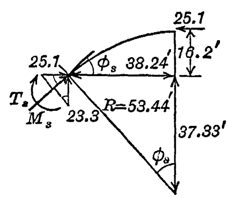
$$M_s = -402.9 + 25.1 \times 16.20 = +3700'\#$$

(See Sheet A4)

$$T_s = 25.1 \times 0.698 + 23.3 \times 0.715 = 34{,}200\#$$

$$e = \frac{3700 \times 12}{34{,}200} = 1.30'' \text{ above axis}$$

$$= 0.11' \text{ above axis}$$



$\sin \phi_s = 0.715$

$\cos \phi_s = 0.698$

Stresses — Crown

$A_c = 199\square''$

$I_c = 3960''^4$

$$Max.\, f_c = \frac{25{,}100}{199} \pm \frac{2200 \times 12 \times 7.5}{3960}$$

$$= 126 \pm 50 = 176\#/\square'' \text{ top fibre.}$$

$$= 76 \quad \text{bottom fibre.}$$

Springing

$A_s = 355\square''$

$I_s = 24{,}700''^4$

$$Max.\, f_c = \frac{34{,}200}{355} \pm \frac{3700 \times 12 \times 14}{24{,}700}$$

$$= 96 \pm 25 = 121\#/\square'' \text{ top fibre.}$$

$$= 71 \quad \text{bottom fibre.}$$

COMPUTATIONS FOR HINGELESS ARCH

Sheet A8

Least Work: Influence Table — Data:

Sect.	$m_L = m_L + m_R = m_L - m_R$	$(m_L + m_R)\frac{1}{I}$	$(m_L - m_R)\frac{x}{I}$	$(m_L + m_R)\frac{y}{I}$
		Load at A_L		
2	2.55	12.7	81	+36.1
3	6.74	30.7	325	66.7
4	10.88	41.6	612	150.6 47.8
5	14.93	47.5	891	−8.1
6	18.84	49.4	1119	89.4
7	22.64	47.8	1264	181.7
8	26.26	42.7	1285	259
9	29.67	37.4	1253	321
10	32.87	32.7	1200	373
		342.5	8030	−1081.6
		at B_L		
4	3.21	12.3	181	+14.1
5	7.26	23.1	433	−3.9
6	11.17	29.3	664	53.0
7	14.97	31.6	836	120.1
8	18.59	30.2	908	183.0
9	22.00	27.7	928	238
10	25.20	25.1	921	286
		179.3	4871	−869.9
		at C_L		
6	3.50	9.2	209	−16.6
7	7.30	15.4	407	58.6
8	10.92	17.8	536	107.9
9	14.33	18.0	603	154.6
10	17.53	17.4	639	198.4
		77.8	2394	−536.1
		at D_L		
8	3.26	5.3	160	−32.1
9	6.67	8.4	281	72.2
10	9.87	9.8	360	111.7
		23.5	801	−216.0
		at E_L		
10	2.20	2.19	80.4	−24.96

Load at A_L:

$$M_0 = \frac{342.5}{60.74} = 5.64$$
$$V_0 = \frac{8030}{19478} = 0.412$$
$$H_0 = \frac{1081.6}{893.6} = 1.210$$

At B_L:

$$M_0 = \frac{179.3}{60.74} = 2.95$$
$$V_0 = \frac{4871}{19478} = 0.250$$
$$H_0 = \frac{869.9}{893.6} = 0.974$$

At C_L:

$$M_0 = \frac{77.8}{60.74} = 1.280$$
$$V_0 = \frac{2394}{19478} = 0.123$$
$$H_0 = \frac{536.1}{893.6} = 0.600$$

At D_L:

$$M_0 = \frac{23.5}{60.74} = 0.387$$
$$V_0 = \frac{801}{19{,}478} = 0.0411$$
$$H_0 = \frac{216.0}{893.6} = 0.242$$

At E_L:

$$M_0 = \frac{2.19}{60.74} = 0.0360$$
$$V_0 = \frac{80.4}{19{,}478} = 0.00413$$
$$H_0 = \frac{24.96}{893.6} = 0.0280$$

COMPUTATIONS FOR HINGELESS ARCH

Sheet A9

Least Work: Influence Table.

	Unit Load at				
	A_L	B_L	C_L	D_L	E_L
M_0	5.64	2.95	1.280	0.387	0.0360
H_0	1.210	0 974	0.600	0 242	0.0280
$V_0 = V_R$	0.412	0 250	0.123	0 0411	0 00413
V_L	0.588	0.750	0.877	0.9589	0.99587
$-3.21H_0$	−3.89	− 3.128	− 1.927	− 0 777	−0 0899
M_{crown}	+ 1.75	− 0 178	− 0 647	− 0.390	−0.0539
$12.90H_0$	15.61	12 56	7.74	3 120	0.3612
$38.24V_0$	15.76	9 56	4.71	1 573	0.1580
	37.01	25 07	13.73	5 080	0.5552
kL	−34.407	−26 740	−19.074	−11.407	−3.741
M_s left	+ 2.61	− 1 67	− 5.34	− 6.33	−3.186
M_s right	+ 5.49	+ 5 95	+ 4 31	+ 1.93	+0.239
$H_0 \cos \phi_s$	0.845	0 680	0.419	0 169	0.0196
$V_L \sin \phi_s$	0.420	0.536	0 627	0 685	0.712
T_s left	1.265	1.216	1.046	0.854	0.732
$H_0 \cos \phi_s$	0.845	0.680	0.419	0.169	0.0196
$V_R \sin \phi_s$	0.295	0 179	0.088	0.030	0.0030
T_s right	1.140	0 859	0 507	0.199	0.0226

$M_c = M_0 - 3.21\, H_0$

For load on left half

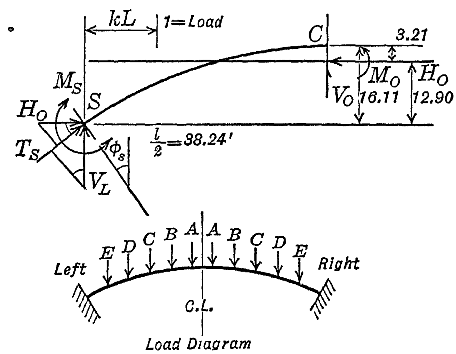
$M_{sLeft} = M_0 + 12.90\, H_0 + 38.24\, V_0 - kL$

$M_{sRight} = M_0 + 12.90\, H_0 - 38.24\, V_0$

$T_s = V \sin \phi_s + H \cos \phi_s$

$= 0.715\, V + 0.698\, H$

(Sheet A7)



COMPUTATIONS FOR HINGELESS ARCH

Sheet A10

Least Work: Maximum Live Thrust and Moment at Crown and Springing.

Data from Influence Table.

		Loading	Unit Load Values	Live Load Stresses
Moment At Crown	(+) (−)	$A_L A_R$ B-C-D-E right and left	+3 50 −2.54	5,160'# 3,740
H_0 with "	$+M_c$ $-M_c$		2.42 3.69	3,570 # 5,440
Moment at Springing	(+) (−)	A_L-A_R-B_R-C_R-D_R-E_R B_L-C_L-D_L-E_L	20 5 16.5	30,200'# 24,400
Thrust with	$+M_s$ $-M_s$		3.99 3.85	5,880 # 5,680

Live panel load = 192 × 7.67 = 1475#

Temperature Stress

For 60° fall: $H_0 = -\dfrac{0.000006 \times 60 \times 10 \times 2{,}500{,}000 \times 144}{446.8}$

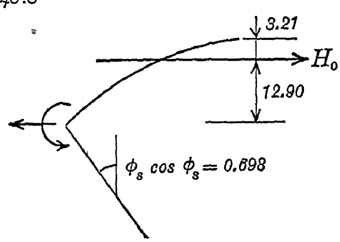
$= -2900\#$

$M_c = 2900 \times 3.21 = +9300'\#$

$T_s = 2900 \times 0.698 = -2030\#$

$M_s = 2900 \times 12.90 = -37{,}400'\#$

For 20° rise divide above values by (−3).



COMPUTATIONS FOR HINGELESS ARCH

Sheet A11

Stress Summary and Unit Stresses.

Crown		Thrust = H_0#	'# Moment = M_C	e [1]	e/h [2]	L [3]	fc [3]	fs [3]
Max +M	Dead Live	+25,100 3,600	+2,200 5,200					
		28,700	7,400	3 1″	0 21	0 110	300	3,000±
	Temp. (fall)	−2,900	+9,300					
		25,800	16,700	7 8″	0 52	0 122	610	7,000±
Max −M	Dead Live	+25,100 5,400	+2,200 −3,700					
		30,500	−1,500	0 6″	0 04			No Tension
	Temp. (rise)	+1,000	−3 100					
		31,500	−4,600	1 8″	0 12			

[1] $e = \dfrac{M_C}{H_0}$. [2] $\dfrac{e}{h} = \dfrac{e}{15}$. [3] From Plate XV $f_C = \dfrac{M}{bh^2L} = \dfrac{M \times 12}{12 \times \overline{15}^2\, L}$.

Data: $p = 0.0050$ *in each face*
$d'/h = 0.10$ *assumed: actual value* $= \frac{2}{15}$.

Springing		Thrust = T_S	Moment = M_S	e	e/h	L	fc	fs
Max. $+M_S$	Dead Live	+34,200 5,900	+3,700 30,200					
		40,100	33,900	10 2″	0 36	0 106	410	4,000±
	Temp. (rise)	+ 700	12,500					
		+40,800	46,400	13 6	0 49	0 100	590	9,000±
Max. $-M_S$	Dead Live	+34,200 5,700	+3,700 −24,400					
		+39,900	−20,900	6 2	0 22	0 101	260	2,000±
	Temp. (fall)	−2,000	−37,400					
		+37,900	−58,100	18 4	0.65	0 096	770	15,500

Data: $h = 28''$
$p = \frac{15}{28} \times 0.0050 = 0.0027$ *in each face*
$d'/h = 0.10$ *assumed: actual value* $= \frac{2}{28}$.

CHAPTER XVII

PLANS AND DETAILS

137. Engineering design is expressed by drawings. They must be clear, complete and free from ambiguity for the sake of the work and must be made as economically as possible. Too often the latter consideration is allowed to outweigh the former.

138. Drawings. Concrete drawings have a double purpose. First, to show the outlines of the concrete, and second, the size, shape and location of the reinforcement. Information regarding both materials is usually combined on a single drawing. Where there are many complications, however, it is good practice to have separate "outline" and "reinforcement" drawings. The criterion for separate drawings is whether or not all necessary dimensions can be placed on a single drawing without confusion. A floor with many openings, or with machine pedestals, or both, will usually require so many dimensions to show properly the size and location of corners in the concrete that there will not be room left to show the reinforcement clearly. This is also often true of large turbine foundations and similar elaborate structures.

The general method of making concrete drawings is similar to that of making structural steel drawings. But concrete drawings, particularly framing plans showing slab steel, are often more complicated than steel drawings for there are more members and they are more closely spaced in concrete than in steel construction. It is therefore particularly necessary that they be made carefully and clearly.

An assembly drawing which gives the size, location and mark of each member (beam, column, wall, etc.) is the first drawing made for concrete as for steel. The individual members are then detailed at larger scale. Slabs are usually detailed on the assembly drawing.

The ordinary principles of drafting suffice to show outlines. In showing reinforcement on drawings there are certain conventions which are helpful and reinforcement drawings are in general diagrammatic in character. For illustration the rods shown in Fig.

117*a* might in fact be as diagrammed in Fig. 117*b* or *c* and properly should be shown as in one of these latter views. Dotted lines are sometimes used to differentiate the reinforcement from outlines, but because of the possibility of ambiguity full heavy lines are better. They are also quicker to draw. Dash lines can then be used to indicate rods extending into the member detailed which are a part of and detailed with some other member.

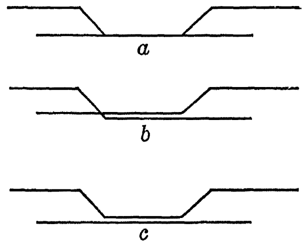


FIG. 117

Assembly drawings of concrete work are usually made at $\frac{1}{4}$ or $\frac{1}{8}$ inch = 1 foot and details at $\frac{1}{2}$ or $\frac{1}{4}$ inch = 1 foot. On account of their complexity it is unwise to use too small a scale and crowd these drawings.

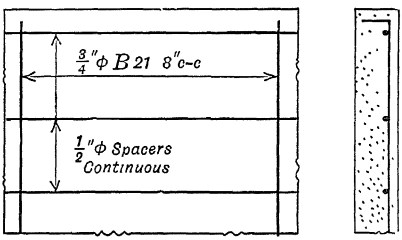


FIG. 118

139. Reinforcement. Slab and wall reinforcement naturally divides into groups or bands of identical parallel rods. Only the two outer rods of such bands are ordinarily shown on a plan, for to show all would confuse the plan to no advantage. These bands are then labelled as shown in Fig. 118. Rods which show in several views may and often should have type marks on them to identify rods in different views, but they should have a complete label and be called for in only one view to avoid duplication in taking off and ordering. It is important that rods be called for in the proper view. They should be listed with the part or member in which they will first be set during construction. For example, an angle rod extending from a wall into a floor as shown in Fig. 119 should be listed in the wall detail rather than in the floor detail, for there will ordinarily be a construction joint at the top of the wall and these rods must be placed in pouring the wall. Other things being equal it is clearer to list rods in a view where they show in elevation as straight lines as shown in Fig. 118. Ambiguity in listing as well as

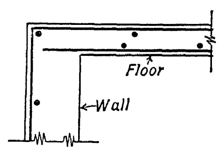


FIG. 119

in drawing should be avoided: for example, listing as shown in *a*, Fig. 120, should be shown as in *b* or *c*, whichever is meant. Fig. 121 shows a part of a concrete floor plan illustrating the foregoing. A wall elevation would be similar. It is often necessary to show a few cross-sections of complicated places in walls and slabs in addition to the plan. The typical reinforcement can usually be shown laid over 90 degrees in the plan as is done in the illustration.

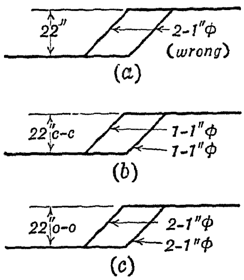


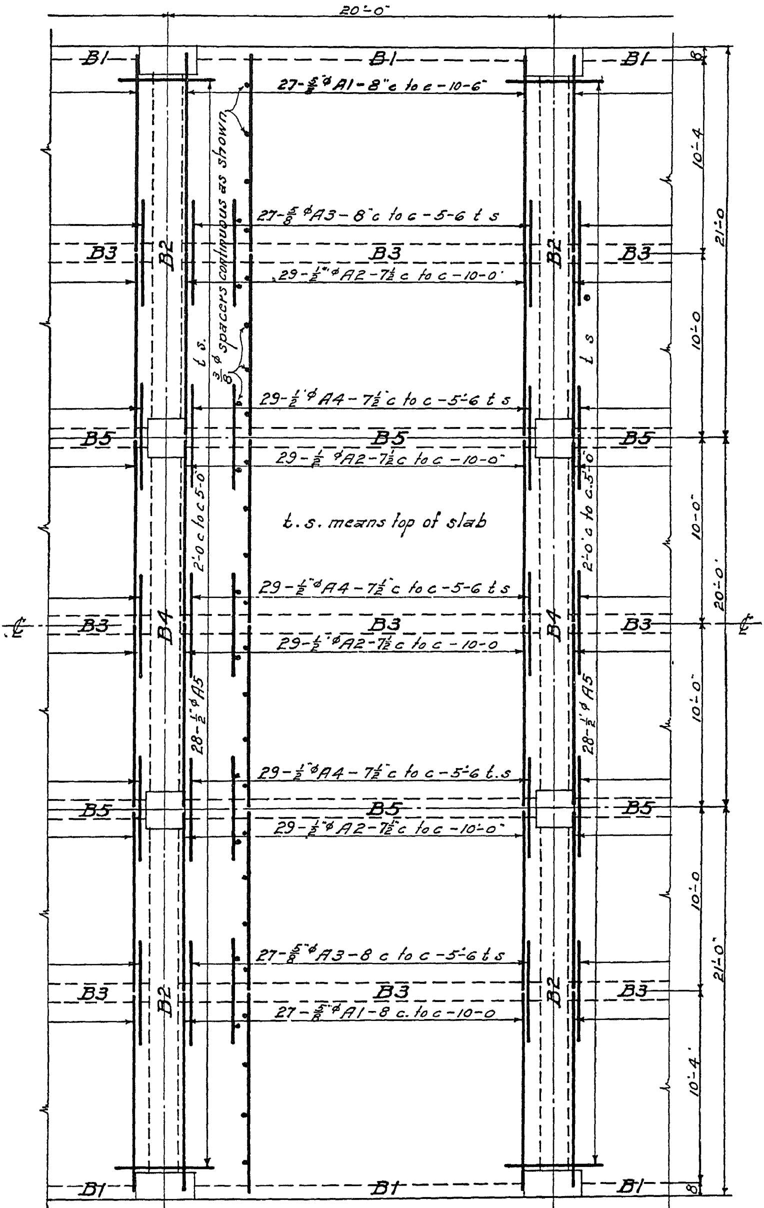
FIG. 120

Beams are detailed by an elevation and section as shown in Fig. 122. It is important that the detail show the location and angle of bends and the lap of rods beyond the centerline. The length of the rods need not ordinarily be figured on the engineer's drawing, nor need the stirrups be dimensioned, as the reinforcing contractor can figure them directly from the concrete dimension. This illustration shows the diagramming of rods over supports with actual location shown in section. Where there are numerous beams which are similar but not identical they can often be detailed by drawing one beam with letters in place of dimensions and then tabulating the value of the various lettered dimensions for each beam.

In ordinary building work it is advisable to detail a typical interior and a typical exterior column in the same general manner that beams are detailed. The balance of the columns are then covered in a schedule similar to that shown in Fig. 123.

Irregular structures should be separated and detailed as slab, beam, and column units so far as possible. Where this is not possible, the foregoing principles, aided by good judgment, will give a satisfactory solution.

The one requirement of bending sketches for steel reinforcement is that they be definite. It is not enough to show the height of a bend as in Fig. 120*a*: it should be indicated whether the figure is the desired dimension out to out or center to center or clear. If the total length occupied by a bar is limited, that limit (with proper allowance for clearance) should be explicitly stated, as otherwise there may be a variation of as much as 2 inches in the wrong direction. The sketches shown in Fig. 124 are those used on his order sheet by a large dealer in reinforcing steel and show the dimen-



PORTION OF A FRAMING PLAN

FIG. 121

sions required for proper bending of the steel. Other companies have similar standards that differ somewhat in detail. It should be remembered that the angle of bend in a bar like Type 9 is not sharp as shown, since the bar is bent around a pin, usually with a diameter of about 3 inches. Precision beyond the nearest whole inch in figuring the length M is not needed. This dimension is required only for computing the length of the bar. There is good reason for requiring that this bend be gradual. Such a bar is heavily stressed and there are heavy bearing stresses brought onto the concrete at these points, tending to split the beam. Hooks, similarly, bring heavy splitting stresses to the concrete and require a large mass for embedment and often cross reinforcement or an enclosing spiral if they are to be effective.

140. Details. The best of designs may be futile if the details are neglected, and furthermore, even if requirements of strength are met, poor details may greatly increase the expense of the work. So careful attention to details cannot be too strongly recommended. Understanding of construction methods, observation and experience are necessary to make a good detailer. An endeavor is made in this section to point out the general requirements and some of the most common problems to be met.

140a. Forms. Concrete outlines must be such that they can be formed and the forms removed with reasonable facility. An example of consideration of form work might be taken as the case of typical beams having a depth of 27 inches framing into typical girders 28 inches deep, a common relation of beam and girder sizes to avoid interference of steel at intersections. In this case it will often be cheaper to put an extra inch of fireproofing on the beam steel making the bottom of the beam flush with the bottom of the girder. Forms are considered in Chapter V which should be read again in this connection.

140b. Rod Spacing. A theoretical minimum spacing for beams, slabs and rods can be deduced from the bond stress and the allowable shear on the concrete between the rods. Except for the largest rods this theoretical spacing is too close for good construction, however. The spacing must be such that coarse aggregate will not arch between rods in pouring and cause voids. The clear distance between rods should be at least $1\frac{1}{4}$ times the size of the maximum aggregate, according to the Joint Committee, but more is preferable where several rods and considerable length are in-

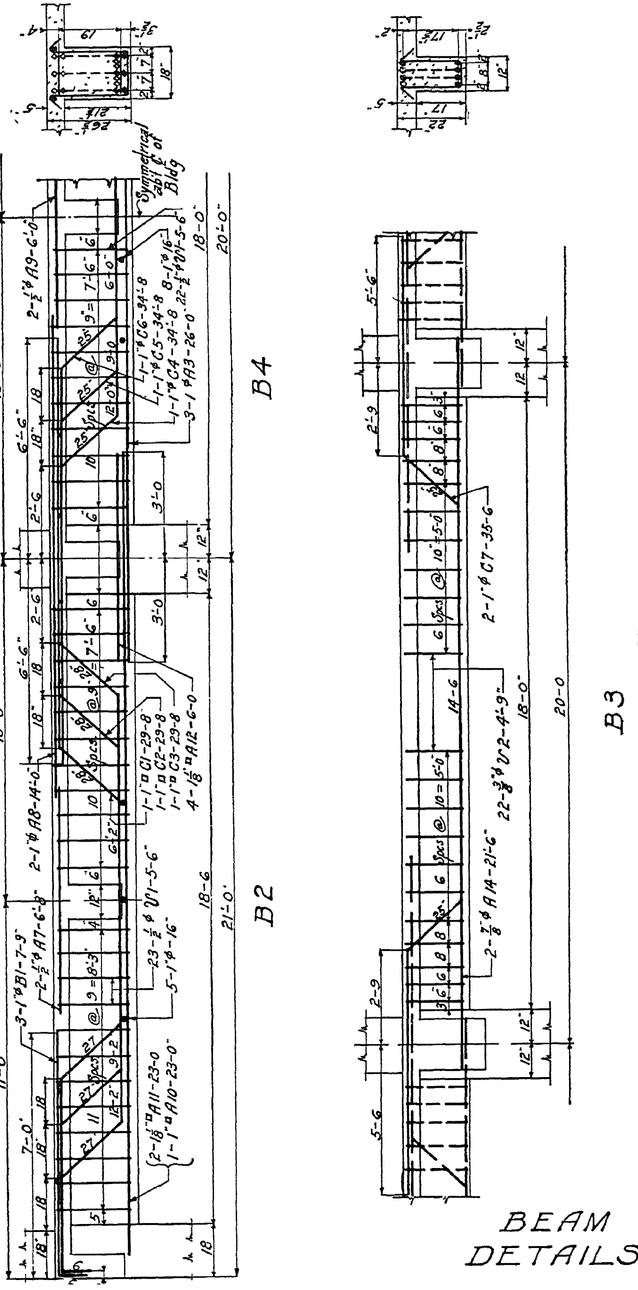


FIG. 122

volved. The maximum spacing for slab rods is $2\frac{1}{2}$ times the slab thickness according to the 1916 Joint Committee, but this is larger than common practice, which seldom exceeds $1\frac{1}{2}$ times the thickness. Maximum spacing is seldom, if ever, a consideration in beam rods. The steel is often in two layers, however, and should in these cases be separated by steel spacers — usually one inch in diameter. Spacing of vertical column steel should be 2 to 3 times the aggregate size in the clear. In large columns it sometimes becomes impossible to get enough steel in a single layer, and if so, the extra steel can usually be more satisfactorily placed along diameters rather than in a double surface layer.

140c. Spacers. Secondary reinforcement in slabs, walls and miscellaneous structures at right angles with the main reinforcing is used for the dual purpose of distributing live load and holding the main reinforcement in place during the pouring of concrete. The latter purpose is of great importance. In ordinary slabs $\frac{3}{8}$-inch round rods 2′0″ c.–c. are commonly used; in walls spacers are often made one size smaller than the main reinforcement and placed 1 to 2 feet apart according to conditions. Stirrups in beams and hoops in columns tie the main steel sufficiently. It is good practice to provide loop bars under the hooks of stirrups as shown in Fig. 122. In miscellaneous structures it is important that sufficient spacers be used to insure rigidity of steel during the pouring of concrete.

140d. Splices. In general it is not considered good practice to splice reinforcing bars under stress. There are many exceptions to this, however. It is necessary to splice rods in columns and other compression members under stress. This is usually done by lapping them a sufficient distance to develop the stress of the rod in bond. Rods from the lower section to the number of those in the section above extend above the floor. Sometimes this leads to such an excessively close spacing of steel as to make the probability of good concrete at the section very small. In such cases only part of the steel is lapped and the concrete is locally over-stressed as the lesser of two evils.

Tension rods carry greater stress than compression rods and when it is necessary to splice them under stress, as in standpipes, they should be fastened together with wire clips or in some equally satisfactory method.

In heavy cantilevers it is often desirable to drop off part of the

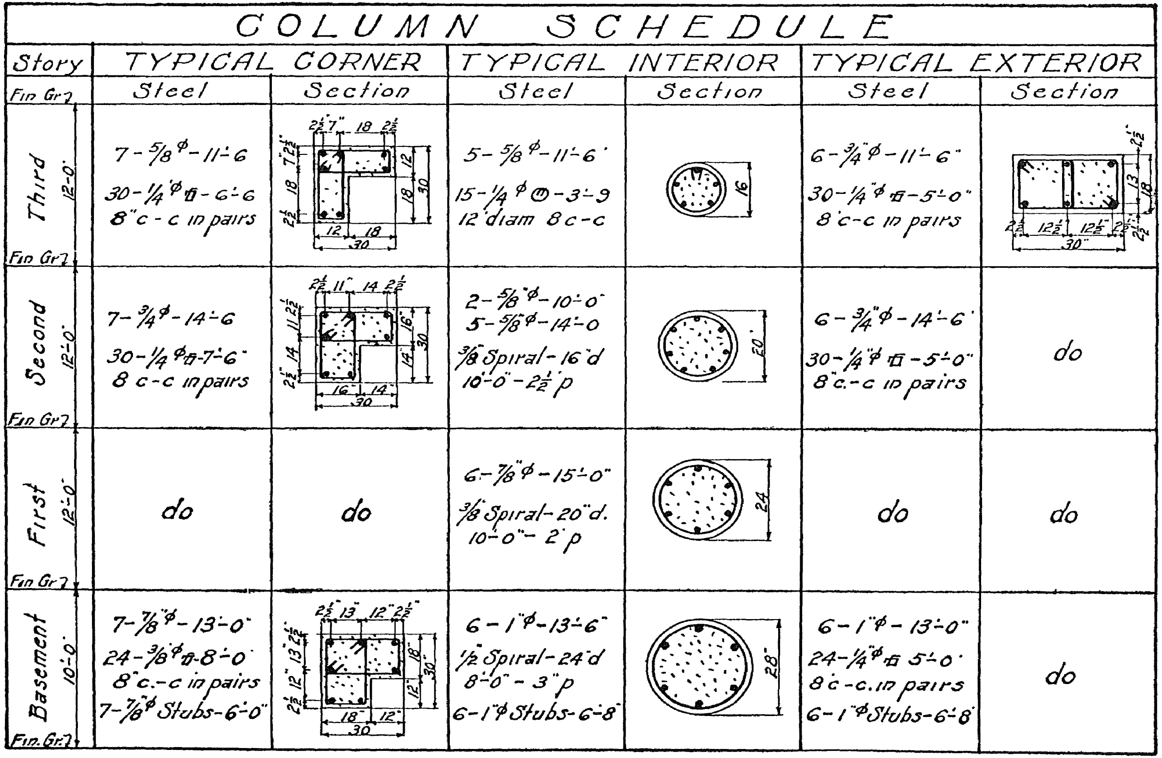

COLUMN SCHEDULE
Story
TYPICAL CORNER
TYPICAL INTERIOR
TYPICAL EXTERIOR
Steel
Section
Steel
Section
Steel
Section
Fin Gr
Third
12'-0"
7-5/8"φ-11'-6
30-1/4"φ ▭-6'-6
8"c-c in pairs
5-5/8"φ-11'-6"
15-1/4"φ ◯-3'-9
12"diam 8c-c
6-3/4"φ-11'-6"
30-1/4"φ ▭-5'-0"
8"c-c in pairs
Second
12'-0"
7-3/4"φ-14'-6
30-1/4"φ ▭-7'-6"
8 c-c in pairs
2-5/8"φ-10'-0"
5-5/8"φ-14'-0
3/8" Spiral-16"d
10'-0"-2½"p
6-3/4"φ-14'-6"
30-1/4"φ ▭-5'-0"
8"c.-c in pairs
do
First
12'-0"
do
do
6-7/8"φ-15'-0"
3/8" Spiral-20"d.
10'-0"-2"p
do
do
Basement
10'-0"
7-7/8"φ-13'-0"
24-3/8"φ ▭-8'-0"
8"c.-c in pairs
7-7/8"φ Stubs-6'-0"
6-1"φ-13'-6"
1/2" Spiral-24"d
8'-0"-3"p
6-1"φ Stubs-6'-8"
6-1"φ-13'-0"
24-1/4"φ ▭ 5'-0"
8"c-c. in pairs
6-1"φ Stubs-6'-8"
do
16
20"
24
28"
30
30"

Fig. 123

steel as the bending moment falls off. It is obviously contrary to theoretical considerations to cut off rods under stress because of bond and it has been common to loop the ends. A better way is to use a hook or to bend the cut rods to the neutral axis at such an angle as to give bond distance between the bend and the neutral axis as shown in the sketch of the retaining wall on page 143. This satisfies both practical and theoretical considerations.

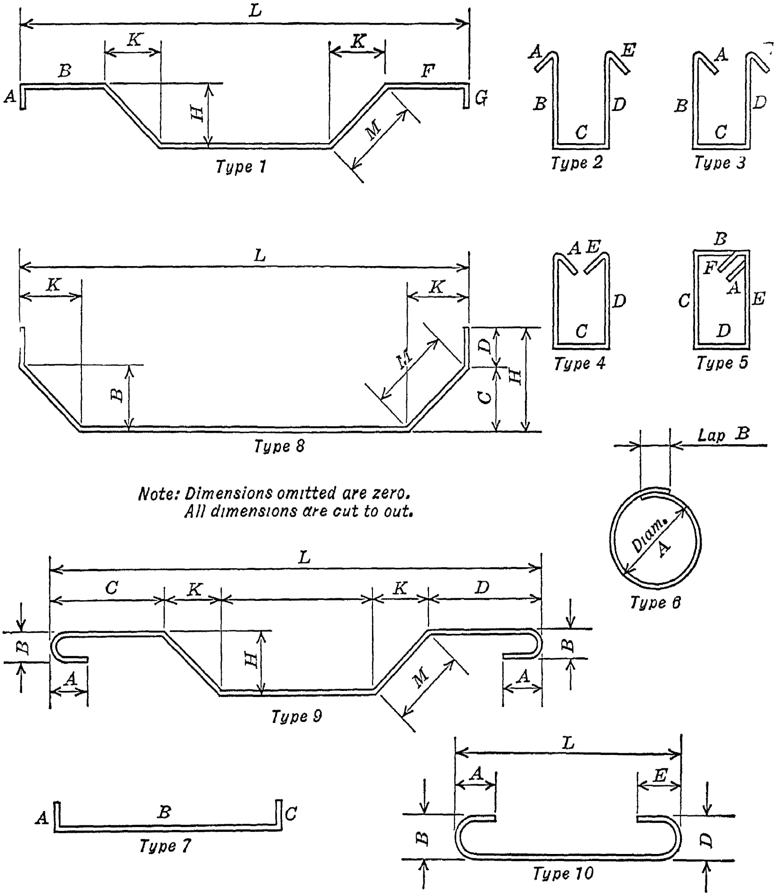
As a general rule splices should be made at construction joints to facilitate field work. That is, rods will project bond distance beyond the joint and then the next tier rods can butt the joint. Short dowel rods, called stubs, which extend bond distance each side of a joint, are used to take steel stress into footings and are often useful in miscellaneous structures.

140e. Connections. In detailing reinforcing, interference at joints often needs attention. Small rods, $\frac{1}{2}$-inch and less, can easily be bent in the field or allowed to sag into place. Larger rods are not pliable; and interfering layers of steel at beam and girder intersections, and beam steel dimensioned so as to intersect column rods, should be avoided.

The location of cambers (or bends) in beam and slab bars is a design consideration, although often left to the detailer, and is considered in Chapter XII.

140f. Construction Joints. In ordinary concrete work it is not possible to pour entire structures in a continuous operation, hence there must be construction joints. The general characteristics of a construction joint are absence of tension value, weakness in shear and reasonable strength in compression. Therefore joints in floors are made as far as possible at mid span. Joints are often toothed or keyed by the insertion of blocks of wood in the first section poured and knocked out after the concrete has set. Horizontal joints are subject to the formation of laitance, a scum of the finest cement, which is soft. It is not possible to avoid this entirely, but it can be lessened by avoiding an excess of water. Laitance should always be cleaned off before proceeding to pour the next layer of concrete.

In miscellaneous structures joints should be located as far as possible with regard to convenience in splicing steel and setting up the next tier of forms: for example, at a point where there is a horizontal break in the structure. Joints in columns are usually fixed by floors or spandrel beams. Joints in floors are a field



Proper allowances for stretch of bars due
to bending are made in shop.

No. Pieces	Size		Length	Mark	Type	Straight Bars Chk. Here	A	B
	Def.	Plain						

FIG. 124

problem subject to a general specification or the approval of the field engineer; but in many structures they should be considered by the designer and shown on the drawings. For example, in a tunnel of rectangular section it is necessary that there be a construction joint at the top of the floor and at the underside of the roof. It is also necessary that these joints be able to sustain in shear the reaction of the wall due to earth pressure. They must, therefore, be designed to resist pressure from one or both sides.

CHAPTER XVIII

ECONOMY IN DESIGN

141. Good engineering design is of course economical design. With reinforced concrete, as with any other engineering material, really good design must of necessity be the product not only of a knowledge of the principles involved but also of considerable experience in their application. Sundry formulas have been published in the past which purported to give economical proportions of concrete members when the relative cost per unit of volume of concrete and steel were known. In the main they were interesting rather than useful. Their lack of practical value was due to the fact that they did not include all the variables. In the opinion of the authors, it is not practical to devise a workable formula of general application for economical proportions of concrete members.

142. Factors to be Considered. The direct cost of a concrete member may be divided into three items: concrete, steel and forms. These vary independently of each other. In addition to these, the effect of changed concrete dimensions on the cost of other parts of the building or structure must be determined. In considering a building column, for example, the value of the extra floor space taken by a lightly reinforced column must be considered in comparing its cost with that of a smaller hooped column.

The minimum clear story height of a building is usually determined by considerations other than structural. In comparing economy of section of typical floor beams, therefore, the additional length of columns, partitions and walls must be debited against deeper beam sections. If the beam under consideration is typical for several floors this may be a large item.

One or more dimensions of concrete members is often directly determined by other than structural requirements. If a large proportion of the beams of a floor are typical, the others, except for secondary headers or trimmers at stairs and shafts, will be made the same depth. This will make forming simpler because "jacks" or supports for forms will be the same length; the ceiling will look better and it will be more convenient for supporting

shafting or pipes. A beam may have its width limited to that of a partition or, if a spandrel, it may have both dimensions determined by architectural considerations.

143. Methods of Comparison. The method to be used in determining economical proportions is to make several designs, compute the cost of each, and select the cheapest which fulfills all fixed conditions, or determine whether some advantage such as appearance or the convenience of a more expensive method is sufficient to offset its extra cost over the cheapest. The design need not be complete in all details nor do the unit costs need to be precise; relative costs are what are needed. This is no royal road but it is the only one which leads to satisfactory results.

In applying this method it is also necessary to consider when and how to apply it. It would be obviously absurd to make such analysis of each member of a building, and equally absurd to omit entirely such computations of an important structure. In practice, consideration of a typical bay of a reasonably regular building or of a few isolated sections of an irregular building are sufficient.

Certain members should be considered as a group in computations of this character because of their interdependence; for example: a two or three-span beam across a building or a complete column. On a structure figured for wind or other loads in addition to floor loads it will usually be best to consider a complete bay or bent as a unit in estimating comparative costs.

This consideration of a complete bay is also necessary in studying the most difficult problem of this type, namely, special or patented types of construction. The great difficulty of this type of problem lies in the personal equation of the contractor, for different contractors will often disagree entirely on costs of unusual or patented systems. Since the economy of patented systems often hinges on savings in certain unit costs, or extra labor which offsets material savings, the best that the engineer can do in such a case is to check over his unit prices with one or more of the contractors who are expected to figure the work, get bids for any patented material or forms, and get estimated costs on any special work the contractor has to do. It is better to get prices from the contractor rather than from a salesman, although the latter may at times be a great help.

The following examples are taken from "Economy in the Design of Reinforced Concrete Building" by Clayton W. Mayer of the

Aberthaw Construction Co., published in the 1918 Proceedings of the American Concrete Institute, to which the student is referred for an excellent treatment of the subject. It must be emphasized that unit prices vary with location and time and it is necessary to use costs which are applicable.

Example 53. Comparative costs of various schemes for the design of a concrete column for a flat slab floor. Fig. 125.

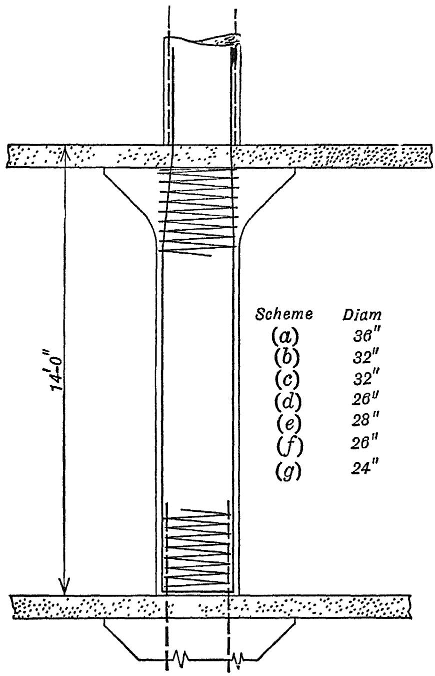


FIG. 125

Design	*Comparative Estimates*		
Scheme (*a*):			
36-in.-dia. column	Concrete	99 cu. ft. at 34¢	$ 33.66
11-1⅛-in. rd. vert. rods	Forms	Round steel	15.00
⅜-in. rd. hoops 12 in. o.c.	Reinfct	716 lb. at 5¢	35.80
Mix 1 : 2 : 4	Lost floor space	5 sq. ft. at $2.75	13.75
	Total		$ 98.21
Scheme (*b*):			
32-in.-dia. column	Concrete	79 cu. ft. at 34¢	$ 26.86
23-1⅛-in. rd. vert. rods	Forms	Round steel	15.00
⅜-in. rd. hoops 12 in. o.c.	Reinfct	1437 lbs. at 5¢	71.85
Mix 1 : 2 : 4	Lost floor space	$3\frac{1}{10}$ sq. ft. at $2.75	8.53
	Total		$122.24

Scheme (*c*):

32-in.-dia. column	Concrete	79 cu. ft. at 36½¢	$ 28.84
12-1⅛-in. rd. vert. rods	Forms	Round steel	15.00
⅜-in. rd. hoops 12 in. o.c.	Reinfct.	770 lbs. at 5¢	38.50
Mix 1 : 1½ : 3	Lost floor space	3 1/10 sq. ft. at $2.75	8.53
		Total	$ 90.87

Scheme (*d*):

26-in.-dia. column	Concrete	52 cu. ft. at 36½¢	$ 18.98
11-1-in. rd. vert. rods	Forms	Round steel	15.00
1 per cent spirals (18½ lb.) per lin. ft.	Reinfct.	514 lb. at 5¢	25.70
	Spirals	264 lb. at 5½¢	14.52
Mix 1 : 1½ : 3	Lost floor space	7/10 sq. ft. at $2.75	1.92
		Total	$ 76.12

Scheme (*e*):

28-in.-dia. column	Concrete	60½ cu. ft. at 43¢	$ 26.02
20-1⅛-in. rd. vert. rods	Forms	Round steel	15.00
⅜-in. rd. hoops 12 in. o.c.	Reinfct.	1255 lb. at 5¢	62.75
Mix 1 : 1 : 2	Lost floor space	1.45 sq. ft. at $2.75	3.99
		Total	$107.76

Scheme (*f*): (Most Economical)

26-in. dia. column	Concrete	52 cu. ft. at 43¢	$ 22.36
7-⅞-in. rd. vert. rods	Forms	Round steel	15.00
1 per cent spirals (18½ lb.) per lin. ft.	Reinfct.	245 lb. at 5¢	12.25
	Spirals	264 lb. at 5½¢	14.52
Mix 1 : 1 : 2	Lost floor space	7/10 sq. ft. at $2.75	1.92
		Total	$ 66.05

Scheme (*g*):

24-in.-dia. column	Concrete	44½ cu. ft. at 43¢	$ 19.14
10-1⅛-in. rd. vert. rods	Forms	Round steel	15.00
1 per cent spirals (16 lb.) per lin. ft.	Reinfct. (Vert.)	606 lb. at 5¢	30.30
	Spirals	229 lb. at 5½¢	12.60
Mix 1 : 1 : 2			
		Total	$ 77.04

Note that forms are the same for all columns using steel forms, and that the floor space figured is the excess over the area of smallest columns of the enclosing squares, which is really the measure of the lost space. The difference in cost of richer concrete mixes is the cost of extra cement. In this case rich concrete is more economical than lean, concrete is more economical than steel, spirals are more economical than vertical steel. These relations are usual.

Example 54. Comparative costs of plain and reinforced concrete footings.

Scheme (*a*), reinforced type (mix 1 : 2 : 4), see Fig. 126.

Concrete 1 : 2 : 4	460 cu. ft. at 34¢	$156.40
Forms (none)		
Reinforcement	420 lb. at 5¢	21.00
Excavation	19¼ cu. yd. at $1.00	19.25
Backfill and level	19¼ cu. yd. at 30¢	5.78
3-in. (close) sheeting	182 sq. ft.	18.20
Total		$220.63

Scheme (*b*), plain type (mix 1 : $2\frac{1}{2}$: 5):

Concrete 1 : $2\frac{1}{2}$: 5	507 cu ft. at 32¢	$162.24
Forms (top block)	84 sq. ft. at 15¢	12.60
Excavation	24 cu yd at $1 00	24.00
Excavation below 5-ft. mark	$5\frac{1}{2}$ cu yd. at $1.50	8.25
Backfill and level	$29\frac{1}{2}$ cu. yd. at 30¢	8.85
3-in. (close) sheeting	270 sq. ft. at 10¢	27.00
Total		$242.94

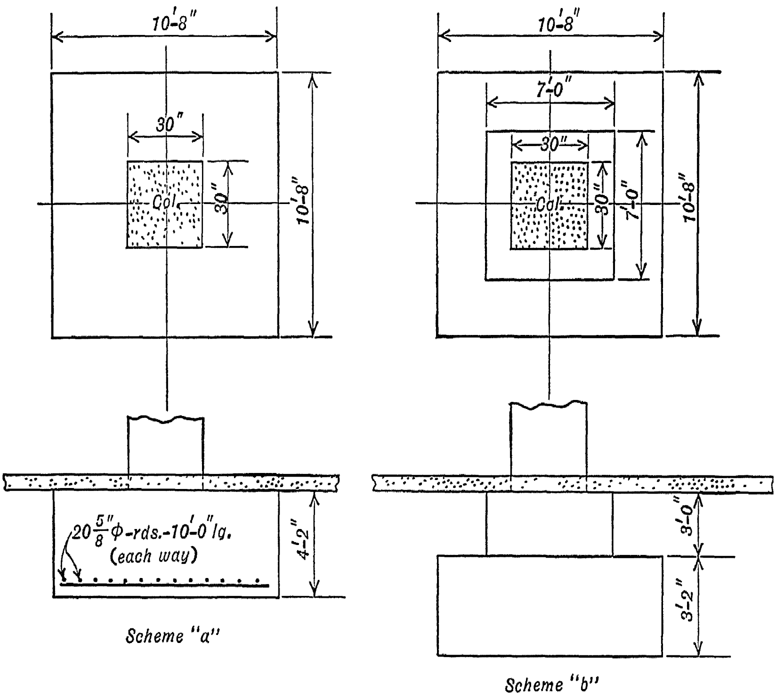


FIG. 126

Note that the deeper excavation is more expensive than the shallow and that the excavation is made so that the sheeting is used as a form.

Problem 38. Compare cost of design of floor in Chapter XII with a similar one using two intermediate beams in each bay, using concrete and steel prices as given in the examples and, for forms, 20 cents per sq. ft. for slabs and beam bottoms and 30 cents for beam sides.

APPENDIX A[1]

PROPORTIONS[2] FOR CONCRETE OF GIVEN COMPRESSIVE STRENGTH AT 28 DAYS

This table gives the Nominal Mix, being based on volumes of dry aggregate compacted by rodding in the measure, as specified in the Standard Method of Test for Unit Weight of Aggregate for Concrete (Serial Designation: C 29–21) of the American Society for Testing Materials. Corrections should be made in the quantities to take account of the bulking effect of mositure in the fine aggregate. The bulking of fine aggregate (swelling) due to contained moisture, and the method of placing it in the measure, may result in a reduction of 25 per cent in the actual quantity of fine aggregate, as compared with that obtained by dry measurement by the standard method.

The table gives the proportions in which Portland cement and a wide range in sizes of fine and coarse aggregates should be mixed to obtain concrete of compressive strengths ranging from 1500 to 3000 lb. per sq. in. at 28 days. Proportions are given for concrete of four different consistencies.

The purpose of the table is twofold:

(1) To furnish a guide in the selection of mixtures to be used in preliminary investigations of the strength of concrete from given materials.

(2) To indicate proportions which may be expected to produce concrete of a given strength under average conditions where control tests are not made.

If the proportions to be used in the work are selected from the table without preliminary tests of the materials, and control tests are not made during the progress of the work, the mixtures in bold-faced type shall be used.

The use of this table as a guide in the selection of concrete mixtures is based on the following:

(1) Concrete shall be plastic;
(2) Aggregates shall be clean and structurally sound;
(3) Aggregates shall be graded between the sizes indicated;
(4) Cement shall conform to the requirements of the Standard Specifications and Tests for Portland Cement (Serial Designation: C 9–21) of the American Society for Testing Materials.

[1] From Appendix 16, 1924 Joint Committee Report.

[2] Based on the 28-day compressive strengths of 6 by 12-in. cylinders, made and stored in accordance with the Tentative Methods of Making Compression Tests of Concrete (Serial Designation: C 39–21 T) of the American Society for Testing Materials. (Appendix 12.)

The plasticity of the concrete shall be determined by the slump test[1] carried out in accordance with the Tentative Method of Test for Consistency of Portland-Cement Concrete for Pavements or for Pavement Base (Serial Designation: D 138–22 T) of the American Society for Testing Materials.

Apply the following rules in determining the size assigned to a given aggregate:

(1) Not less than 15 per cent shall be retained between the sieve which is considered the maximum size[2] and the next smaller sieve.

(2) Not more than 15 per cent of a coarse aggregate shall be finer than the sieve considered as the minimum size.[2]

(3) Only the sieve sizes given in the table shall be considered in applying rules (1) and (2).

(4) Sieve analysis shall be made in accordance with the Standard Method of Test for Sieve Analysis of Aggregates for Concrete (Serial Designation: C 41–24) of the American Society for Testing Materials.

Proportions may be interpolated for concrete strengths, aggregate sizes and consistencies not covered by the table or determined by test.

[1] This test is made by filling an open-ended metal form, 12 in. high, made in the form of the frustum of a right cone, with wet concrete, the form being placed upright on a flat surface. The mold is carefully lifted and the height of the concrete measured after it ceases settling. The slump is the difference between this height and 12 in.

[2] For example: a graded sand with 16 per cent retained on the No. 8 sieve would fall in the 0-No. 4 size; if 14 per cent or less were retained, the sand would fall in the 0-No. 8 size. A coarse aggregate having 16 per cent coarser than 2-in. sieve would be considered as 3-in. aggregate.

Proportions for 2000 Lb. per Sq. In. Concrete

Proportions are expressed by volume as follows: Portland Cement : Fine Aggregate : Coarse Aggregate.

Thus 1 : 2.6 : 4.6 indicates 1 part by volume of Portland cement, 2.6 parts by volume of fine aggregate and 4.6 parts by volume of coarse aggregate

Size of Coarse Aggregate	Slump, in.	Size of Fine Aggregate				
		0–No. 28	0–No. 14	0–No. 8	0–No. 4	0–⅜ in.
None	½ to 1	1 : 2.2	1 : 2.6	1 : 3.0	1 : 3.5	1 : 4.1
	3 " 4	1 : 1.9	1 : 2.2	1 : 2.6	1 : 3.0	1 : 3.5
	6 " 7	**1 : 1.5**	**1 : 1.7**	**1 : 2.0**	**1 : 2.3**	**1 : 2.7**
	8 " 10	**1 : 1.0**	**1 : 1.1**	**1 : 1.3**	1 : 1.6	1 : 1.8
No. 4 to ¾ in.	½ to 1	1 : 2.1 : 3.8	1 : 2.3 : 3.7	1 : 2.6 : 3.5	1 : 3.0 : 3.1	1 : 3.6 : 2.8
	3 " 4	1 : 1.7 : 3.3	1 : 1.9 : 3.2	1 : 2.2 : 3.1	1 : 2.6 : 2.8	1 : 3.0 : 2.4
	6 " 7	**1 : 1.3 : 2.7**	**1 : 1.4 : 2.6**	**1 : 1.7 : 2.5**	**1 : 1.9 : 2.3**	**1 : 2.3 : 2.1**
	8 " 10	1 : 0.8 : 1.9	1 : 0.9 : 1.9	1 : 1.0 : 1.8	1 : 1.2 : 1.7	1 : 1.5 : 1.6
No. 4 to 1 in.	½ to 1	1 : 1.9 : 4.5	1 : 2.2 : 4.3	1 : 2.5 : 4.2	1 : 2.8 : 3.9	1 : 3.4 : 3.6
	3 " 4	1 : 1.6 : 3.9	1 : 1.8 : 3.8	1 : 2.1 : 3.7	1 : 2.4 : 3.5	1 : 2.8 : 3.2
	6 " 7	**1 : 1.2 : 3.1**	**1 : 1.3 : 3.1**	**1 : 1.5 : 3.0**	**1 : 1.8 : 2.9**	**1 : 2.1 : 2.7**
	8 " 10	1 : 0.7 : 2.2	1 : 0.8 : 2.2	1 : 1.0 : 2.3	1 : 1.1 : 2.1	1 : 1.3 : 2.0
No. 4 to 1½ in.	½ to 1	1 : 1.9 : 5.0	1 : 2.1 : 4.9	1 : 2.4 : 4.9	1 : 2.7 : 4.6	1 : 3.2 : 4.4
	3 " 4	1 : 1.6 : 4.4	1 : 1.7 : 4.3	1 : 2.0 : 4.2	1 : 2.4 : 4.0	1 : 2.7 : 3.8
	6 " 7	**1 : 1.1 : 3.5**	**1 : 1.3 : 3.5**	**1 : 1.4 : 3.5**	**1 : 1.7 : 3.4**	**1 : 2.0 : 3.2**
	8 " 10	1 : 0.7 : 2.5	1 : 0.8 : 2.5	1 : 0.9 : 2.5	1 : 1.0 : 2.4	1 : 1.2 : 2.3
No. 4 to 2 in.	½ to 1	1 : 1.7 : 5.8	1 : 1.9 : 5.7	1 : 2.1 : 5.8	1 : 2.4 : 5.6	1 : 2.8 : 5.5
	3 " 4	1 : 1.4 : 5.0	1 : 1.5 : 5.0	1 : 1.8 : 5.0	1 : 2.0 : 4.9	1 : 2.3 : 4.7
	6 " 7	**1 : 1.0 : 4.1**	**1 : 1.1 : 4.1**	**1 : 1.2 : 4.1**	**1 : 1.4 : 4.1**	**1 : 1.7 : 3.9**
	8 " 10	1 : 0.6 : 2.9	1 : 0.7 : 2.9	1 : 0.7 : 3.0	1 : 0.8 : 2.9	1 : 1.0 : 2.9
⅜ to 1 in.	½ to 1	1 : 2.2 : 4.4	1 : 2.5 : 4.2	1 : 2.8 : 4.1	1 : 3.3 : 3.8	1 : 3.8 : 3.4
	3 " 4	1 : 1.9 : 3.8	1 : 2.1 : 3.7	1 : 2.4 : 3.6	1 : 2.8 : 3.4	1 : 3.2 : 3.1
	6 " 7	**1 : 1.4 : 3.1**	**1 : 1.5 : 3.0**	**1 : 1.8 : 3.0**	**1 : 2.1 : 2.8**	**1 : 2.4 : 2.5**
	8 " 10	1 : 0.9 : 2.2	1 : 1.0 : 2.2	1 : 1.1 : 2.2	1 : 1.3 : 2.0	1 : 1.5 : 1.9
⅜ to 1½ in.	½ to 1	1 : 2.2 : 4.9	1 : 2.5 : 4.8	1 : 2.8 : 4.7	1 : 3.2 : 4.6	1 : 3.7 : 4.2
	3 " 4	1 : 1.9 : 4.3	1 : 2.1 : 4.2	1 : 2.4 : 4.1	1 : 2.7 : 4.0	1 : 3.1 : 3.7
	6 " 7	**1 : 1.4 : 3.5**	**1 : 1.5 : 3.4**	**1 : 1.7 : 3.4**	**1 : 2.0 : 3.3**	**1 : 2.3 : 3.1**
	8 " 10	1 : 0.9 : 2.5	1 : 1.0 : 2.5	1 : 1.1 : 2.4	1 : 1.3 : 2.4	1 : 1.5 : 2.3
⅜ to 2 in.	½ to 1	1 : 2.1 : 5.6	1 : 2.3 : 5.5	1 : 2.6 : 5.5	1 : 3.0 : 5.4	1 : 3.5 : 5.1
	3 " 4	1 : 1.7 : 4.8	1 : 2.0 : 4.8	1 : 2.2 : 4.8	1 : 2.5 : 4.7	1 : 2.9 : 4.4
	6 " 7	**1 : 1.3 : 4.0**	**1 : 1.4 : 3.9**	**1 : 1.6 : 3.9**	**1 : 1.8 : 3.9**	**1 : 2.1 : 3.8**
	8 " 10	1 : 0.8 : 2.9	1 : 0.9 : 2.9	1 : 1.0 : 2.9	1 : 1.2 : 2.9	1 : 1.3 : 2.8
¾ to 1½ in.	½ to 1	1 : 2.6 : 4.5	1 : 2.9 : 4.5	1 : 3.3 : 4.4	1 : 3.8 : 4.2	1 : 4.3 : 3.9
	3 " 4	1 : 2.2 : 3.9	1 : 2.5 : 3.9	1 : 2.8 : 3.8	1 : 3.2 : 3.6	1 : 3.6 : 3.3
	6 " 7	**1 : 1.6 : 3.2**	**1 : 1.8 : 3.2**	**1 : 2.1 : 3.1**	**1 : 2.4 : 3.0**	**1 : 2.7 : 2.8**
	8 " 10	1 : 1.0 : 2.3	1 : 1.2 : 2.3	1 : 1.4 : 2.2	1 : 1.6 : 2.2	1 : 1.8 : 2.1
¾ to 2 in.	½ to 1	1 : 2.5 : 5.2	1 : 2.8 : 5.2	1 : 3.2 : 5.1	1 : 3.6 : 5.0	1 : 4.1 : 4.7
	3 " 4	1 : 2.1 : 4.5	1 : 2.4 : 4.5	1 : 2.7 : 4.4	1 : 3.1 : 4.3	1 : 3.5 : 4.0
	6 " 7	**1 : 1.6 : 3.7**	**1 : 1.8 : 3.7**	**1 : 2.0 : 3.7**	**1 : 2.3 : 3.6**	**1 : 2.6 : 3.5**
	8 " 10	1 : 1.0 : 2.6	1 : 1.1 : 2.7	1 : 1.3 : 2.6	1 : 1.5 : 2.7	1 : 1.7 : 2.6
1 to 3 in.	½ to 1	1 : 2.5 : 6.0	1 : 2.9 : 5.9	1 : 3.2 : 5.9	1 : 3.6 : 5.8	1 : 4.1 : 5.6
	3 " 4	1 : 2.1 : 5.1	1 : 2.4 : 5.2	1 : 2.7 : 5.2	1 : 3.1 : 5.1	1 : 3.5 : 4.9
	6 " 7	**1 : 1.5 : 4.1**	**1 : 1.7 : 4.2**	**1 : 2.0 : 4.2**	**1 : 2.3 : 4.2**	**1 : 2.5 : 4.0**
	8 " 10	1 : 1.0 : 2.9	1 : 1.1 : 3.0	1 : 1.3 : 3.0	1 : 1.5 : 3.0	1 : 1.7 : 3.0

In the Joint Committee Report will be found tables for 1500, 2500 and 3000 lbs./sq. in. in addition to the one given here.

APPENDIX B

From the Report of the Joint Committee [1] on Standard Specifications for Concrete and Reinforced Concrete, submitted August 14, 1924.

Note: Art. 63, PLACING, is reprinted on page 158; Art. 67, MOISTURE PROTECTION, is reprinted on page 40; Art. 68, FIRE PROTECTION, is reprinted on page 41.

CHAPTER XI. DESIGN

A. General Assumptions

103. **General Assumptions.** The design of reinforced concrete members under these specifications shall be based on the following assumptions:

(*a*) Calculations are made with reference to working stresses and safe loads rather than with reference to ultimate strength and ultimate loads.

(*b*) A plane section before bending remains plane after bending, shearing distortions being neglected.

(*c*) The modulus of elasticity of concrete in compression is constant within the limits of working stresses and the distribution of compressive stress in beams is rectilinear.

(*d*) The moduli of elasticity of concrete in computations for the position of the neutral axis, for the resisting moment of beams, and for compression of concrete in columns, are as follows:

(1) $\frac{1}{15}$ that of steel, when the compressive strength of the concrete at 28 days exceeds 1500 and does not exceed 2200 lb. per sq. in.;

(2) $\frac{1}{12}$ that of steel, when the compressive strength of the concrete at 28 days exceeds 2200 and does not exceed 2900 lb. per sq. in.;

(3) $\frac{1}{10}$ that of steel, when the compressive strength of the concrete at 28 days is greater than 2900 lb. per sq. in.

(*e*) In calculating the moment of resistance of reinforced concrete beams and slabs the tensile resistance of the concrete is neglected.

(*f*) The bond between the concrete and the metal reinforcement remains unbroken throughout the range of working stresses. Under compression the two materials are therefore stressed in proportion of their moduli of elasticity.

(*g*) Initial stress in the reinforcement due to contraction or expansion of the concrete is neglected, except in the design of reinforced concrete columns.[2]

[1] Composed of affiliated committees of the American Society of Civil Engineers, American Society for Testing Materials, American Railway Engineering Association, American Concrete Institute, Portland Cement Association.

[2] Formula 43 for the permissible compressive stress in reinforced concrete columns takes into account the effect of shrinkage in the concrete on the stress in the longitudinal reinforcement. It is not required, however, that the designer consider shrinkage stresses in columns, except through the use of that formula.

B. Flexure of Rectangular Reinforced Concrete Beams and Slabs

104.[1] **Flexure Formulas.** (The formulas given are in general the same as those presented in Chap. VIII.)

105. **Notation.** (See page 374, Appendix C.)

106. **Span Length.** The span length, l, of freely supported beams and slabs, shall be the distance between centers of the supports, but shall not exceed the clear span plus the depth of beam or slab. The span length for continuous or restrained beams built to act integrally with supports shall be the clear distance between faces of supports. Where brackets having a width not less than the width of the beam and making an angle of 45 degrees or more with the horizontal axis of a restrained beam are built to act integrally with the beam and support, the span shall be measured from the section where the combined depth of the beam and bracket is at least one-third more than the depth of the beam, but no portion of such a bracket shall be considered as adding to the effective depth of the beam. Maximum negative moments are to be considered as existing at the ends of the span, as defined above.

107. **Slightly Restrained Beams of Equal Span.** Beams and slabs of equal spans built to act integrally with beams, girders, or other slightly restraining supports and carrying uniformly distributed loads shall be designed for the following moments at critical sections:

(*a*) Beams and slabs of one span,
Maximum positive moment near center,

$$M = \frac{wl^2}{8}. \tag{12}$$

(*b*) Beams and slabs continuous for two spans only,
(1) Maximum positive moment near center,

$$M = \frac{wl^2}{10}. \tag{13}$$

(2) Negative moment over interior support,

$$M = \frac{wl^2}{8}. \tag{14}$$

(*c*) Beams and slabs continuous for more than two spans,
(1) Maximum positive moment near center and negative moment at support of interior spans,

$$M = \frac{wl^2}{12}. \tag{15}$$

(2) Maximum positive moment near centers of end spans and negative moment at first interior support,

$$M = \frac{wl^2}{10}. \tag{16}$$

[1] For f_s = 16,000 to 18,000 lbs. per sq. in. and f_c = 800 to 900 lbs. per sq. in., j may be assumed as 0.86. For values of pn varying from 0.04 to 0.24, jk is approximately equal to $0.67\sqrt[3]{pn}$.

(*d*) Negative moment at end supports for cases (*a*), (*b*), (*c*) of this section,

$$M = \text{not less than } \frac{wl^2}{16}. \tag{16a}$$

108. **Beams Built Into Brick or Masonry Walls.** Beams and slabs built into brick or masonry walls in a manner which develops partial end restraint shall be designed for a negative moment at the support of

$$M = \text{not less than } \frac{wl^2}{16}. \tag{17}$$

109. **Freely Supported Beams of Equal Span.** Beams and slabs of equal spans freely supported and assumed to carry uniformly distributed loads shall be designed for the moments specified in Section 107, except that no reinforcement for negative moment need be provided at end supports where effective measures are taken to prevent end restraint. The span shall be taken as defined in Section 106 for freely supported beams.

110. **Restrained Beams of Equal Span.** Beams and slabs of equal span built to act integrally with columns, walls, or other restraining supports and assumed to carry uniformly distributed loads, shall (except as provided in Section 107) be designed for the following moments at critical sections:

(*a*) Interior spans,

(1) Negative moment at interior supports except the first,

$$M = \frac{wl^2}{12}. \tag{18}$$

(2) Maximum positive moment near centers of interior spans,

$$M = \frac{wl^2}{16}. \tag{19}$$

(*b*) End spans of continuous beams and beams of one span in which I/l is less than twice the sum of the values of I/h for the exterior columns above and below which are built into the beams:

(1) Maximum positive moment near center of span and negative moment at first interior supports,

$$M = \frac{wl^2}{12}. \tag{20}$$

(2) Negative moment at exterior supports,

$$M = \frac{wl^2}{12}. \tag{21}$$

(*c*) End spans of continuous beams, and beams of one span, in which I/l is equal to or greater than twice the sum of the values of I/h for the exterior column above and below which are built into the beams:

(1) Maximum positive moment near center of span and negative moment at first interior support,

$$M = \frac{wl^2}{10}. \tag{22}$$

(2) Negative moment at exterior support,

$$M = \frac{wl^2}{16}. \tag{23}$$

111. **Continuous Beams of Unequal Spans or with Non-uniform Loading.** Continuous beams with unequal spans, or with other than uniformly distributed loading, whether freely-supported or restrained, shall be designed for the actual moments under the conditions of loading and restraint.

Provision shall be made where necessary for negative moment near the center of short spans which are adjacent to long spans, and for the negative moment at the end supports, if restrained.

112. **Unsupported Flange Length.** The distance between lateral supports of the compression area of a beam shall not exceed 24 times the least width of compression flange.

C. Flexure of Reinforced Concrete T-Beams

113. **Flexure Formulas.** (See page 111.)

114. **Notation.** (See Appendix C.)

115. **Flange Width.** Effective and adequate bond and shear resistance shall be provided in beam-and-slab construction at the junction of the beam and slab; the slab shall be built and considered an integral part of the beam; the effective flange width to be used in the design of symmetrical T-beams shall not exceed one-fourth of the span length of the beam, and its overhanging width on either side of the web shall not exceed eight times the thickness of the slab nor one-half the clear distance to the next beam.

For beams having a flange on one side only, the effective flange width to be used in design shall not exceed one-tenth of the span length of the beam, and its overhanging width from the face of the web shall not exceed six times the thickness of the slab nor one-half the clear distance to the next beam.

116. **Transverse Reinforcement.** Where the principal slab reinforcement is parallel to the beam, transverse reinforcement, not less in amount than 0.3 per cent of the sectional area of the slab, shall be provided in the top of the slab and shall extend across the beam and into the slab not less than two-thirds of the width of the effective flange overhang. The spacing of the bars shall not exceed 18 in.

117. **Compressive Stress at Supports.** Provision shall be made for the compressive stress at the support in continuous T-beam construction.

118. **Shear.** The flange of the beam shall not be considered as effective in computing the shear and diagonal tension resistance of T-beams.

119. **Isolated Beams.** Isolated beams in which the T-form is used only for the purpose of providing additional compression area, shall have a flange thickness not less than one-half the width of the web and a total flange width not more than 4 times the web thickness.

D. Diagonal Tension and Shear

120. **Notation.** (See Appendix C.)

121. **Formula for Shear.** The shearing unit-stress, v, in reinforced concrete beams shall be taken as not less than that computed by Formula 29.[1]

$$v = \frac{V}{bjd}. \tag{29}$$

122. **Variation of Shear in Beams with Uniform Load.** For purpose of design of beams carrying uniform loads, not less than one-fourth of the total shearing resistance required at either end of span shall be provided at the section where the computed shearing stress is zero; from that section to the ends of span the required shearing resistance shall be assumed to vary uniformly.

123. **Width of Beams in Shear Computations.** The shearing unit stress shall be computed on the minimum width of rectangular beams and on the minimum thickness of the web in beams of I or T-section.

124. **Shear in Beam-and-Tile Construction.** The width of the effective section for shear as governing diagonal tension shall be assumed as the thickness of the concrete web plus one-half the thickness of the vertical webs of the concrete or clay tile in contact with the beam. (For typical design see Appendix C, Fig. 13.)

125. **Types and Spacing of Web Reinforcement.** Web reinforcement may consist of:

(*a*) Vertical stirrups or web reinforcing bars;
(*b*) Inclined stirrups or web reinforcing bars forming an angle of 30 degrees or more with the longitudinal bars;
(*c*) Longitudinal bars bent up at an angle of 15 degrees or more with the direction of the longitudinal bars.

Stirrups or bent-up bars which are not anchored at both ends, according to the provisions of Section 141, shall not be considered effective as web reinforcement. When the shearing stress is not greater than $0.06\,f'_c$, the distance s measured in the direction of the axis of the beam between two successive stirrups, or between two successive points of bending up of bars, or from the point of bending up of a bar to the edge of the support, shall not be greater than

$$s = \frac{45\,d}{\alpha + 10} \tag{30}$$

where the angle α is in degrees.

When the shearing stress is greater than $0.06\,f'_c$, the distance s shall not be greater than two-thirds of the values given by Formula 30.

126. **Anchorage of Web Reinforcement.** See Section 141.

[1] For I or T-beams b is the width of the stem as given in Section 123.

127. Beams Without Special Anchorage of Longitudinal Reinforcement. The shearing unit stress computed by Formula 29 in beams in which the longitudinal reinforcement is without special anchorage shall not exceed the values given by Formulas 31 and 32 and in no case shall it exceed $0.06\,f'_c$.

When α is between 45 and 90 degrees,

$$v = 0.02 f'_c + \frac{f_v A_v}{bs \sin \alpha}. \tag{31}$$

When α is less than 45 degrees,

$$v = 0.02 f'_c + \frac{f_v A_v}{bs} (\sin \alpha + \cos \alpha). \tag{32}$$

128. Beams with Special Anchorage of Longitudinal Reinforcement. The shearing unit stress computed by Formula 29 in beams in which longitudinal reinforcement is anchored by means of hooked ends or otherwise, as specified in Section 140, shall not exceed the value given by Formulas 31 and 32, when $0.03\,f'_c$ is substituted for $0.02\,f'_c$ in those formulas; in no case shall the shearing unit stress exceed $0.12\,f'_c$.

129. Beams with Bars Bent up at a Single Point. Where the web reinforcement consists of bars bent up at a single point, the point of bending shall be at a distance s from the edge of the support, not greater than that given in Section 125 and the value of the quantity $\frac{f_v A_v}{bs}(\sin \alpha + \cos \alpha)$ used in the design shall not exceed 75 lb. per sq. in. (See Appendix C, Fig. 10.)

130. Combined Web Reinforcement. Where two or more types of web reinforcement are used in conjunction, the total shearing resistance of the beam shall be assumed as the sum of the shearing resistances computed for the various types separately. In such computations the shearing resistance of the concrete (the term $0.02\,f'_c$ or $0.03\,f'_c$ in Formulas 31 and 32) shall be included only once. In no case shall the maximum shearing stresses be greater than the limiting values given in Sections 127 and 128.

131. Shearing Stress in Flat Slabs. The shearing unit stress in flat slabs shall not exceed the value of v as given by Formula 33,

$$v = 0.02 f'_c (1 + r) \tag{33}$$

and shall not in any case exceed $0.03\,f'_c$.

The shearing unit stress shall be computed on:

(*a*) A vertical section which has a depth in inches of $\frac{7}{8}(t_1 - 1\frac{1}{2})$ and which lies at a distance in inches of $t_1 - 1\frac{1}{2}$ from the edge of the column capital; and

(*b*) A vertical section which has a depth in inches of $\frac{7}{8}(t_2 - 1\frac{1}{2})$ and which lies at a distance in inches of $t_2 - 1\frac{1}{2}$ from the edge of the dropped panel.

In no case shall r be less than 0.25. Where the shearing stress computed as in (*a*) is being considered, r shall be assumed as the proportional amount of the negative reinforcement, within the column strip, crossing the column capital. Where the shearing stress computed as in (*b*) is being considered, r shall be assumed as the proportional amount of the negative reinforcement,

within the column strip, crossing entirely over the dropped panel.[1] (For typical flat slab and designation of principal design sections see Appendix C, Figs. 14 and 15.)

132. **Shear and Diagonal Tension in Footings.** The shearing stress shall be taken as not less than that computed by Formula 29. The stress on the critical section shall not exceed 0.02 f'_c for footings with straight reinforcement bars, nor 0.03 f'_c for footings in which the reinforcement bars are anchored at both ends by adequate hooks or otherwise as specified in Section 140.

133. **Critical Section for Soil Footings.** The critical section for diagonal tension in footings on soil shall be computed on a vertical section through the perimeter of the lower base of a frustum of a cone or pyramid which has a base angle of 45 degrees, and which has for its top the base of the column or pedestal and for its lower base the plane at the centroid of longitudinal reinforcement.

134. **Critical Section for Pile Footings.** The critical section for diagonal tension in footings on piles shall be computed on a vertical section at the inner edge of the first row of piles entirely outside a section midway between the face of the column or pedestal and the section described in Section 133 for soil footings, but in no case outside of the section described in Section 133. The critical section for piles not arranged in rows shall be taken midway between the face of the column and the perimeter of the base of the frustum described in Section 133.

E. Bond and Anchorage

135. **Bond Stresses by Beam Action.** Where bar reinforcement is used to resist tensile stresses developed by beam action, the bond stress shall be taken as not less than that computed by Formula 34,

$$u = \frac{V}{\Sigma ojd}. \tag{34}$$

For continuous or restrained members, the critical section for bond for the positive reinforcement shall be assumed to be at the point of inflection; that for the negative reinforcement shall be assumed to be at the face of the support, and at the point of inflection. For simple beams or freely supported end spans of continuous beams, the critical section for bond shall be assumed to be at the face of the support.

Bent-up longitudinal bars which, at the critical section, are within a distance $\frac{d}{3}$ from horizontal reinforcement under consideration, may be included with the straight bars in computing Σo.

In footings only the bars specified in Section 177 as effective in resisting bending moment shall be considered as resisting bond stresses. Special investigation shall be made of bond stresses in footings with stepped or sloping

[1] In special cases, where supported by satisfactory engineering analysis, diagonal tension reinforcement may be used and increased shearing stresses allowed in accordance with Sections 127 to 130.

upper surface, as maximum bond stresses may occur at the vertical plane of the steps or near the edges of the footing.

136. **Bond Stress for Ordinary Anchorage.** In beams where the ordinary anchorage described in Section 139 is provided, the bond stress computed by Formula 34 at any section shall not exceed the following values:

For plain bars . . . . $u = 0.04 f'_c$
For deformed bars meeting the requirements of Section 23 . $u = 0.05 f'_c$

137. **Bond Stresses for Special Anchorage.** In beams where special anchorage of the bars is provided as specified in Section 140, bond stresses exceeding those specified in Section 136 may be used, provided the total tensile stress at a point of abrupt change in stress or at the point of maximum stress, does not exceed the value of F given by Formula 35,

$$F = Qu\Sigma oy + u\Sigma ox \tag{35}$$

where F = total tension in the bar;
Σo = the perimeter of the bar under consideration;
Q = ratio of the average to the maximum bond stress computed by Formula 34 within the distance y;
u = permissible bond stress = 0.04 f'_c for plain and 0.05 f'_c for deformed bars meeting the requirements of Section 23;
x = the length of bar added for anchorage, including the hook, if any;
y = distance from the point at which the tension is computed to the point of beginning of anchorage.

The length of bar added for anchorage may be either straight or bent. The radius of bend shall not be less than four bar diameters.

138. **Bond Stress for Reinforcement in Two or More Directions.** The permissible bond stress for footings and similar members in which reinforcement is placed in more than one direction shall not exceed 75 per cent of the values in Sections 136 and 137.

139. **Ordinary Anchorage Requirements.** In continuous, restrained or cantilever beams, anchorage of the tensile negative reinforcement beyond the face of the support shall provide for the full maximum tension with bond stresses not greater than those specified in Section 136. Such anchorage shall provide a length of bar not less than the depth of the beam. In the case of end supports which have a width less than three-fourths of the depth of the beam, the bars shall be bent down toward the support a distance not less than the effective depth of the beam. The portion of the bar so bent down shall be as near to the end of the beam as protective covering permits. (See Fig. 9.) In continuous or restrained beams, negative reinforcement shall be carried to or beyond the point of inflection. Not less than one-fourth of the area of the positive reinforcement shall extend into the support to provide an embedment of ten or more bar diameters.

In simple beams or freely supported end spans of continuous beams at least one-fourth of the area of the tensile reinforcement shall extend along the

tension side of the beam and beyond the face of the support to provide an embedment of ten or more bar diameters.

140. **Special Anchorage Requirements.** Where increased shearing stresses are used as provided in Sections 128 and 132 or increased bond stresses as provided in Section 137, special anchorage of all reinforcement in addition to that required in Section 139 shall be provided as follows:

(*a*) In continuous and restrained beams, anchorage beyond points of inflection of one-third the area of the negative reinforcement and beyond the face of the support of one-third the area of the positive reinforcement, shall be provided to develop one-third of the maximum working stress in tension, with bond stresses not greater than those specified in Section 136.

(*b*) At the edges of footings, anchorage for all the bars for one-third the maximum working stress in tension shall be provided within a region where the tension in the concrete, computed as an unreinforced beam, does not exceed 40 lb. per sq. in.

(*c*) In simple beams or freely supported end spans of continuous beams, at least one-half of the tensile reinforcement shall extend along the tension side of the beam to provide an anchorage beyond the face of the support for one-third of the maximum working stress in tension.

141. **Anchorage of Web Reinforcement.** Web bars shall be anchored at both ends by:

(*a*) providing continuity with the longitudinal reinforcement; or
(*b*) bending around the longitudinal bar; or
(*c*) a semi-circular hook which has a radius not less than four times the diameter of the web bar.

Stirrup anchorage shall be so provided in the compression and tension regions of a beam as to permit the development of safe working tensile stress in the stirrup at a point 0.3 d from either face.[1]

The end anchorage of a web member not in bearing on the longitudinal reinforcement shall be such as to engage an amount of concrete sufficient to prevent the bar from pulling out. In all cases the stirrups shall be carried as close to the upper and lower surfaces as fireproofing requirements permit. (For typical designs see Appendix C, Figs. 8 and 12.)

F. Flat Slabs

(Two-Way and Four-Way Systems with Rectangular Panels)

142. **Moments in Interior Panels.** The moment coefficients, moment distribution and slab thicknesses specified herein are for slabs which have three or more rows of panels in each direction, and in which the panels are approximately uniform in size. Slabs with paneled ceiling or with depressed paneling in the floor shall be considered as coming under the requirements herein given.

[1] Generally a properly-anchored stirrup whose diameter does not exceed $\frac{1}{50}$ of the depth of the beam will meet these requirements.

The symbols used in Formulas 36 to 41 are defined in Appendix C except as indicated in Sections 142, 145 and 155.

In flat slabs in which the ratio of reinforcement for negative moment in the column strip is not greater than 0.01, the numerical sum of the positive and negative moments in the direction of either side of the panel for which tension reinforcement must be provided, shall be assumed as not less than that given by Formula 36,

$$M_0 = 0.09\, Wl\left(1 - \frac{2}{3}\frac{c}{l}\right)^2 \tag{36}$$

where M_0 = sum of positive and negative bending moments[1] in either rectangular direction at the principal design sections of a panel of a flat slab;

c = base diameter of the largest right circular cone, which lies entirely within the column (including the capital) whose vertex angle is 90 degrees and whose base is $1\frac{1}{2}$ inches below the bottom of the slab or the bottom of the dropped panel (see Fig. 14);

l = span length[2] of flat slab, center to center of columns in the rectangular direction in which moments are considered;

l_1 = span length[2] of flat slab, center to center of columns perpendicular to the rectangular direction in which moments are considered; and

W = total dead and live load uniformly distributed over a single panel area.

143. Principal Design Sections. In computing the critical moments in flat slabs subjected to uniform load the following principal design sections shall be used:

(*a*) *Section for Negative Moment in Middle Strip:* The section beginning at a point on the edge of the panel $l_1/4$ from the column center and extending in a rectangular direction a distance $l_1/2$ toward the center of the adjacent column on the same panel edge. (See Fig. 15, Appendix C.)

[1] The sum of the positive and negative moments provided for by this equation is about 72 per cent of the moment found by rigid analysis based upon the principles of mechanics. Extensive tests and experience with existing structures have shown that the requirements here stated will give adequate strength. See " Statical Limitations upon the Steel Requirement in Reinforced Concrete Flat Slab Floors," by John R. Nichols, *Transactions*, Am. Soc. Civil Engrs., Vol. LXXVII (1914), and " Moments and Stresses in Slabs," by Westergaard and Slater, *Proceedings*, Am. Concrete Inst., Vol. XVII (1921).

[2] The column strip and the middle strip to be used when considering moments in the direction of the dimension l are located and dimensioned as shown in Fig. 15. The dimension l_1 does not always represent the short length of the panel. When moments in the direction of the shorter panel length are considered, the dimensions l and l_1 are to be interchanged and the strips corresponding to those shown in Fig. 15 but extending in the direction of the shorter panel length are to be considered.

TABLE VI

MOMENTS TO BE USED IN DESIGN OF FLAT SLABS [1]

Strip	Flat Slabs without Dropped Panels		Flat Slabs with Dropped Panels	
	Negative	Positive	Negative	Positive
SLABS WITH 2-WAY REINFORCEMENT				
Column strip	0 23 M_0	0 11 M_0	0 25 M_0	0 10 M_0
2 Column strips	0 46 M_0	0 22 M_0	0 50 M_0	0 20 M_0
Middle strip	0 16 M_0	0 16 M_0	0 15 M_0	0 15 M_0
SLABS WITH 4-WAY REINFORCEMENT				
Column strip	0 25 M_0	0 10 M_0	0 27 M_0	0 095 M_0
2 Column strips	0 50 M_0	0 20 M_0	0 54 M_0	0 190 M_0
Middle strip	0 10 M_0	0 20 M_0	0 08 M_0	0 190 M_0

[1] These are approximately the values which would be obtained by considering one-third of the total moment, M_0, as positive and two-thirds of it as negative moment.

(*b*) *Section for Negative Moment in Column Strip:* The section beginning at a point on the edge of the panel $l_1/4$ from the center of a column and extending in a rectangular direction toward the column to a point $c/2$ therefrom and thence along a one-quarter circumference about the column center to the adjacent edge of the panel.

(*c*) *Section for Positive Moment in Middle Strip:* The section of a length $l_1/2$ extending in a rectangular direction across the center of the middle strip.

(*d*) *Section for Positive Moment in Column Strip:* The section of length $l_1/4$ extending in a rectangular direction across the center of the column strip.

144. **Moments in Principal Design Sections.** The moments in the principal design sections shall be those given in Table VI, except as follows:

(*a*) The sum of the maximum negative moments in the two column strips may be greater or less than the values given in Table VI by not more than 0.03 M_0.

(*b*) The maximum negative moment and the maximum positive moments in the middle strip and the sum of the maximum positive moments in the two column strips may each be greater or less than the values given in Table VI by not more than 0.01 M_0.

145. **Thickness of Flat Slabs and Dropped Panels.** The total thickness,[1] t_1, of the dropped panel in inches, or of the slab if a dropped panel is not used, shall be not less than:

$$t_1 = 0.038\left(1 - 1.44\frac{c}{l}\right)l\sqrt{Rw'\frac{l_1}{b_1}} + 1\tfrac{1}{2} \qquad (37)^2$$

[1] The thickness will be in inches regardless of whether l and w' are in feet and pounds per square foot or in inches and pounds per square inch.

[2] The values of R used in this formula are the coefficients of M_0 for negative moment in two column strips in Table VI.

where R = ratio of negative moment in the two column strips to M_0;
w' = uniformly distributed dead and live load per unit of area of floor; and
b_1 = dimension of the dropped panel in the direction parallel to l_1,

For slabs with dropped panels the total thickness[1] in inches at points beyond the dropped panel shall be not less than

$$t_2 = 0.02\, l\sqrt{w'} + 1. \tag{38}$$

The slab thickness t_1 or t_2 shall in no case be less than $l/32$ for floor slabs, and not less than $l/40$ for roof slabs. In determining minimum thickness by Formulas 37 and 38, the value of l shall be the panel length center to center of the columns, on long side of panel, l_1 shall be the panel length on the short side of the panel, and b_1 shall be the width or diameter of dropped panel in the direction of l_1, except that in a slab without dropped panel b_1 shall be 0.5 l_1.

146. **Minimum Dimensions of Dropped Panels.** The dropped panel shall have a length or diameter in each rectangular direction of not less than one-third the panel length in that direction, and a thickness not greater than 1.5 t_2.

147. **Wall and Other Irregular Panels.** In wall panels and other panels in which the slab is discontinuous at the edge of the panel, the maximum negative moment one panel length away from the discontinuous edge and the maximum positive moment between shall be increased as follows:

(*a*) Column strip perpendicular to the wall or discontinuous edge, 15 per cent greater than that given in Table VI;
(*b*) Middle strip perpendicular to wall or discontinuous edge, 30 per cent greater than that given in Table VI.

In these strips the bars used for positive moments perpendicular to the discontinuous edge shall extend to the edge of the panel at which the slab is discontinuous.

148. **Panels with Marginal Beams.** In panels having a marginal beam on one edge or on each of two adjacent edges, the beam shall be designed to carry at least the load superimposed directly upon it, exclusive of the panel load. A beam which has a depth greater than the thickness of the dropped panel into which it frames, shall be designed to carry, in addition to the load superimposed upon it, at least one-fourth of the distributed load for which the adjacent panel or panels are designed, and each column strip adjacent to and parallel with the beam shall be designed to resist a moment at least one-half as great as that specified in Table VI for a column strip.[2]

Each column strip adjacent to and parallel with a marginal beam which has a depth less than the thickness of the dropped panel into which it frames

[1] The thickness will be in inches regardless of whether l and w' are in feet and pounds per square foot or in inches and pounds per square inch.

[2] In wall columns, brackets are sometimes substituted for capitals or other changes are made in the design of the capital. Attention is directed to the necessity for taking into account the change in the value of c in the moment formula for such cases.

shall be designed to resist the moments specified in Table VI for a column strip. Marginal beams on opposite edges of a panel and the slab between them shall be designed for the entire load and the panels shall be designed as simple beams.

149. **Discontinuous Panels.** The negative moments on sections at and parallel to the wall, or discontinuous edge of an interior panel, shall be determined by the conditions of restraint.[1]

150. **Flat Slabs on Bearing Walls.** Where there is a beam or a bearing wall on the center line of columns in the interior portion of a continuous flat slab, the negative moment at the beam or wall line in the middle strip perpendicular to the beam or wall shall be taken as 30 per cent greater than the moment specified in Table VI for a middle strip. The column strip adjacent to and lying on either side of the beam or wall shall be designed to resist a moment at least one-half of that specified in Table VI for a column strip.

151. **Point of Inflection.** The point of inflection in any line parallel to a panel edge in interior panels of symmetrical slabs without dropped panels shall be assumed to be at a distance from the center of the span equal to three-tenths of the distance between the two sections of critical negative moment at opposite ends of the line; for slabs having dropped panels, the coefficient shall be 0.25.

152. **Reinforcement.** The reinforcement bars which cross any section and which fulfill the requirements given in Section 153 may be considered as effective in resisting the moment at the section. The sectional area of a bar multiplied by the cosine of the angle between the direction of the axis of the bar and any other direction may be considered effective as reinforcement in that direction.

153. **Arrangement of Reinforcement.** The design shall include adequate provision for securing the reinforcement in place so as to take not only the critical moments but the moments at intermediate sections. Provision shall be made for possible shifting of the point of inflection by carrying all bars in rectangular or diagonal directions, each side of a section of critical moment, either positive or negative, to points at least 20 diameters beyond the point of inflection as specified in Section 151. Lapped splices shall not be permitted at or near regions of maximum stress except as described above. At least four-tenths of all bars in each direction shall be of such length and shall be so placed as to provide reinforcement at two sections of critical negative moment and at the intermediate section of critical positive moment. Not less than one-third of the bars used for positive reinforcement in the column strip shall extend into the dropped panel not less than 20 diameters of the bar, or in case no dropped panel is used, shall extend to a point not less than one-eighth of the span length from the center line of the column or the support.

154. **Reinforcement at Construction Joints.** See Section 72.

[1] The committee is not prepared to make a more definite recommendation at this time.

155. **Tensile Stress in Reinforcement.** The tensile stress f_s in the reinforcement in flat slabs shall be taken as not less than that computed by Formula 39,

$$f_s = \frac{RM_0}{A_s jd} \tag{39}$$

where RM_0 = moment specified in Section 144 for two column strips or for one middle strip; and

A_s = effective cross-sectional area of the reinforcement which crosses any of the principal design sections and which meets the requirements of Section 153.

The stress so computed shall not at any of the principal design sections exceed the values specified in Section 194.

156. **Compressive Stress in Concrete.** The compressive stress in the concrete in flat slabs shall be taken as not less than that computed by Formulas 40 and 41, but the stress so computed shall not exceed $0.4 f'_c$.

Compression due to negative moment, RM_0, in the two column strips,

$$f_c = \frac{3.5\ RM_0}{0.67 \sqrt[3]{pn}\, b_1 d^2}\left(1 - 1.2\frac{c}{l}\right) \tag{40}$$

where b_1 is as specified in Section 145.

Compression due to positive moment, RM_0, in the two column strips, or negative or positive moment in the middle strip,

$$f_c = \frac{6\ RM_0}{0.67 \sqrt[3]{pn}\, l_1 d^2}. \tag{41}$$

In special cases where supported by satisfactory engineering analysis, approved by the Engineer, compression reinforcement may be used to increase the resistance to compression in accordance with other provisions of these specifications.

157. **Shearing Stress.** See Section 131.

158. **Unusual Panels.** For structures having a width of one or two panels, and also for slabs having panels of markedly different sizes, an analysis shall be made of the moments developed in both slab and columns, and the values given in Sections 142 to 157 modified accordingly.

159. **Bending Moments in Columns.** See Section 171.

G. Reinforced Concrete Columns

160. **Limiting Dimensions.** The following sections on reinforced concrete columns are based on the assumption of a short column. Where the unsupported length is greater than 40 times the least radius of gyration (40 R), the safe load shall be determined by Formula 47. Principal columns in buildings shall have a minimum diameter or thickness of 12 in. Posts that are not continuous from story to story shall have a minimum diameter or thickness of 6 in.

161. **Unsupported Length.** The unsupported length of reinforced concrete columns shall be taken as:

(*a*) In flat slab construction the clear distance between the floor and under side of the capital;

(*b*) In beam-and-slab construction, the clear distance between the floor and the under side of the shallowest beam framing into the column at the next higher floor level;

(*c*) In floor construction with beams in one direction only, the clear distance between floor slabs;

(*d*) In columns supported laterally by struts or beams only, the clear distance between consecutive pairs (or groups) of struts or beams, provided that to be considered an adequate support, two such struts or beams shall meet the column at approximately the same level and the angle between the two planes formed by the axis of the column and the axis of each strut respectively is not less than 75 degrees nor more than 105 degrees.

When haunches are used at the junction of beams or struts with columns, the clear distance between supports may be considered as reduced by two-thirds of the depth of the haunch.

162. **Safe Load on Spiral Columns.** The safe axial load on columns reinforced with longitudinal bars and closely spaced spirals enclosing a circular core shall be not greater than that determined by Formula 42.

The symbols used in Formulas 42 to 49 are defined in Appendix C, except as indicated in Sections 162, 165, 168, 170, 176 and 182.

$$P = A_c f_c + n f_c p A \tag{42}$$

where

P = total safe axial load on column whose h/R is less than 40;

A = area of the concrete core enclosed within the spiral; the diameter of the core (or of the spiral) shall be taken as the distance center to center of the spiral wire;

p = ratio of effective area of longitudinal reinforcement to area of the concrete core;

$A_c = A(1 - p)$ = net area of concrete core; and

f_c = permissible compressive stress in concrete =

$$300 + (0.10 + 4\,p)f'_c. \tag{43}$$

The longitudinal reinforcement shall consist of at least six bars of minimum diameter of $\frac{1}{2}$ in., and its effective cross-sectional area shall not be less than 1 per cent nor more than 6 per cent of that of the core.

163. **Spiral Reinforcement.** The spiral reinforcement shall be not less than one-fourth the volume of the longitudinal reinforcement. It shall consist of evenly spaced continuous spirals held firmly in place and true to line by at least three vertical spacer bars. The spacing of the spirals shall be not greater than one-sixth of the diameter of the core and in no case more than 3 in. The spiral reinforcement shall meet the requirements of the Tentative Specifications for Cold-Drawn Steel Wire for Concrete Reinforcement.

164. **Protection of Spirally Reinforced Column.** Reinforcement shall be protected everywhere by a covering of concrete cast monolithic with the core, which shall have a minimum thickness of $1\frac{1}{2}$ in. in square columns and 2 in. in round or octagonal columns.

165. **Safe Load on Columns with Lateral Ties.** The safe axial load on columns reinforced with longitudinal bars and separate lateral ties shall be not greater than that determined by Formula 44,

$$P = (A'_c + A_s n) f_c \tag{44}$$

where A'_c = net area of concrete in the column (total column area minus area of reinforcement);
A_s = effective cross-sectional area of longitudinal reinforcement; and
f_c = permissible compressive stress in concrete and shall not exceed $0.20 f'_c$.

The amount of longitudinal reinforcement considered in the calculations shall be not more than 2 per cent nor less than 0.5 per cent of the total area of the column. The longitudinal reinforcement shall consist of not less than four bars of minimum diameter of $\frac{1}{2}$ in., placed with clear distance from the face of the column not less than 2 in.

166. **Lateral Ties.** Lateral ties shall be not less than $\frac{1}{4}$ in. in diameter, spaced not more than 8 in. apart.

167. **Bending in Columns.** Reinforced concrete columns subject to bending stresses shall be treated as follows:

(a) *With Spiral Reinforcement.* The compressive unit stress on the concrete within the core area under combined axial load and bending shall not exceed by more than 20 per cent the value given for axial load by Formula 43.

(b) *With Lateral Ties.* Additional longitudinal reinforcement may be used if required and the compressive unit stress on the concrete under combined axial load and bending may be increased to $0.30 f'_c$. The total amount of reinforcement considered in the computations shall be not more than 4 per cent of the total area of the column.

Tension in the longitudinal reinforcement due to bending of the column shall not exceed 16,000 lb. per sq. in.

168. **Composite Columns.** The safe load on composite columns in which a structural steel or cast-iron column is thoroughly encased in a circumferentially reinforced concrete core shall be based on a certain unit stress for the steel or cast-iron core plus a unit stress of $0.25 f'_c$ on the area within the spiral core.

The unit compressive stress on the steel section shall be not greater than that determined by Formula 45,

$$f_r = 18{,}000 - 70\, h/R \tag{45}$$

but shall not exceed 16,000 lb. per sq. in.

The unit stress on the cast-iron section shall be not greater than that determined by Formula 46,

$$f_r = 12{,}000 - 60\, h/R \tag{46}$$

but shall not exceed 10,000 lb. per sq. in.

In Formulas 45 and 46,

f_r = compressive unit stress in metal core, and
R = least radius of gyration of the steel or cast-iron section.

The diameter of the cast-iron section shall not exceed one-half of the diameter of the core within the spiral. The spiral reinforcement shall be not less than 0.5 per cent of the volume of the core within the spiral and shall conform in quality, spacing and other requirements to the provisions for spirals in Section 163.

Ample section of concrete and continuity of reinforcement shall be provided at the junction with beams or girders. The area of the concrete between the spiral and the metal core shall be not less than that required to carry the total floor load of the story above on the basis of a stress in the concrete of $0.35\, f'_c$, unless special brackets are arranged on the metal core to receive directly the beam or slab load.

169. **Structural Steel Columns.** The safe load on a structural steel column of a section which fully encases an area of concrete, and which is protected by an outside shell of concrete at least 3 in. thick, shall be computed in the same manner as for composite columns in Section 168, allowing $0.25\, f'_c$ on the area of the concrete enclosed by the steel section. The outside shell shall be reinforced by wire mesh, ties or spiral hoops weighing not less than 0.2 lb. per sq. ft. at the surface of the mesh and with a maximum spacing of 6 in. between strands or hoops. Special brackets shall be used to receive the entire floor load at each story. The safe load in steel columns calculated by Formula 45 shall not exceed 16,000 lb. per sq. in.

170. **Long Columns.** The permissible working load on the core in axially loaded columns which have a length greater than 40 times the least radius of gyration of the column core (40 R) shall be not greater than that determined by Formula 47,

$$\frac{P'}{P} = 1.33 - \frac{h}{120\, R} \tag{47}$$

where P' = total safe axial load on long column;
P = total safe axial load on column of the same section whose h/R is less than 40, determined as in Sections 162 and 165; and
R = least radius of gyration of column core.

171. **Bending Moments in Columns.** The bending moments in interior and exterior columns shall be determined on the basis of loading conditions and end restraint, and shall be provided for in the design. The recognized methods shall be followed in calculating the stresses due to combined axial load and bending. In spiral columns the area to be considered as resisting the stress is the area within the spiral.

H. Footings

172. **General.** The requirements for tension, compression, shear and bond in Sections 103 to 141, inclusive, shall govern the design of footings, except as hereinafter provided.

173. **Soil Footings.** The load per unit of area on soil footings shall be computed by dividing the column load by the area of base of the footing.

174. **Pile Footings.** Footings on piles shall be treated in the same manner as footings on soil, except that the load shall be considered as concentrated at the pile centers.

175. **Sloped or Stepped Footings.** Footings in which the thickness has been determined by the requirements for shear as specified in Sections 133 and 134 may be sloped or stepped between the critical section and the edge of the footing, provided that the shear on no section outside the critical section exceeds the value specified, and provided further that the thickness of the footing above the reinforcement at the edge shall not be less than 6 in. for footings on soil nor less than 12 in. for footings on piles. Sloped or stepped footings shall be cast as a unit.

176. **Critical Section for Bending.** The critical section for bending in a concrete footing which supports a concrete column or pedestal, shall be considered to be at the face of the column or pedestal. Where steel or cast-iron column bases are used, the moment in the footing shall be computed at the middle and at the edge of the base; the load shall be considered as uniformly distributed over the column or pedestal base.

The bending moment at the critical section in a square footing supporting a concentric square column, shall be computed from the load on the trapezoid bounded by one face of the column, the corresponding outside edge of the footing, and the portions of the two diagonals. The load on the two corner triangles of this trapezoid shall be considered as applied at a distance from the face equal to six-tenths of the projection of the footing from the face of the column. The load on the rectangular portion of the trapezoid shall be considered as applied at its center of gravity. The bending moment is expressed by Formula 48,

$$M = \frac{w}{2}(a + 1.2\,c)c^2 \qquad (48)$$

where M = bending moment at critical section of footing;
a = width of face of column or pedestal;
c = projection of footing from face of column; and
w = upward reaction per unit of area of base of footing.

For a round or octagonal column, the distance a shall be taken as equal to the side of a square of an area equal to the area enclosed within the perimeter of the column. (For typical footing designs, see Appendix C, Figs. 16 and 17.)

177. **Reinforcement.** The reinforcement in each direction in the footing shall be determined as for a reinforced concrete beam; the effective depth shall be the distance from the top of the footing to the plane of the reinforce-

ment. The sectional area of reinforcement shall be distributed uniformly across the footing unless the width is greater than the side of the column or pedestal plus twice the effective depth of the footing, in which case the width over which the reinforcement is spread may be increased to include one-half the remaining width of the footing. In order that no considerable area of the footing shall remain unreinforced, additional reinforcement shall be placed outside of the width specified, but such reinforcement shall not be considered as effective in resisting the calculated bending moment. For the extra reinforcement a spacing double that within the effective belt may be used.

178. **Concrete Stress.** The extreme fiber stress in compression in the concrete shall be kept within the limits specified in Section 189. The extreme fiber stress in sloped or stepped footings shall be based on the exact shape of the section for a width not greater than that assumed effective for reinforcement.

179. **Irregular Footings.** A rectangular or irregularly shaped footing shall be computed by dividing it into rectangles or trapezoids tributary to the sides of the column, using the distance to the center of gravity of the area as the moment arm of the upward forces. Outstanding portions of combined footings shall be treated in the same manner. Other portions of combined footings shall be designed as beams or slabs.

180. **Shearing Stresses.** See Sections 132 to 134.

181. **Bond Stress.** See Sections 135 to 141.

182. **Transfer of Stress at Base of Column.** The compressive stress in longitudinal reinforcement at the base of a column shall be transferred to the pedestal or footing by either dowels or distributing bases. When dowels are used, there shall be at least one for each column bar, and the total sectional area of the dowels shall be not less than the sectional area of the longitudinal reinforcement in the column. The dowels shall extend into the column and into the pedestal or footing not less than 50 diameters of the dowel bars for plain bars, or 40 diameters for deformed bars.

When metal distributing bases are used, they shall have sufficient area and thickness to transmit safely the load from the longitudinal reinforcement in compression and bending. The permissible compressive unit stress on top of the pedestal or footing directly under the column shall be not greater than that determined by Formula 49,

$$r_a = 0.25 f'_c \sqrt[3]{\frac{A}{A'}} \qquad (49)$$

where r_a = permissible working stress over the loaded area;
A = total area at the top of the pedestal or footing;
A' = loaded area at the column base;
f'_c = ultimate compressive strength of concrete. (See Section 120.)

In sloped or stepped footings A may be taken as the area of the top horizontal surface of the footing or as the area of the lower base of the largest frustum of a pyramid or cone contained wholly within the footing and having

for its upper base the loaded area A', and having side slopes of 1 vertical to 2 horizontal.

183. **Pedestals Without Reinforcement.** The allowable compressive unit stress on the gross area of a concentrically loaded pedestal or on the minimum area of a pedestal footing shall not exceed 0.25 f'_c, unless reinforcement is provided and the member designed as a reinforced concrete column.

The depth of a pedestal or pedestal footing shall be not greater than three times its least width and the projection on any side from the face of the supported member shall be not greater than one-half the depth. The depth of a pedestal whose sides are sloped or stepped shall not exceed three times the least width or diameter of the section midway between the top and bottom. A pedestal footing supported directly on piles shall have a mat of reinforcing bars having a cross-sectional area of not less than 0.20 sq. in. per foot in each direction, placed 3 in. above the top of the piles

I. Reinforced Concrete Retaining Walls

184. **Loads and Unit Stresses.** Reinforced concrete retaining walls shall be so designed [1] that the permissible unit stresses specified in Sections 186 to 197 are not exceeded. The heels of cantilever, counterforted and buttressed retaining walls shall be proportioned for maximum resultant vertical loads, but when the foundation reaction is neglected the permissible unit stresses shall not be more than 50 per cent greater than the normal permissible stresses.

185. **Details of Design.** The following principles shall be followed in the design of reinforced concrete retaining walls:

(*a*) The unsupported toe and heel of the base slabs shall be considered as cantilever beams fixed at the edge of the support.

(*b*) The vertical section of a cantilever wall shall be considered as a cantilever beam fixed at the top of the base.

(*c*) The vertical sections of counterforted and buttressed walls and parts of base slabs supported by the counterforts or buttresses shall be designed in accordance with the requirements for a continuous slab in Section 110.

(*d*) The exposed faces of walls without buttresses shall preferably be given a batter of not less than $\frac{1}{4}$ in. per ft.

(*e*) Counterforts shall be designed in accordance with the requirements for T-beams in Sections 113 to 115. Stirrups shall be provided in the counterforts to take the reaction when the tension reinforcement of the face walls and heels of bases is designed to span between the counterforts. Stirrups shall be anchored as near the exposed face of the longitudinal wall and as close to the lower face of the base as the requirements for protective covering permit.

(*f*) Buttresses shall be designed in accordance with the requirements specified for rectangular beams.

[1] In proportioning retaining walls consideration should be given to:

(*a*) Bearing value of soil;

(*b*) Stability against sliding.

(*g*) The shearing stress at the junction of the base with counterforts or buttresses shall not exceed the values specified in Sections 120 to 130.

(*h*) Horizontal metal reinforcement shall be of such form and so distributed as to develop the required bond. To prevent temperature and shrinkage cracks in exposed surface not less than 0.25 sq. in. of horizontal metal reinforcement per foot of height shall be provided.

(*i*) Grooved lock joints shall be placed not over 60 ft. apart to care for temperature changes.

(*j*) Counterforts and buttresses shall be located under all points of concentrated loading, and at intermediate points, as may be required by the design.

(*k*) The walls shall be cast as a unit between expansion joints, unless construction joints formed in accordance with Sections 69 and 73 are provided.

(*l*) Drains or "weep holes" not less than 4 in. in diameter and not more than 10 ft. apart, shall be provided. At least one drain shall be provided for each pocket formed by counterforts.

J. Summary of Working Stresses

186. **General.** The following working stresses shall be used:

where f'_c = ultimate compressive strength of concrete at age of 28 days, based on tests of 6 by 12-in. or 8 by 16-in. cylinders made and tested in accordance with the Standard Methods of Making and Storing Specimens of Concrete in the Field and the Tentative Methods of Making Compression Tests of Concrete.

Direct Stress in Concrete

187. **Direct Compression.**

(*a*) Columns whose length does not exceed 40 R:

(1) With spirals.....varies with amount of longitudinal reinforcement. (See Section 162.)

(2) Longitudinal reinforcement and lateral ties. (See Section 165.)

(*b*) Long columns............................ (See Section 170.)

(*c*) Piers and Pedestals.................................. $0.25 f'_c$
(See Section 183.)

188. **Compression in Extreme Fiber.**

(*a*) Extreme fiber stress in flexure........................ $0.40 f'_c$

(*b*) Extreme fiber stress in flexure adjacent to supports of continuous beams.............................. $0.45 f'_c$

189. **Tension.** In Concrete Members.................... None

Shearing Stresses in Concrete

190. Longitudinal Bars without Special Anchorage.

(*a*) Beams without web reinforcement .... $0.02 f'_c$
(*b*) Beams with stirrups or bent-up bars or combination of the two .... $0.06 f'_c$

191. Longitudinal Bars Having Special Anchorage.

(*a*) Beams without web reinforcement .... $0.03 f'_c$
(*b*) Beams with stirrups or bent-up bars or a combination of the two .... $0.12 f'_c$

192. Flat Slabs.

(*a*) Shear at distance *d* from capital or dropped panel .... $0.03 f'_c$
(*b*) Other limiting cases in flat slabs .... (See Section 131.)

193. Footings.

(*a*) Longitudinal bars without special anchorage .... $0.02 f'_c$
(*b*) Longitudinal bars having special anchorage .... $0.03 f'_c$

Stresses in Reinforcement

194. Tension in Steel.

(*a*) Billet-steel bars:

(1) Structural steel grade .... 16,000 lb. per sq. in.
(2) Intermediate grade .... 18,000 lb. per sq. in.
(3) Hard grade .... 18,000 lb. per sq. in.

(*b*) Rail-steel bars .... 18,000 lb. per sq. in.
(*c*) Structural steel .... 16,000 lb. per sq. in.
(*d*) Cold-drawn steel wire:

(1) Spirals .... Stress not calculated
(2) Elsewhere .... 18,000 lb. per sq. in.

195. Compression in Steel.

(*a*) Bars .... same as Section 194 (*a*) and (*b*)
(*b*) Structural steel core of composite column .... 16,000 lb. per sq. in. reduced for slenderness ratio .... (see Section 168)
(*c*) Structural steel column .... 16,000 lb. per sq. in. reduced for slenderness ratio .... (see Section 169)

196. Compression in Cast Iron.

Composite cast-iron column .... 10,000 lb. per sq. in. reduced for slenderness ratio .... (see Section 168)

197. Bond Between Concrete and Reinforcement.

(*a*)	Beams and slabs, plain bars	$0.04 f'_c$
(*b*)	Beams and slabs, deformed bars	$0.05 f'_c$
(*c*)	Footings, plain bars, one way	$0.04 f'_c$
(*d*)	Footings, deformed bars, one way	$0.05 f'_c$
(*e*)	Footings, bars two ways	(*c*) or (*d*) reduced by 25 per cent

APPENDIX C[1]

NOTATIONS AND FIGURES

All symbols used in the Standard Specifications for Concrete and Reinforced Concrete have been collected here for convenience of reference. The symbols are in general defined in the text near the formulas in which they are used. In a few instances the same symbol is used in two distinct senses; however, there is little danger of confusion from this source.

NOTATION

a = width of face of column or pedestal;

α = angle between inclined web bars and longitudinal bars;

A = total net area of column, footing, or pedestal, exclusive of fireproofing;

A' = loaded area of pedestal, pier or footing;

A_c = $A(1 - p)$ = net area of concrete core of column (core area minus reinforcement);

A'_c = net area of concrete in columns with lateral ties (total column area minus area of reinforcement);

A_s = effective cross-sectional area of metal reinforcement in tension in beams or compression in columns; and the effective cross-sectional area of metal reinforcement which crosses any of the principal design sections of a flat slab and which meets the requirements of Section 153;

A_v = total area of web reinforcement in tension within a distance of s (s_1, s_2, s_3, etc.) or the total area of all bars bent up in any one plane (see Fig. 9);

b = width of rectangular beam or width of flange of T-beam;

b' = width of stem of T-beam;

b_1 = dimension of the dropped panel of a flat slab in the direction parallel to l_1;[2]

c = base diameter of the largest right circular cone which lies entirely within the column (including the capital) whose vertex angle is 90 degrees and whose base is $1\frac{1}{2}$ in. below the bottom of the slab or the bottom of the dropped panel (see Fig. 14);

[1] Appendix I of the 1924 Joint Committee Report.

[2] In flat slab design, the column strip and the middle strip to be used when considering moments in the direction of the dimension l are located and dimensioned as shown in Fig. 15. The dimension l_1 does not always represent the short length of the panel. When moments in the direction of the shorter panel length are considered, the dimensions l and l_1 are to be interchanged and strips corresponding to those shown in Fig. 15 but extending in the direction of the shorter panel length are to be considered.

c = projection of footing from face of column;
C = total compressive stress in concrete;
C' = total compressive stress in reinforcement;
d = depth from compression surface of beam or slab to center of longitudinal tension reinforcement;
d' = depth from compression surface of beam or slab to center of compression reinforcement;
E_c = modulus of elasticity of concrete in compression;
E_s = modulus of elasticity of steel in tension = 30,000,000 lb. per sq. in.;
f_c = compressive unit stress in extreme fiber of concrete;
f'_c = ultimate compressive strength of concrete at age of 28 days, based on tests of 6 by 12-in. or 8 by 16-in. cylinders made and tested in accordance with the Standard Methods of Making and Storing Specimens of Concrete in the Field and the Tentative Methods of Making Compression Tests of Concrete;
f_r = compressive unit stress in metal core;
f_s = tensile unit stress in longitudinal reinforcement;
f'_s = compressive unit stress in longitudinal reinforcement;
f_v = tensile unit stress in web reinforcement;
F = total tension in a bar;
h = unsupported length of column;
I = moment of inertia of a section about the neutral axis for bending;
j = ratio of lever arm of resisting couple to depth d;
$jd = d - z$ = arm of resisting couple;
k = ratio of depth of neutral axis to depth d;
l = span length of beam or slab (generally distance from center to center of supports; for special cases, see Sections 106 and 145);
l = span length of flat slab, center to center of columns, in the rectangular direction in which moments are considered; [1]
l_1 = span length of flat slab, center to center of columns, perpendicular to the rectangular direction in which moments are considered; [1]
M = bending moment or moment of resistance in general;
M_0 = sum of positive and negative bending moments in either rectangular direction, at the principal design sections of a panel of a flat slab;
$n = E_s/E_c$ = ratio of modulus of elasticity of steel to that of concrete;
Σo = sum of perimeters of bars in one set;
p = ratio of effective area of tension reinforcement to effective area of concrete in beams = A_s/bd; and the ratio of effective area of longitudinal reinforcement to the area of the concrete core in columns;
p' = ratio of effective area of compression reinforcement to effective area of concrete in beams;
P = total safe axial load on column whose h/R is less than 40;
P' = total safe axial load on long column;
Q = ratio of the average to the maximum bond stress computed by Formula 34 within the distance y;
r = ratio of cross-sectional area of negative reinforcement which crosses

[1] See footnote regarding b_1 in foregoing notation, p. 374.

entirely over the column capital of a flat slab or over the dropped panel, to the total cross-sectional area of the negative reinforcement in the two column strips;

r_a = permissible working stress in concrete over the loaded area of a pedestal, pier or footing;

R = ratio of positive or negative moment in two column strips or one middle strip of a flat slab, to M_0;

R = least radius of gyration of a section;

s = spacing of web members, measured at the plane of the lower reinforcement and in the direction of the longitudinal axis of the beam;

t = thickness of flange of T-beam;

t_1 = thickness of flat slab without dropped panels or thickness of a dropped panel (see Fig. 14);

t_2 = thickness of flat slab with dropped panels at points away from the dropped panel (see Fig. 14);

T = total tensile stress in longitudinal reinforcement;

u = bond stress per unit of area of surface of bar;

v = shearing unit stress;

V = total shear;

w = uniformly distributed load per unit of length of beam or slab;

w = upward reaction per unit of area of base of footing;

w' = uniformly distributed dead and live load per unit of area of a floor roof;

W = total dead and live load uniformly distributed over a single panel area;

x = length of bar added for anchorage, including the hook, if any;

y = distance from the point at which the tension is computed to the point of beginning of anchorage;

z = depth from compression surface of beam or slab to resultant of compressive stresses.

Figures

For explanation of symbols used in figures, see the foregoing Notation.

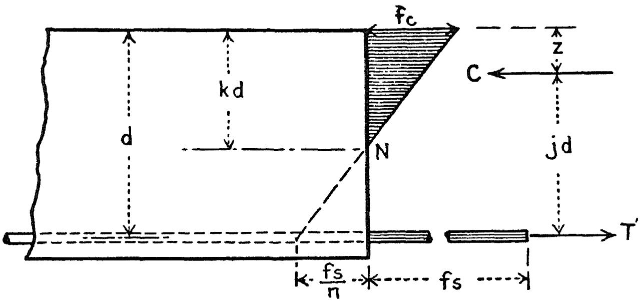


FIG. 1 — Nomenclature for Concrete Beam Reinforced for Tension.

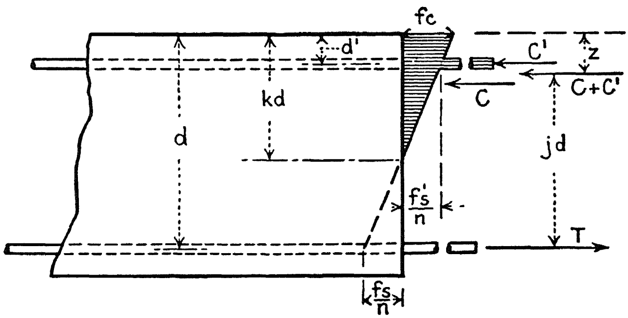


FIG. 2 — Nomenclature for Concrete Beam Reinforced for Tension and Compression.

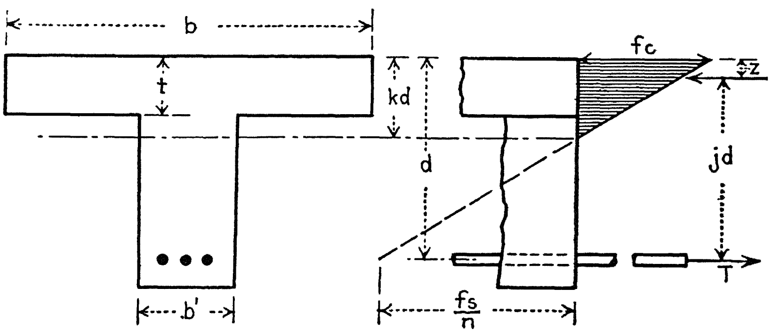


FIG. 3 — Nomenclature for Reinforced Concrete T-Beam.

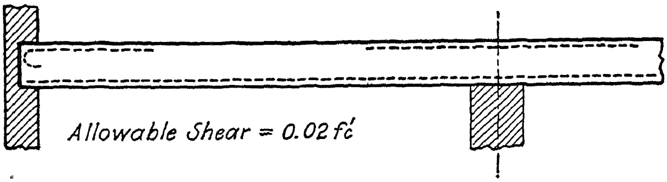


Fig. 4 — Typical Reinforced Concrete Beam; Principal Longitudinal Bars Without Special Anchorage.

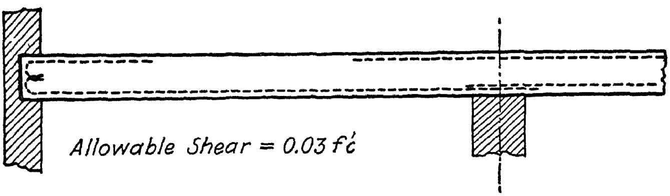


Fig. 5. — Typical Reinforced Concrete Beam; Special Anchorage of Longitudinal Bars.

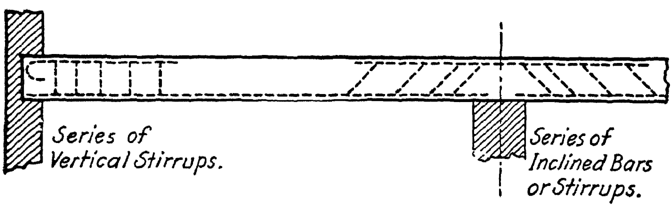


Fig. 6. — Typical Reinforced Concrete Beam without Special Anchorage; Web Reinforced by Means of Series of Vertical Stirrups; or Series of Inclined Bars or Stirrups.

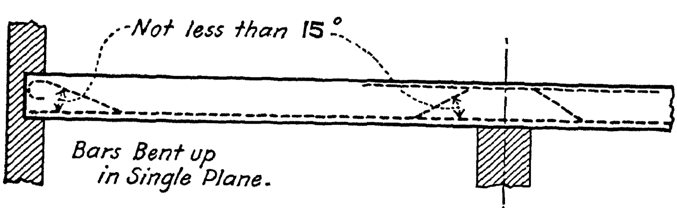


Fig. 7. — Typical Reinforced Concrete Beam; Principal Longitudinal Bars Bent Up in Single Plane.

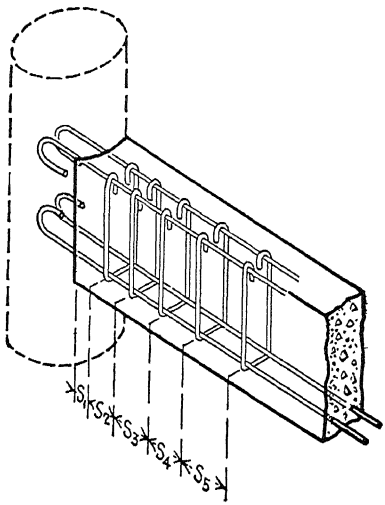


FIG. 8. — Typical Reinforced Concrete Beam with Anchored Longitudinal Bars and Vertical Stirrups.

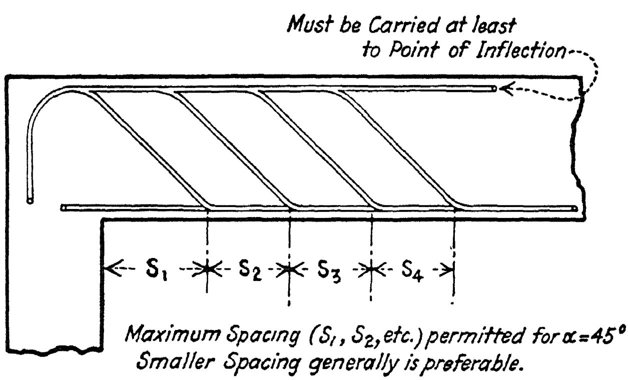


FIG. 9. — Typical Beam with Web Reinforced by Means of Series of Inclined Bars.

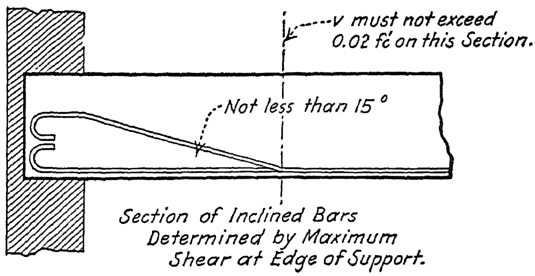


FIG. 10. — Typical Beam with Web Reinforced by Means of Bars Bent Up in a Single Plane.

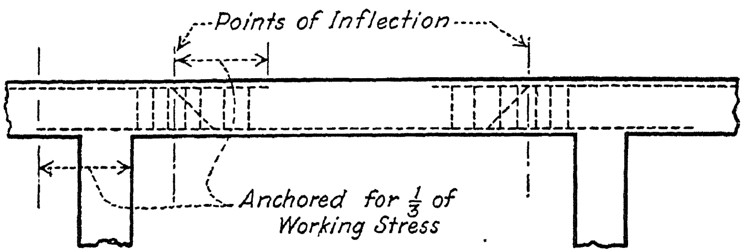


FIG. 11. — Typical Web Reinforcement for Continuous Beams with Special Anchorage.

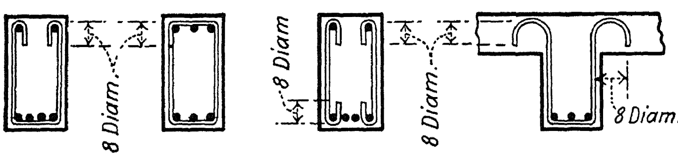


FIG. 12. — Typical Methods of Anchoring Vertical Stirrups.

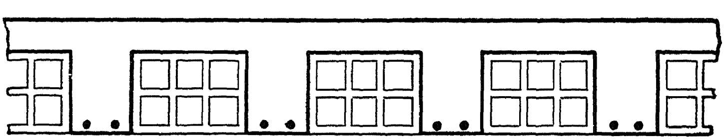


FIG. 13. — Typical Reinforced Concrete Beam-and-Tile Construction.

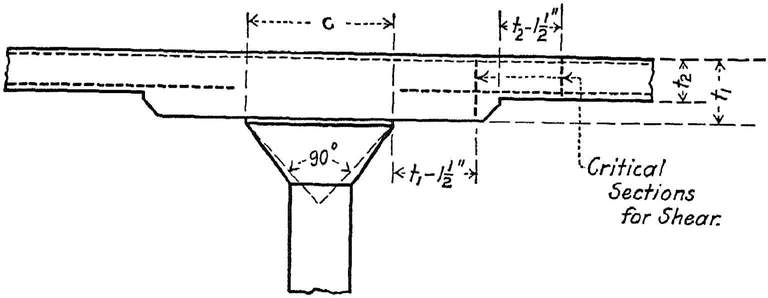


FIG. 14. — Typical Column Capital and Sections of Flat Slab with Dropped Panel.

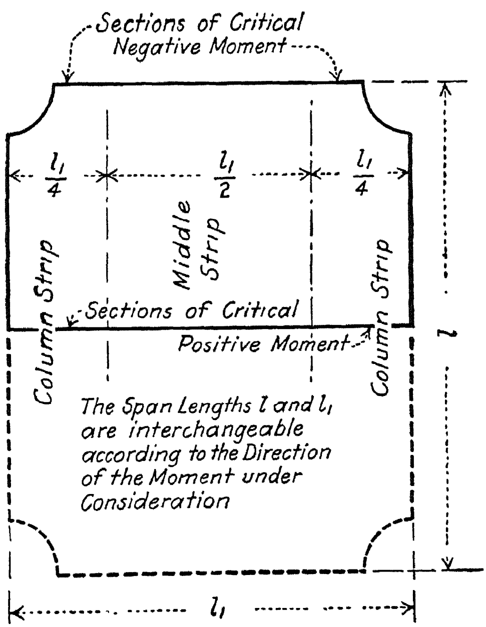
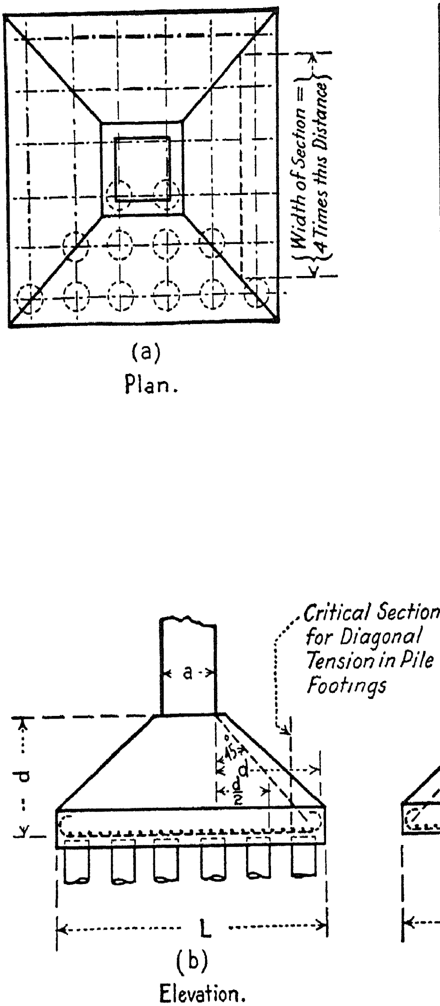


FIG. 15. — Principal Design Sections of a Flat Slab.



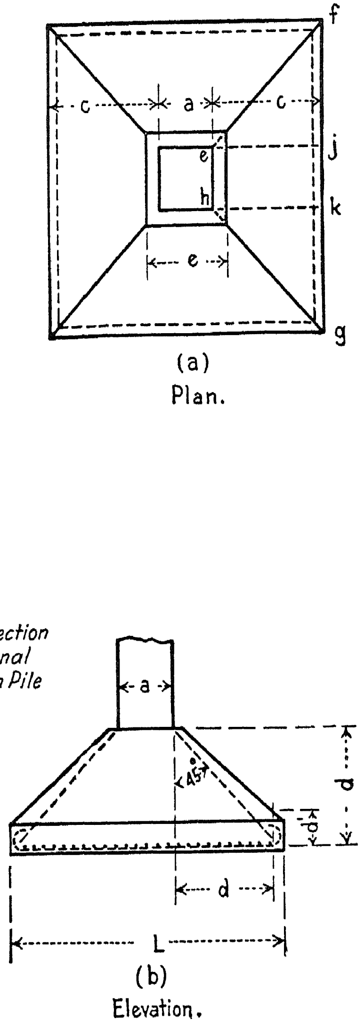


FIG. 16. — Typical Sloped Reinforced Concrete Footing on Piles.

FIG. 17. — Typical Sloped Reinforced Concrete Footing on Soil.

APPENDIX D

RANKINE'S THEORY OF EARTH PRESSURE

Rankine's analysis of the earth pressure problem assumes a uniform mass of dry granular material, without cohesion and without limit in extent. Consider the forces that must act upon the small element *abcd* shown in Fig. 127, whose weight (W) is equal to $wh\,dA$, where w is the unit weight of the material and dA is the area of a horizontal cross-section. Since the mass is of infinite extent the state of stress on face *ad* must be identical with that on *bc*, and so P_1 is equal and parallel to P_2 and both cut the vertical planes on which they act at the same distance from the surface.

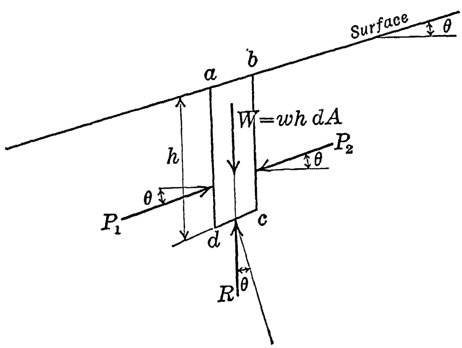


Fig. 127

Since the horizontal components of P_1 and P_2 are equal and since W acts vertically, the pressure R on the face *cd* can have no horizontal component and so must act vertically. Since the intensities of pressure on *cd* at *c* and *d* must be equal, this force, R, acts at the center of *cd*, and its line of action coincides with that of W. Therefore, for equilibrium, the lines of action of P_1 and P_2 must coincide, and since they both act at the same distance from the surface they are both parallel to it. So it may be concluded that on a vertical plane through any point, as *c*, the resultant pressure is parallel to the surface, and on any plane parallel to the surface through the same point, the stress is vertical.

That is, these are conjugate stresses and the expression for the relation between the intensities of conjugate stresses, derived in the texts on strength of materials, may be used:

$$\frac{p}{p_1} = \frac{\cos\theta - \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta + \sqrt{\cos^2\theta - \cos^2\phi}}$$

where p is less than p_1. If p is greater than p_1 obviously the relation is the same with the signs in numerator and denominator interchanged. Here

θ = the common angle of obliquity of the stresses, *i.e.*, that between the stress and the normal to the plane on which it acts; in this case equal to angle made by earth surface with the horizontal.

ϕ = the maximum possible angle of obliquity; in this case the angle of internal friction of the material, generally taken as the angle of repose, the steepest angle of surface slope the loose material will maintain.

The intensity of pressure upon plane *cd* equals

$$p_1 = \frac{W}{Area\ cd} = \frac{wh\,dA}{dA/\cos\theta} = wh\cos\theta.$$

The intensity of pressure (p) at the same point upon a vertical plane, then, is given by the following:

$$p = Cwh \tag{38}$$

where

$$C = \cos\theta\left(\frac{\cos\theta \mp \sqrt{\cos^2\theta - \cos^2\phi}}{\cos\theta \pm \sqrt{\cos^2\theta - \cos^2\phi}}\right) \tag{39}$$

The pressure actively exerted by earth upon a support is far less than the passive resistance that may be developed by pushing the support against the earth. Accordingly the value of C (Equation (39)) with the negative sign in the numerator and the positive in the denominator may be considered to give the value of the active pressure and the same equation, with signs reversed, the passive resistance.

APPENDIX E

ANALYSIS OF THE HINGELESS ARCH

The following outline of the common theory of the hingeless arch, based on the flexure of a curved bar, is that presented by Professors Turneaure and Maurer in their "Principles of Reinforced Concrete Construction." Abridgement and a few changes in order have been made but the same words have been retained in large part.

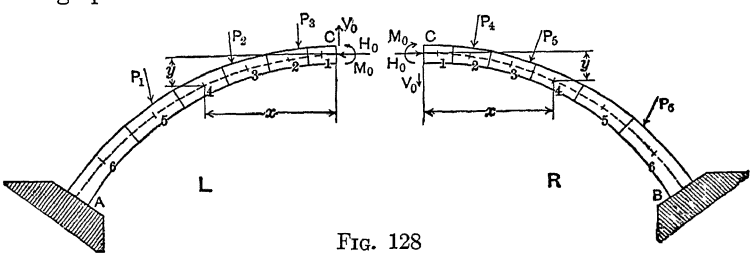


FIG. 128

Notation. (See Fig. 128.)

H_0 = thrust at the crown;

V_0 = shear at the crown;

M_0 = bending moment at the crown;

N, V, and M = thrust, shear, and moment at any other section;

R = resultant pressure at any section = resultant of N and V;

δs = length of a division of the arch ring measured along the arch axis;

n = number of divisions in one-half of the arch;

I = moment of inertia of any section $= I_{\text{concrete}} + (n-1)\, I_{\text{steel}}$; where $n = E_s/E_c$;

P = any load on the arch;

x, y = co-ordinates of any point on the arch axis referred to the crown as origin, and all to be considered as positive in sign;

m = bending moment at any point in the cantilever, Fig. 128, due to external loads.

In order to obtain the three additional equations necessary for its solution the arch is considered to be cut at the crown and the moment, thrust and shear exerted by each part upon the other represented as M_0, H_0 and V_0, Fig. 128. For convenience of analysis the arch is divided into an even number of divisions (ten to twenty usually) so proportioned that the ratio of length, measured along the axis, to average moment of inertia, is the same for all divisions.

"Consider the left-hand cantilever of Fig. 128. Under the forces acting the point C will deflect and the tangent to the axis at this point will change direction (the abutment at A being fixed). Let Δy, Δx, and $\Delta\phi$ be, respectively, the vertical and horizontal components of this motion and the change in angle of the tangent. Then according to the principles relating to curved beams[1] we have the values

$$\Delta y = \Sigma Mx\frac{\delta s}{EI}, \quad \Delta x = \Sigma My\frac{\delta s}{EI}, \quad \text{and} \quad \Delta\phi = \Sigma M\frac{\delta s}{EI}. \qquad (a)$$

"In like manner, referring to the right cantilever, let $\Delta y'$, $\Delta x'$, and $\Delta\phi'$ represent the components of the movement of C and the change of angle of the tangent. These may be expressed in terms similar to Eq. (a).

"Now evidently

$$\Delta y = \Delta y', \quad \Delta x = -\Delta x', \quad \text{and} \quad \Delta\phi = -\Delta\phi'. \qquad (b)$$

Furthermore, since $\delta s/I$ is constant and likewise E, the quantity $\delta s/EI$ may be placed outside the summation sign.

"Using the subscript L to denote left side and R to denote right side we then derive the relations

$$\left.\begin{aligned} \Sigma M_L x &= \Sigma M_R x, \\ \Sigma M_L y &= -\Sigma M_R y, \\ \Sigma M_L &= -\Sigma M_R. \end{aligned}\right\} \qquad (c)$$

"The moment M may in general be expressed in terms of known and unknown quantities thus:

$$M_L = m_L + M_0 + H_0 y + V_0 x \text{ for the left side.}$$

and $$M_R = m_R + M_0 + H_0 y - V_0 x \text{ for the right side.}$$

[1] "Modern Framed Structures," Johnson, Bryan, Turneaure: Part II, pages 112–119.

Hence, substituting in (*c*) and combining terms, and noting that ΣM_0 for one half is equal to nM_0, we have

$$\Sigma m_L x - \Sigma m_R x + 2\, V_0 \Sigma x^2 = 0, \tag{d}$$
$$\Sigma m_L y + \Sigma m_R y + 2\, M_0 \Sigma y + 2\, H_0 \Sigma y^2 = 0, \tag{e}$$
$$\Sigma m_L + \Sigma m_R + 2\, nM_0 + 2\, H_0 \Sigma y = 0. \tag{f}$$

"The solution of these equations gives:

$$H_0 = \frac{n\Sigma my - \Sigma m \Sigma y}{2\,[(\Sigma y)^2 - n\Sigma y^2]}, \tag{1}$$
$$V_0 = \frac{\Sigma(m_R - m_L)x}{2\,\Sigma x^2}, \tag{2}$$
$$M_0 = -\frac{\Sigma m + 2\, H_0 \Sigma y}{2\, n}. \tag{3}$$

"In these equations the summations Σy, Σy^2, and Σx^2 are for one-half of the arch only; the summation Σm is for the entire arch and is equal to $\Sigma m_R + \Sigma m_L$; the summation $\Sigma(m_R - m_L)x$ is a summation of the products $(m_R - m_L)x$, in which m_R and m_L are the bending moments at corresponding points in the right and left halves which have equal abscissas x; and the summation Σmy is for the entire arch, but since symmetrical points have equal y's this quantity may be calculated as $\Sigma(m_R + m_L)y$. A positive result for V_0 indicates action as shown in Fig. 128.

"The loads and their points of application have been considered apart from the divisions of the arch ring, as the two things are in no wise related. Where no spandrel arches are used and the entire load is applied continuously along the arch ring, the load may for convenience be divided to correspond with the arch divisions and applied at the center points, 1, 2, 3, etc. This division is, however, of no importance, the only requirement being a sufficiently small subdivision of the arch ring and of the load so that the errors of approximation will be negligible. Where spandrel arches are used, the live load and a large part of the dead load will be applied at the centers of the arch piers. The weight of the main arch ring may also be considered as concentrated at these same points.

"If calculations are to be made for more than one loading it will be noted that the denominators of the values for H_0, V_0, and M_0 do not change. The quantities involving m are the only ones requiring recalculation, and if the load on but one-half of

the arch is changed, then the values of m for that half only need be recalculated. In the case of a symmetrical loading, or a load on one-half only, the calculation of m is also necessary for one-half the arch only. For symmetrical loads, $V_0 = 0$."

Division of Arch Ring to Give Constant $\delta s/I$. "In most cases the depth of the arch ring increases from crown towards springing line, giving a variable moment of inertia. Considering the concrete only, the moment of inertia will increase as d^3 so that a comparatively small change in depth will cause a large change in moment of inertia. To maintain $\delta s/I$ constant, the value of δs will therefore be much greater near the springing line than at the crown, and hence to secure the desired accuracy the length of division at the crown will need to be made fairly short. The value of $\delta s/I$ to adopt so that there will be no fractional division may be determined as follows:

"Let $i = \frac{1}{I}$;

i_a = mean value of i;

s = half length of the arch ring measured along the axis;

n = desired number of divisions in one-half the arch.

"Calculate first the mean value of i for the half arch ring by determining several values at equal intervals along the arch. Then the desired value of $\delta s/I$ is

$$\frac{\delta s}{I} = \frac{s i_a}{n}. \tag{5}$$

"The value of $\delta s/I$ being known, the proper length of δs for any part of the arch ring can readily be determined. Beginning at the crown, the length of the first division is determined, then the second, third, etc., to the end. The length of a division not being exactly known beforehand, the value of I for that division will not be exactly known, but the necessary adjustment is very simple.

"In determining the value of I the steel reinforcement must be duly considered."

A graphical method of making this division proceeds thus. Lay off the length of the half axis on a horizontal line and plot as ordinates a sufficient number of values of the moment of inertia to draw the curve of I variation. Construct by trial a continuous

series of similar isosceles triangles with bases on the horizontal line and vertices on the I curve, the series beginning and ending as closely as possible at the crown and springing line points. The ratio of base to altitude will be the same for all triangles and so the several base lengths give the required lengths of the divisions of the arch axis.

Temperature Stresses. For temperature stresses, $\Delta\phi$ of Eq. (a), above, is zero and Δx is equal to the change in length of the half-span, $= \frac{1}{2}\,ctl$ where t is the temperature rise in degrees, c is the coefficient of expansion and l the span. Therefore

$$\Sigma M_L y \frac{\delta s}{EI} = \tfrac{1}{2}\,ctl$$

and

$$\Sigma M_L = 0.$$

Since there are no external loads $m = 0$; from symmetry $V_0 = 0$; hence $M = M_0 + H_0 y$. Substituting this value of M in the above equations there results

$$M_0 \Sigma y + H_0 \Sigma y^2 = \frac{ctl}{2} \cdot \frac{EI}{\delta s},$$

and

$$nM_0 + H_0 \Sigma y = 0.$$

Whence

$$H_0 = \frac{EI}{\delta s}\left(\frac{ctln}{2\,[n\Sigma y^2 - (\Sigma y)^2]}\right) \tag{6}$$

$$M_0 = -\frac{H_0 \Sigma y}{n}. \tag{7}$$

The summations refer to one-half the arch. The bending moment at any point is

$$M = M_0 + H_0 y. \tag{8}$$

Graphically, the true equilibrium polygon is a horizontal line drawn a distance below the crown equal to $M_0/H_0 = \Sigma y/n$.

Stresses Due to Shortening of Arch from Thrust. A thrust throughout the arch producing an average stress on the concrete equal to f_c pounds per square inch would shorten the arch span an amount equal to $f_c l/E$ if unrestrained. This action develops horizontal reactions in the same manner as a lowering of tempera-

ture. The value of the resulting reactions, or the crown thrust, may then be found by substituting $f_c l/E$ for ctl of Eq. (6). There results

$$H_0 = -\frac{I}{\delta s}\cdot\frac{f_c l n}{2\,[n\Sigma y^2 - (\Sigma y)^2]}\cdot \qquad (9)$$

The moments at crown and elsewhere are given by Eqs. (7) and (8), using the value of H_0 from Eq. (9).

The thrusts and moments due to arch shortening will not usually be large. They may be applied as corrections to the thrusts and moments found before.

Deflection of the Crown. The downward deflection of the crown under a load is given by Eq. (a), above. It is

$$\Delta y = -\frac{\delta s}{EI}\Sigma Mx. \qquad (10)$$

If M is not determined for all points, use the value of M from Eq. (4), deriving

$$\Delta y = -\frac{\delta s}{EI}[\Sigma mx + M_0\Sigma x + H_0\Sigma xy + V_0\Sigma x^2]. \qquad (11)$$

The summations are for one-half only.

The *rise* of crown due to an increase of temperature is obtained from Eq. (11) by substituting from Eqs. (6) and (7). There results

$$\Delta y = \frac{ctl}{2}\cdot\frac{n\Sigma xy - \Sigma x\Sigma y}{n\Sigma y^2 - (\Sigma y)^2}\cdot \qquad (12)$$

APPENDIX F

DESIGN DATA

(See Chapter VIII for illustration and explanation of the use of these data.)

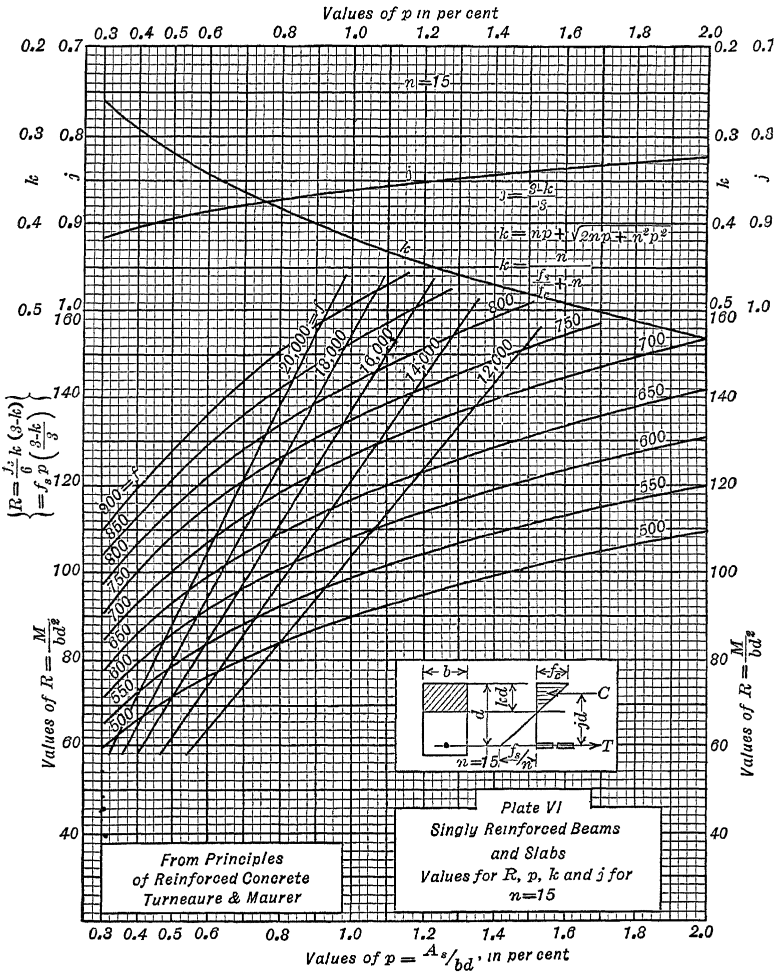
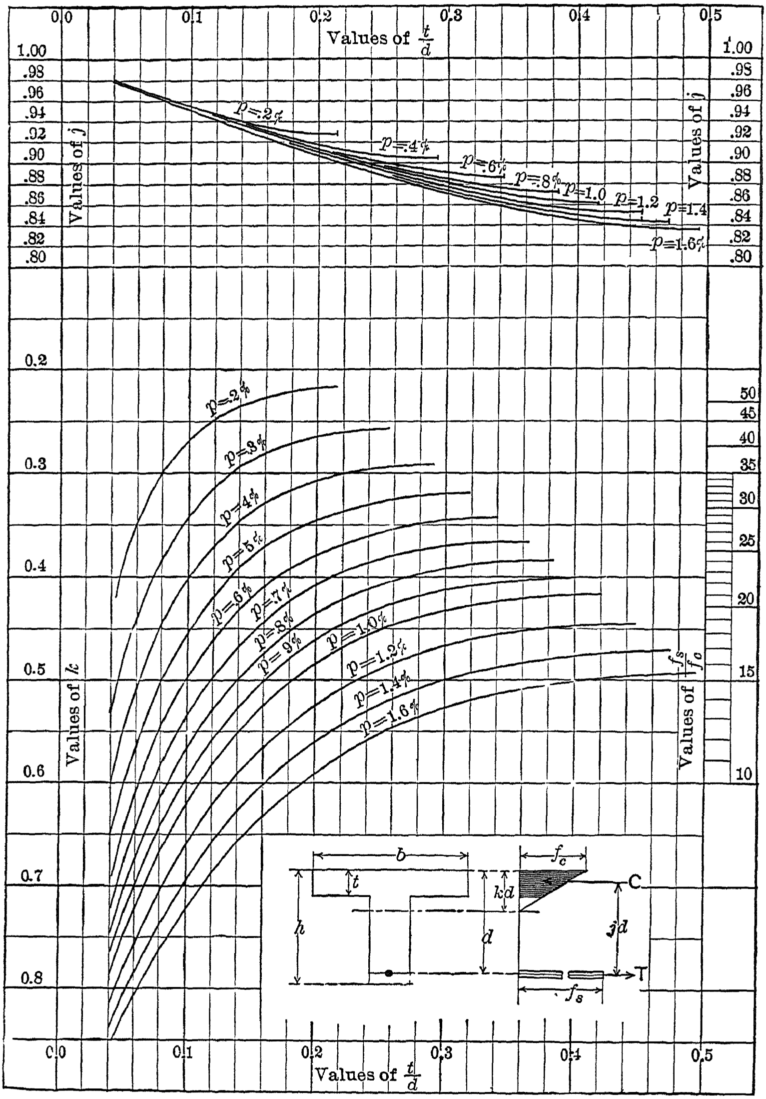


PLATE VI

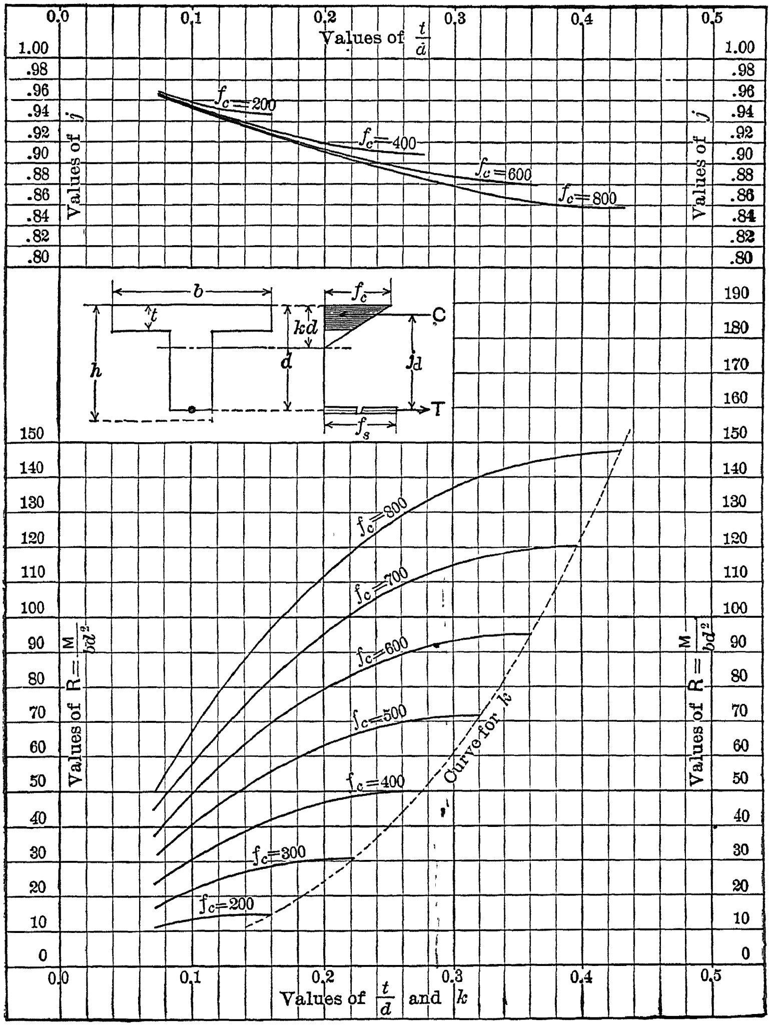


Values of k and j for Tee Beams ($n = 15$)

PLATE VII[1]

[1] Reproduced by permission from "Principles of Reinforced Concrete Construction" by Turneaure and Maurer.

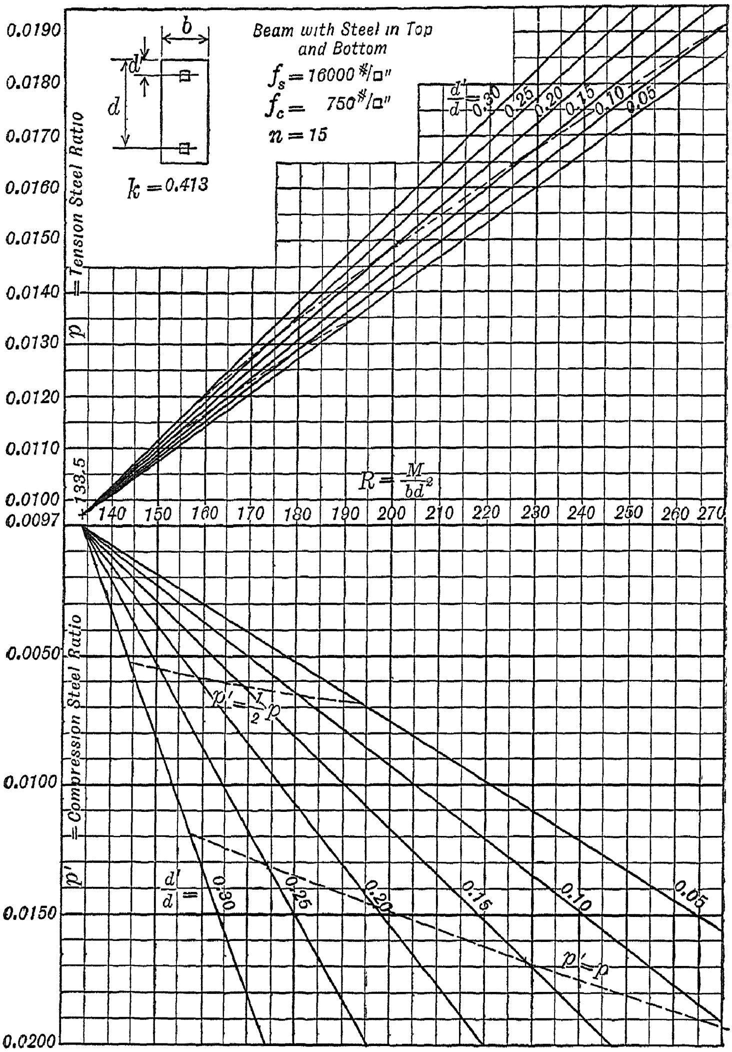
$f_s = 16{,}000$



Coefficients of Resistance of Tee Beams ($n = 15$)

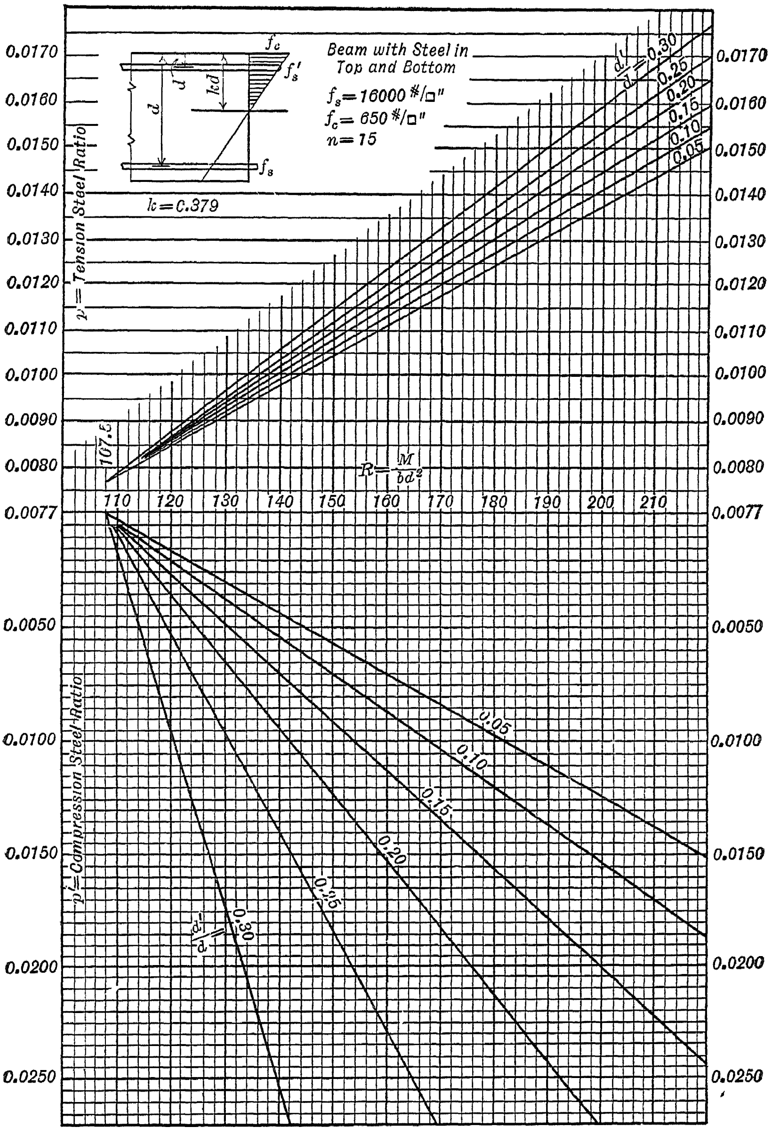
PLATE VIII[1]

[1] Reproduced by permission from "Principles of Reinforced Concrete Construction" by Turneaure and Maurer.



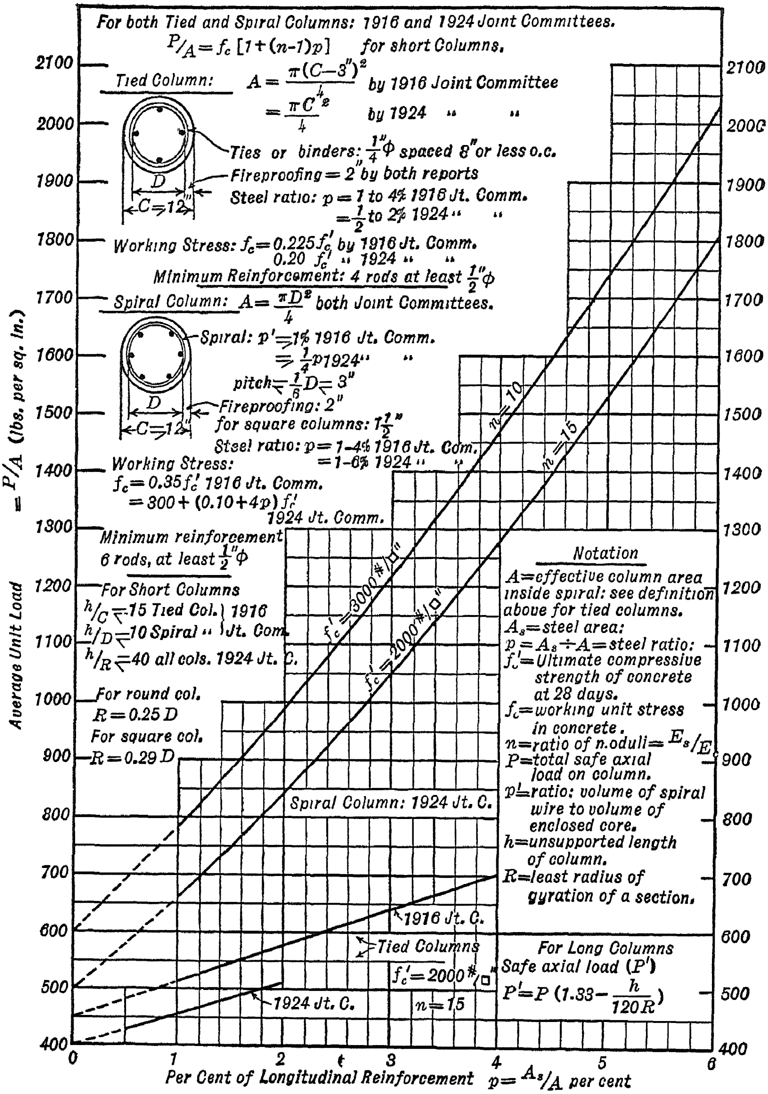
Moments of Resistance for Doubly Reinforced Beams ($f_c = 750$)

PLATE IX



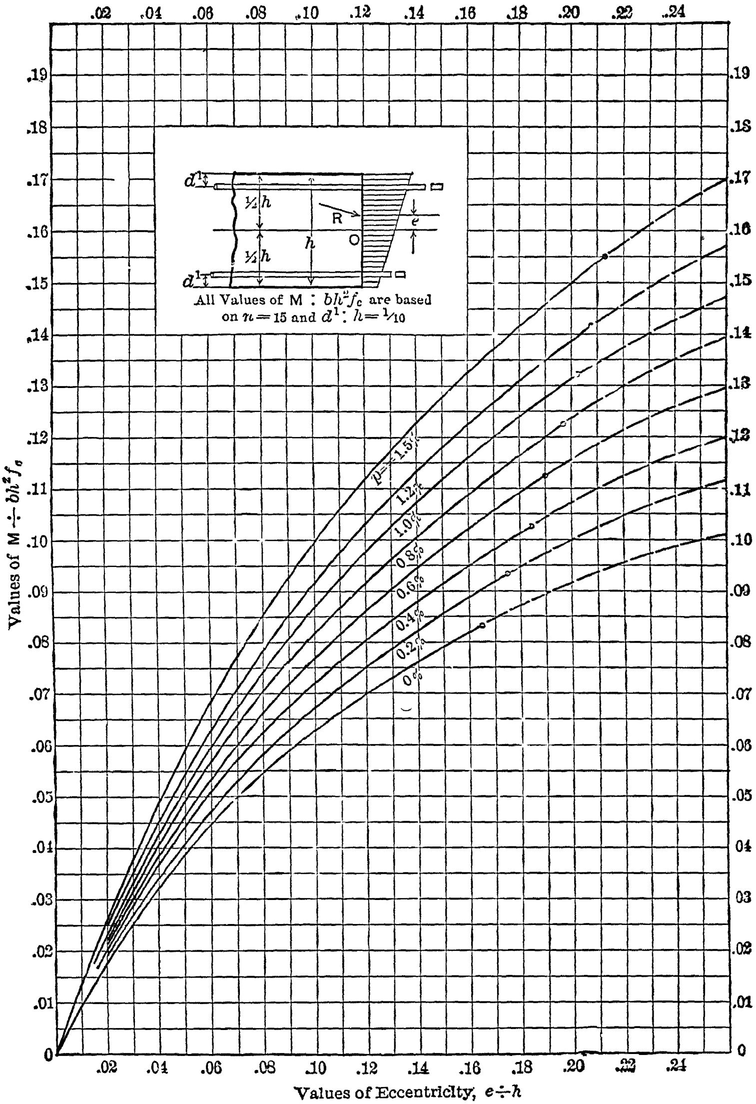
Moments of Resistance for Doubly Reinforced Beams ($f_c = 650$)

PLATE X



Average Column Stresses

PLATE XI

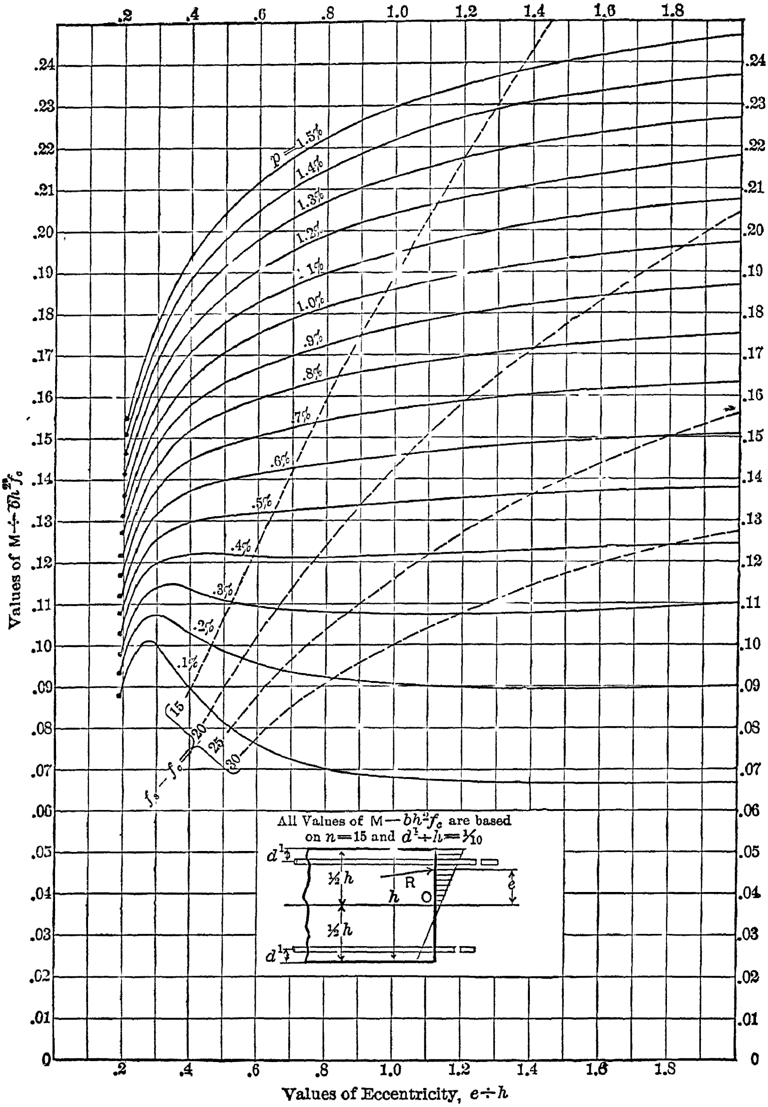


Flexure and Direct Stress (Compression over Whole Section) $n = 15$

$p = p'$

PLATE XII[1]

[1] Reproduced by permission from "Principles of Reinforced Concrete Construction" by Turneaure and Maurer.

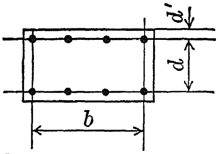


Flexure and Direct Stress (Tension on Part of Section) $n = 15$

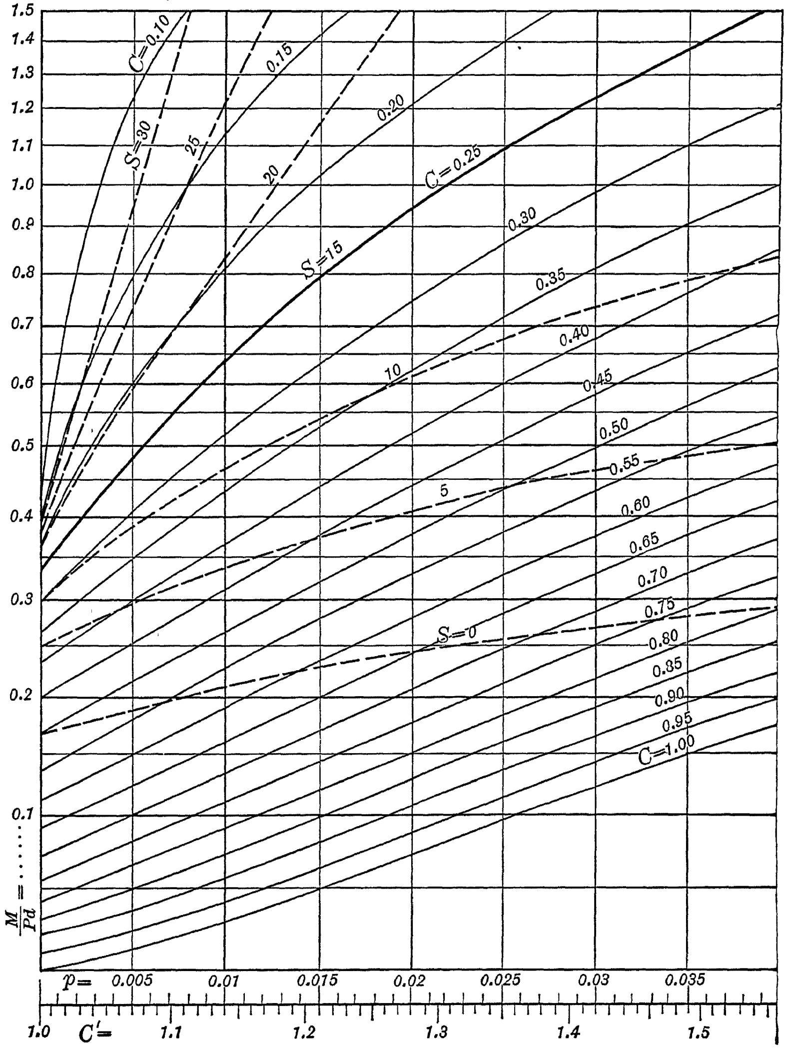
$p = p'$

PLATE XIII[1]

[1] Reproduced by permission from "Principles of Reinforced Concrete Construction" by Turneaure and Maurer.



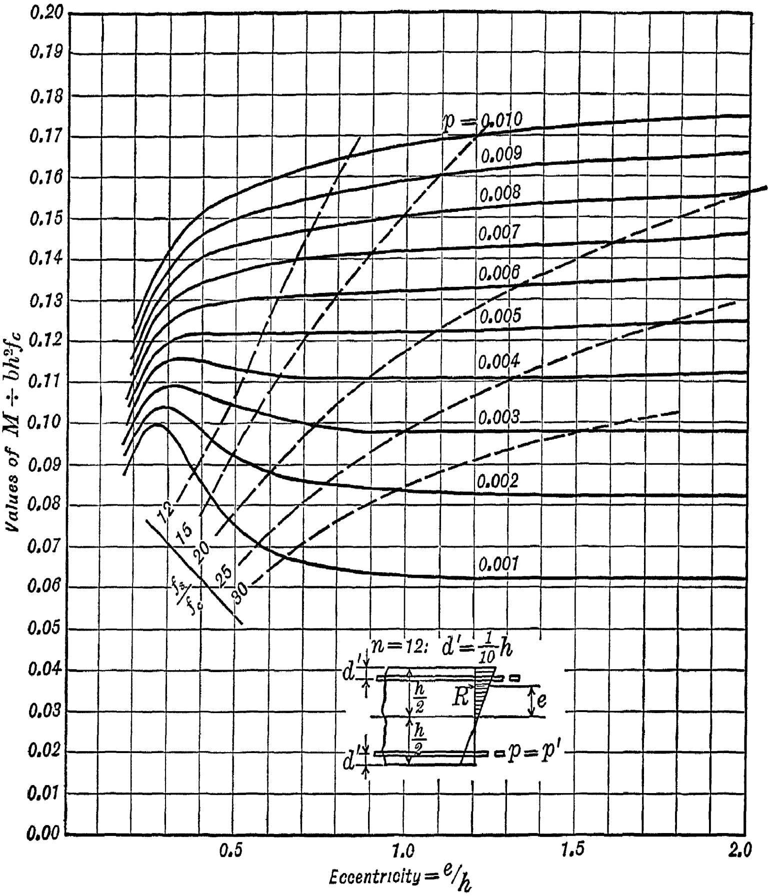
M = bending moment (inch-lbs.);
P = direct load p = ratio of *total* steel
Area ($A_s = pbd$) $f_c = \frac{P}{bdC'}$; $f_s = Sf_c$ $\frac{d'}{d} = 0$; $n = 15$



Flexure and Direct Stress

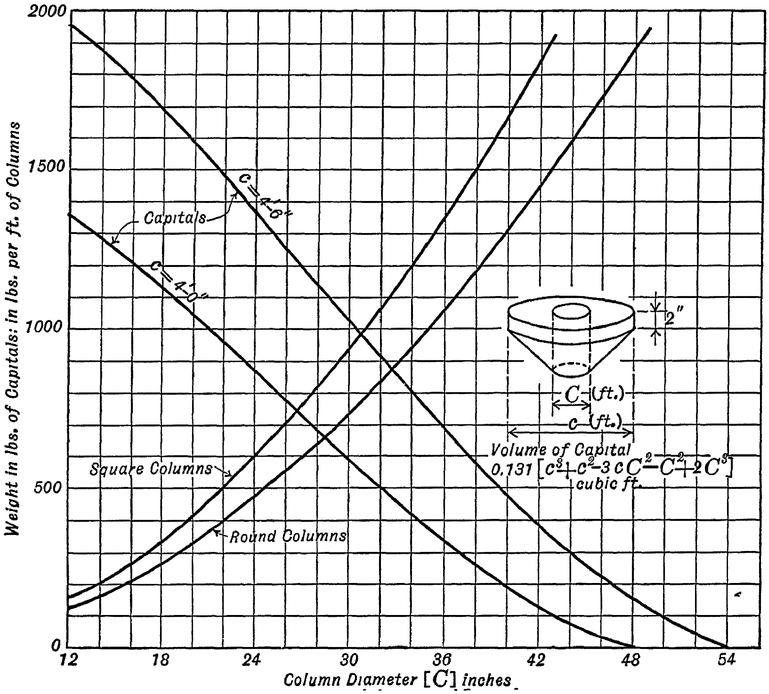
PLATE XIV[1]

[1] Devised by Mr. S. G. Roeblad, Consulting Engineer, Boston.



Flexure and Direct Stress (Tension on Part of Section), $n = 12$

PLATE XV



Weight of Columns and Capitals

PLATE XVI

Table 9
SLABS AND SIMPLE BEAMS
(Bending moments in pound feet and area of steel per foot of width)

$R = 107.5$ $f_s = 16{,}000$ $n = 15$

$p =$ 0 00769 $f_c = 650$

Depth, in inches	Moment	Area	Depth, in inches	Moment	Area	Depth, in inches	Moment	Area
1	108	0.09	10	10,750	0.92	28	84,300	2 58
¼	168	0.12	¼	11,290	0.95	29	90,400	2.68
½	242	0.14	½	11,850	0.97	30	96,700	2.77
¾	330	0.16	¾	12,420	0.99	31	103,000	2.86
2	430	0.18	11	13,010	1.02	32	110,000	2.95
¼	544	0.21	¼	13,610	1.04	33	117,000	3.05
½	672	0 23	½	14,220	1.06	34	124,000	3 14
¾	813	0.25	¾	14,840	1.08	35	132,000	3.23
3	968	0.28	12	15,500	1.11	36	139,000	3 32
¼	1,136	0 30	½	16,800	1.15	37	147,000	3 41
½	1,317	0.32	13	18,200	1.20	38	155,000	3.51
¾	1,512	0.35	½	19,600	1.25	39	164,000	3.60
4	1,720	0.37	14	21,100	1.29	40	172,000	3.69
¼	1,940	0.39	½	22,600	1.34	41	181,000	3.78
½	2,180	0.42	15	24,200	1.38	42	190,000	3.88
¾	2,430	0.44	½	25,800	1.42	43	199,000	3.97
5	2,690	0.46	16	27,500	1.47	44	208,000	4.06
¼	2,960	0.48	½	29,300	1.51	45	218,000	4.15
½	3,250	0.51	17	31,100	1.56	46	227,000	4.24
¾	3,550	0.53	½	32,900	1.61	47	237,000	4.34
6	3,870	0.55	18	34,800	1.65	48	248,000	4.43
¼	4,200	0.58	½	36,800	1.70	49	258,000	4.52
½	4,540	0.60	19	38,800	1.75	50	269,000	4.61
¾	4,900	0.62	½	40,900	1.79	51	280,000	4.71
7	5,270	0.65	20	43,000	1.84	52	291,000	4.80
¼	5,650	0.67	½	45,200	1.88	53	302,000	4.89
½	6,050	0.69	21	47,400	1.93	54	313,000	4.98
¾	6,460	0.72	½	49,700	1.98	55	325,000	5.08
8	6,880	0.74	22	52,000	2.02	56	337,000	5.17
¼	7,320	0.76	½	54,400	2.07	57	349,000	5.26
½	7,770	0.78	23	56,900	2.11	58	362,000	5 35
¾	8,230	0.81	½	59,400	2.16	59	374,000	5.44
9	8,710	0.83	24	61,900	2.21	60	387,000	5.54
¼	9,200	0.85	25	67,200	2.31	61	400,000	5.63
½	9,700	0.88	26	72,700	2.40	62	413,000	5 72
¾	10,220	0.90	27	78,400	2.49	63	427,000	5.81

From Thomas & Nichols' Reinforced Concrete Design Tables.
[Abridged.]

TABLE 10

SLABS AND SIMPLE BEAMS

(Bending moments in pound feet and area of steel per foot of width)

R = 133.5 $\quad f_s$ = 16,000 $\quad n$ = 15

p = 0 00967 $\quad f_c$ = 750

Depth, in inches	Moment	Area	Depth, in inches	Moment	Area	Depth, in inches	Moment	Area
1	136	0 12	10	13,350	1.16	28	105,000	3.25
¼	209	0 15	¼	14,030	1.19	29	112,000	3.37
½	300	0 17	½	14,720	1 22	30	120,000	3.48
¾	409	0 20	¾	15,400	1.25	31	128,000	3.60
2	534	0 23	11	16,200	1 28	32	137,000	3.71
¼	676	0.26	¼	16,900	1 31	33	145,000	3.83
½	834	0.29	½	17,700	1 33	34	154,000	3.95
¾	1,010	0.32	¾	18,400	1.36	35	164,000	4.06
3	1,201	0 35	12	19,200	1.39	36	173,000	4.18
¼	1,410	0 38	½	20,900	1.45	37	183,000	4.29
½	1,640	0 41	13	22,600	1 51	38	193,000	4.41
¾	1,880	0.44	½	24,300	1.57	39	203,000	4.53
4	2,140	0 46	14	26,200	1.62	40	214,000	4.64
¼	2,410	0.49	½	28,100	1.68	41	224,000	4.76
½	2,700	0.52	15	30,000	1.74	42	235.000	4.87
¾	3,010	0.55	½	32,100	1.80	43	247,000	4 99
5	3,340	0 58	16	34,200	1.86	44	258,000	5.11
¼	3,680	0.61	½	36,300	1.91	45	270,000	5 22
½	4,040	0.64	17	38,600	1.97	46	282,000	5.34
¾	4,410	0.67	½	40,900	2.03	47	295,000	5.45
6	4,810	0 70	18	43,300	2.09	48	308,000	5.57
¼	5,210	0 73	½	45,700	2.15	49	321,000	5.69
½	5,640	0 75	19	48,200	2.20	50	334,000	5.80
¾	6,080	0.78	½	50,800	2.26	51	347,000	5.92
7	6,540	0.81	20	53,400	2.32	52	361,000	6.03
¼	7,020	0.84	½	56,100	2.38	53	375,000	6.15
½	7,510	0.87	21	58.900	2.44	54	389,000	6.27
¾	8,020	0.90	½	61,700	2.49	55	404,000	6.38
8	8,540	0.93	22	64,600	2.55	56	419,000	6 50
¼	9,090	0.96	½	67,600	2.61	57	434,000	6.61
½	9,650	0.99	23	70,600	2 67	58	449,000	6.73
¾	10,220	1.02	½	73,700	2.73	59	465,000	6.85
9	10,810	1.04	24	76,900	2.78	60	481,000	6.96
¼	11,420	1.07	25	83,400	2.90	61	497,000	7.08
½	12,050	1.10	26	90,200	3 02	62	513,000	7 19
¾	12,690	1.13	27	97,300	3.13	63	530,000	7.31

From Thomas & Nichols' REINFORCED CONCRETE DESIGN TABLES.

[Abridged.]

Table 11

TEE BEAMS

(Bending moment in pound feet and area of steel per foot width)

$f_s = 16{,}000$ $f_c = 650$ $n = 15$

Depth, in inches	3½-in. Slab		4-in. Slab		4½-in. Slab		5-in. Slab		5½-in. Slab		6-in. Slab	
	Moment	Area	Moment	Area	Moment	Area	Moment	Area	Moment	Area	Moment	Area
10	10,710	0.92	...	...	...	...	...			...		...
11	12,870	0.99	13,000	1.01			...					...
12	14,850	1.05	15,300	1.09	15,500	1.11		...				...
13	17,000	1.10	17,700	1.16	18,100	1.19	...					...
14	19,100	1.14	20,100	1.21	20,700	1.26	21,000	1.29	...		...	...
15	21,300	1.18	22,500	1.26	23,400	1.32	23,900	1.36	24,200	1.38	...	...
16	23,400	1.21	25,000	1.31	26,100	1.38	26,900	1.43	27,400	1.46	27,500	1.48
17	25,600	1.24	27,400	1.34	28,800	1.43	29,900	1.49	30,600	1.54	31,000	1.56
18	27,800	1.27	29,900	1.38	31,600	1.47	32,900	1.54	33,800	1.60	34,500	1.64
19	30,000	1.29	32,400	1.41	34,400	1.51	35,900	1.59	37,100	1.65	38,000	1.71
20	32,200	1.31	34,900	1.43	37,200	1.54	39,000	1.63	40,500	1.71	41,600	1.77
21	34,500	1.33	37,400	1.46	39,900	1.57	42,100	1.67	43,800	1.75	45,200	1.82
22	36,700	1.35	39,900	1.48	42,800	1.60	45,200	1.71	47,200	1.80	48,800	1.87
23	38,900	1.36	42,500	1.50	45,600	1.63	48,300	1.74	50,600	1.83	52,500	1.92
24	41,100	1.38	45,000	1.52	48,400	1.65	51,400	1.77	54,000	1.87	56,200	1.96
25	43,400	1.39	47,500	1.54	51,200	1.67	54,500	1.79	57,400	1.90	59,800	2.00
26	45,600	1.40	50,000	1.55	54,100	1.69	57,700	1.82	60,800	1.93	63,600	2.03
27	47,800	1.41	52,600	1.57	57,000	1.71	60,800	1.84	64,300	1.96	67,300	2.07
28	50,100	1.42	55,200	1.58	59,800	1.73	64,000	1.86	67,700	1.99	71,000	2.10
29	52,300	1.43	57,700	1.59	62,700	1.74	67,100	1.88	71,200	2.01	74,800	2.13
30	54,600	1.44	60,200	1.61	65,500	1.76	70,300	1.90	74,600	2.03	78,500	2.15
31	56,800	1.45	62,800	1.62	68,400	1.77	73,500	1.92	78,100	2.05	82,300	2.18
32	59,100	1.46	65,400	1.63	71,300	1.79	76,700	1.93	81,600	2.07	86,100	2.20
33	61,300	1.47	68,000	1.64	74,200	1.80	79,800	1.95	85,100	2.09	89,800	2.22
34	63,600	1.47	70,500	1.65	77,000	1.81	83,000	1.96	88,600	2.11	93,600	2.24
35	65,800	1.48	73,100	1.66	79,900	1.82	86,200	1.98	92,000	2.12	97,400	2.26
36	68,100	1.49	75,700	1.66	82,800	1.83	89,400	1.99	95,600	2.14	101,000	2.28
37		...	78,300	1.67	85,700	1.84	92,600	2.00	99,100	2.16	105,000	2.30
38	...	...	80,900	1.68	88,500	1.85	95,800	2.01	103,000	2.17	109,000	2.32
39	...	...	83,400	1.69	91,500	1.86	99,000	2.02	106,000	2.18	113,000	2.33
40	...		86,000	1.69	94,300	1.87	102,000	2.04	110,000	2.19	116,000	2.35
41	...		88,600	1.70	97,300	1.88	105,000	2.05	113,000	2.21	120,000	2.36
42	...		91,100	1.71	100,000	1.88	109,000	2.05	117,000	2.22	124,000	2.37
43			...	...	...	...	112,000	2.06	120,000	2.23	128,000	2.39
44	...	...	...	...	...	...	115,000	2.07	124,000	2.24	132,000	2.40
45	...	...	...	...	...	...	118,000	2.08	127,000	2.25	135,000	2.41
46	...		...	...	...	...	121,000	2.09	131,000	2.26	139,000	2.42
47	...		...	...	...	...	125,000	2.10	134,000	2.27	143,000	2.43
48	...		...	...	...	...	128,000	2.10	138,000	2.28	147,000	2.44
49	...		...	...	...	...	...	...	...	...	151,000	2.45
50	...		...	...	...	...	...	...	...	...	155,000	2.46
51	...		...	...	...	...	...		...	...	159,000	2.47
52	...			...	...	...	...		...	...	163,000	2.48
53	...			...	...	...	...		...	...	166,000	2.49
54	...	...	...		...	...	...	...	...	...	170,000	2.50

From Thomas & Nichols' Reinforced Concrete Design Tables.
[Abridged and rearranged.]

TABLE 12

BEAMS WITH COMPRESSION REINFORCEMENT

(Moments of resistance of 1 sq. in. of compression steel)

$f_s = 16{,}000$ $n = 15$

(d' = depth of compression steel from top of beam)

Depth of beam, ins.	f_c							
	$d' = 2''$				$d' = 3''$			
	650	750	800	900	650	750	800	900
8	1,550	2,070	2,330	2,860	.....	....	....	....
9	2,190	2,830	3,150	3,780	......	....	....	....
10	2,860	3,610	3,980	4,730	1,100	1,670	1,960	2,530
11	3,550	4,410	4,840	5,690	1,700	2,380	2,720	3,390
12	4,240	5,220	5,700	6,680	2,320	3,110	3,500	4,290
13	4,950	6,040	6,580	7,670	2,960	3,860	4,310	5,200
14	5,670	6,870	7,470	8,670	3,620	4,630	5,130	6,140
15	6,390	7,700	8,360	9,670	4,290	5,410	5,970	7,090
16	7,110	8,540	9,250	10,680	4,980	6,210	6,820	8,060
17	7,840	9,380	10,160	11,700	5,670	7,010	7,690	9,030
18	8,570	10,230	11,060	12,720	6,370	7,830	8,550	10,010
19	9,310	11,080	11,970	13,740	7,070	8,640	9,430	11,000
20	10,040	11,930	12,880	14,770	7,780	9,470	10,310	12,000
21	10,780	12,790	13,790	15,800	8,500	10,300	11,200	13,000
22	11,520	13,650	14,710	16,800	9,220	11,130	12,090	14,000
23	12,270	14,500	15,600	17,900	9,940	11,970	12,980	15,000
24	13,010	15,400	16,500	18,900	10,670	12,810	13,880	16,000
25	13,760	16,200	17,500	19,900	11,400	13,650	14,790	17,000
26	14,500	17,100	18,400	21,000	12,120	14,510	15,700	18,100
27	15,300	17,900	19,300	22,000	12,860	15,400	16,600	19,100
28	16,000	18,800	20,200	23,000	13,600	16,200	17,500	20,100
29	16,700	19,700	21,100	24,100	14,330	17,100	18,400	21,100
30	17,500	20,600	22,100	25,100	15,100	17,900	19,300	22,200
31	18,200	21,400	23,000	26,100	15,800	18,800	20,200	23,200
32	19,000	22,300	23,900	27,200	16,500	19,600	21,200	24,200
33	19,800	23,100	24,800	28,200	17,300	20,500	22,100	25,200
34	20,500	24,000	25,800	29,300	18,000	21,300	23,000	26,300
35	21,200	24,900	26,700	30,300	18,800	22,200	23,900	27,300
36	22,000	25,700	27,600	31,400	19,500	23,000	24,800	28,300
37	22,800	26,600	28,500	32,400	20,300	23,900	25,700	29,400
38	23,500	27,500	29,500	33,400	21,000	24,800	26,600	30,400
39	24,300	28,400	30,400	34,500	21,700	25,600	27,600	31,400
40	25,000	29,200	31,300	35,500	22,500	26,500	28,500	32,500
41	25,800	30,100	32,300	36,600	23,300	27,400	29,400	33,500
42	26,500	31,000	33,200	37,600	24,000	28,200	30,300	34,600
43	27,300	31,800	34,100	38,700	24,700	29,100	31,300	35,600
44	28,000	32,700	35,000	39,700	25,500	30,000	32,200	36,600
45	28,800	33,600	36,000	40,800	26,200	30,800	33,100	37,700

From Thomas & Nichols' REINFORCED CONCRETE DESIGN TABLES.
[Abridged and rearranged.]

TABLE 13

SQUARE COLUMNS

(Total load in pounds on net area)

$f_c = 450$

Size of column in ins.	Area of column in sq. in.	½% of vertical steel			1% of vertical steel			1½% of vertical steel		
		Area of steel	Load		Area of steel	Load		Area of steel	Load	
			$n = 15$	$n = 12$		$n = 15$	$n = 12$		$n = 15$	$n = 12$
10	100	0 50	48,100	47,500	1.00	51,300	49,900	1.50	54,400	52,400
11	121	0.61	58,300	57,400	1.21	62,100	60,400	1.82	65,900	63,400
12	144	0.72	69,300	68,400	1 44	73,900	71,900	2.16	78,400	75,500
13	169	0.85	81,400	80,200	1.69	86,700	84,400	2 54	92,000	88,600
14	196	0.98	94,400	93,100	1.96	100,500	97,900	2.94	106,700	102,800
15	225	1.13	108,300	106,800	2 25	115,400	112,400	3 38	122,500	118,000
16	256	1.28	123,300	121,500	2.56	131,300	127,900	3.84	139,400	134,200
17	289	1.45	139,200	137,200	2 89	148,300	144,400	4 34	157,400	151,500
18	324	1.62	156,000	153,800	3.24	166,200	161,800	4 86	176,400	169,900
19	361	1.81	173,800	171,400	3.61	185,200	180,300	5 42	196,600	189,300
20	400	2.00	192,600	189,900	4.00	205,200	199,800	6.00	217,800	209,700
21	441	2.21	212,300	209,400	4.41	226,200	220,300	6.62	240,100	231,200
22	484	2 42	233,000	229,800	4 84	248,300	241,800	7.26	263,500	253,700
23	529	2.65	254,700	251,100	5 29	271,400	264,200	7.94	288,000	277,300
24	576	2.88	277,300	273,500	5.76	295,500	287,700	8.64	313,600	302,000
25	625	3.13	300,900	296,700	6.25	320,600	312,200	9.38	340,300	327,700
26	676	3.38	325,500	320,900	6.76	346,800	337,700	10.14	368,100	354,400
27	729	3.65	351,000	346,100	7.29	374,000	364,100	10.94	396,900	382,200
28	784	3.92	377,500	372,200	7 84	402,200	391,600	11.76	426,900	411,000
29	841	4.21	404,900	399,300	8.41	431,400	420,100	12.62	457,900	440,900
30	900	4.50	433,300	427,300	9.00	461,700	449,500	13.50	490,000	471,800
31	961	4.81	462,700	456,200	9.61	493,000	480,000	14.42	523,000	504,000
32	1,024	5.12	493,100	486,100	10.24	525,000	511,000	15.36	558,000	537,000
33	1,089	5.44	524,000	517,000	10.89	559,000	544,000	16.34	593,000	571,000
34	1,156	5.78	557,000	549,000	11.56	593,000	577,000	17.34	629,000	606,000
35	1,225	6.13	590,000	582,000	12.25	628,000	612,000	18.38	667,000	642,000
36	1,296	6.48	624,000	615,000	12.96	665,000	647,000	19.44	706,000	679,000
37	1,369	6.85	659,000	650,000	13.69	702,000	684,000	20.54	745,000	718,000
38	1,444	7.22	695,000	686,000	14.44	741,000	721,000	21.66	786,000	757,000
39	1,521	7.61	732,000	722,000	15.21	780,000	760,000	22 82	828,000	797,000
40	1,600	8.00	770,000	760,000	16.00	821,000	799,000	24.00	871,000	839,000
41	1,681	8.41	809,000	798,000	16.81	862,000	840,000	25.22	915,000	881,000
42	1,764	8.82	849,000	837,000	17.64	905,000	881,000	26.46	960,000	925,000
43	1,849	9.25	890,000	878,000	18.49	949,000	924,000	27.74	1,007,000	969,000
44	1,936	9.68	932,000	919,000	19.36	993,000	967,000	29.04	1,054,000	1,015,000
45	2,025	10.13	975,000	961,000	20.25	1,039 000	1,011,000	30.38	1,103,000	1,062,000
46	2,116	10.58	1,019,000	1,005,000	21.16	1,086,000	1,057,000	31.74	1,152,000	1,109,000

From Thomas & Nichols' REINFORCED CONCRETE DESIGN TABLES.
[Abridged.]

TABLE 14

$D = XL\sqrt{x}$

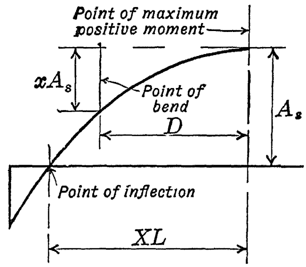


TABLE SHOWING WHERE BOTTOM (POSITIVE) RODS CAN BE BENT UP

Total No. of Rods in Bottom of Beam								x % Rods bent up	Maximum Distance of Point of Bend from Point of Maximum Positive Moment = D			
									Maximum Positive Moments			
12	10	8	6	5	4	3	2		$\frac{wL^2}{8}$ [a]	$\frac{wL^2}{12}$ [a]	$\frac{wL^2}{16}$ [a]	$\frac{wL^2}{10}$ [b]
...	1	...	...	...	...	...	...	10	0.16L	0.13L	0.11L	0.14L
...	...	1	...	...	...	...	...	12.5	0.18L	0.14L	0.13L	0.16L
2	...	...	1	...	...	...	...	16.7	0.20L	0.17L	0.14L	0.18L
...	2	...	...	1	...	...	...	20	0.22L	0.18L	0.16L	0.20L
3	...	2	...	...	1	...	...	25	0.25L	0.20L	0.18L	0.22L
...	3	...	...	...	...	...	...	30	0.27L	0.22L	0.19L	0.24L
4	...	...	2	...	...	1	...	33.3	0.29L	0.24L	0.20L	0.26L
...	...	3	...	...	...	...	...	37.5	0.31L	0.25L	0.22L	0.27L
...	4	...	...	2	...	...	...	40	0.32L	0.26L	0.22L	0.28L
6	5	4	3	...	2	...	1	50	0.35L	0.29L	0.25L	0.32L
...	...	...	...	3	...	...	...	60	0.39L	0.32L	0.27L	0.35L
8	...	...	...	...	...	2	...	66.7	0.41L	0.33L	0.29L	0.36L
Distance Max. Pos. Mom. to Inflection Point = XL									0.5L [a]	0.408L [a]	0.354L [a]	0.447L [b]

[a] End moments assumed equal. Maximum positive moment at center of beam.
[b] One end moment assumed 0. Maximum positive moment 0.447 L from zero moment end.

INFLECTION POINTS FOR VARIOUS NEGATIVE MOMENTS

Moment at Left Support	Distance Inflection Point from Left Support	Distance Inflection Point from Right Support	Moment at Right Support
$wL^2/12$	0.21L	0.21L	$wL^2/12$
$wL^2/16$	0.17L	0.24L	$wL^2/10$
0	0.00L	0.20L	$wL^2/10$
$wL^2/12$	0.22L	0.26L	$wL^2/10$

Note: In computing camber points it is safe and convenient to consider the negative part of the moment curve as a straight line.

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